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C - APPROACH OF ABS ALGORITHM TO SOLVE LINEAR FRACTIONAL PROGRAMMING PROBLEM BY CONVERTING IT INTO SINGLE LINEAR PROGRAMMING PROBLEM

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ABSTRACT

ABS algorithm is being used worldwide now a days to solve system of equations (either linear or non-linear) comprising large number of constraints and variables. In this paper, we proposed C-approach oriented ABS algorithm to solve linear fractional programming problem. First of all, we reduced the linear fractional programming problem to linear programming problem by the method given by Hasan, M.B. and Acharjee, S. After that, the solution of linear fractional programming problem can be obtained by the reduced linear programming problem provided that degeneracy has been treated properly. An illustration is given at the end to demonstrate and it has also been verified by graphical method, simplex method and the method given by Hasan, M.B. and Acharjee, S.

Keywords: ABS algorithm, linear fractional programming problem, linear programming problem, C- approach.

AMS Subject Classification: 65F30

INTRODUCTION

ABS algorithm has been introduced by Abaffy, Broyden and Spedicato to solve linear systems (determined or undetermined). Non-linear equations, integer equations, problems based on linear least squares, linear programming problems and optimization problems have also been solved by ABS algorithm.

ABS algorithm had been applied to solve simplex method and the dual simplex method by Feng, E. et al.[1]. A method had been developed to find solutions for a system of m linear integer inequalities in n variables where $m \leq n$, with full rank coefficient matrix by Hamid Esmaeili et al.[2].

They also applied this result to solve integer linear programming problems with $m \leq n$ inequalities. IABS-MPVT algorithm was developed by Emilio Spedicato et al.[3] to solve a system of linear equations and linear inequalities. Hamid Esmaeili et al.4 had been applied ABS algorithm for linear real systems. They applied ABS algorithm to solve full rank linear inequalities and linear programming problems where the number of inequalities is less than or equal to the number of variables. They obtained the both the optimality and unboundedness conditions in the context of ABS algorithm also. Emilio Spedicato et al.[5] developed ABS methods for continuous and integer linear equations and optimization problems.

The linear fractional programming problems have great importance in the field of non-linear programming. It is frequently encountered in business and economics. Linear fractional programming problem is very useful in production planning, financial planning, corporate planning, health care and hospital planning. Linear fractional programming deals with that class of mathematical programming problems in which the relations among the variables are linear; the constraint relations must be in linear form and the objective function to be optimized must be a ratio of two linear functions. Charnes-Cooper[6], Kantiswarup[7], Chadha[8-9], Jain et al.[10], Hasan M.B. and Acharjee, S.[11] and many more researchers gave different methods for solving linear fractional programming problems.

PRELIMINARIES

Let the fractional programming problem under consideration is of the form :

Max. $Z = \frac{cx+\alpha}{dx+\beta}$

Subject to $Ax = b$ ....(1)

and $x \geq 0$.

Constraints in fractional programming problem may include any of the sign ($\leq$, =, $\geq$). But by introducing slack and surplus variables, we can always convert them in strict equations. So, we have constraints as $A\,x = b$, where x, c and d are $n\times 1$column vector. A is an activity matrix of order $m\times n$ and b is a column vector of order $m\times 1$. α and β are some scalars. Further it is assumed that the constraint set $\{ x: A\,x = b; x \geq 0 \} = S$ is nonempty and bounded and $dx + \beta > 0$.

Here, we are solving the fractional programming problem by the given methods namely:

(1) Graphical method
(2) Simplex method
(3) Method given by Hasan, M.B. and Acharjee,S.
(4) ABS algorithm
(5) C-approach

The above methods are explaining one by one by taking an example. One can easily see that if the degeneracy is treated properly then the feasible solution given by ABS method becomes optimal solution.

METHOD

We can convert Linear Fractional Programming into a Linear Programming by the method given by Hasan, M.B. and Acharjee, S. assuming that $\beta \neq 0$.

We have $Z = \frac{cx+\alpha}{dx+\beta}$

Or $\quad Z = \frac{(cx+\alpha)\beta}{(dx+\beta)\beta} = \frac{cx\beta+\alpha\beta}{(dx+\beta)\beta}$

$$= \frac{cx\beta-dx\alpha+dx\alpha+\alpha\beta}{(dx+\beta)\beta}$$

$$= \frac{(c\beta-d\alpha)x+\alpha(dx+\beta)}{(dx+\beta)\beta}$$

$$= \frac{(c\beta-d\alpha)x}{\beta(dx+\beta)} + \frac{\alpha}{\beta}$$

$$= \frac{\left(c-d\frac{\alpha}{\beta}\right)x}{(dx+\beta)} + \frac{\alpha}{\beta}$$

$$= py+g$$

Where $\quad p = \left(c - d\ \frac{\alpha}{\beta}\right)$, $y = \frac{x}{dx+\beta}$ and $g = \frac{\alpha}{\beta}$

so that $\quad F(y) = py + g$

also, from the constraints

$Ax - b \leq 0$

$\frac{(Ax-b)}{(dx+\beta)} \leq 0$

$\frac{(Ax-b)\beta}{(dx+\beta)\beta} \leq 0$

$\frac{Ax\beta-\beta b}{(dx+\beta)\beta} \leq 0$

$\frac{Ax\beta+bdx-bdx-b\beta}{(dx+\beta)\beta} \leq 0$

$\frac{\beta\left(A+\frac{b}{\beta}d\right)x}{(dx+\beta)\beta} - \frac{b(dx+\beta)}{\beta(dx+\beta)} \leq 0$

$$\left(A+\frac{b}{\beta}d\right)\frac{x}{(dx+\beta)} \leq \frac{b}{\beta}$$

$$Gy \leq h$$

Where $\quad G = \left(A+\frac{b}{\beta}d\right)$ and $h = \frac{b}{\beta}$

From the above transformations, finally we get a Linear Programming Problem of the given Linear Fractional Programming Problem (1) as follows :

Max. $\quad F(y) = py + g$

Subject to $\quad Gy \leq h \quad$ ....(2)

And $\quad y \geq 0$

By solving this Linear Programming problem for y, we can recover the solution of original Linear Fractional Programming Problem.

ABS ALGORITHM

1. Let $x_1 \epsilon R^n$ be an arbitrary vector. Let $H_1 \epsilon R^{\mathrm{n},n}$ be an arbitrary nonsingular matrix.
2. Cycle for $i = 1 - - - n$

   (a) Let $z_i \epsilon R^n$, be a vector arbitrary save for the condition;
3. $z_i^T H_i a_i \neq 0$

   Compute search vector P_i:
4. $P_i = H_i^T z_i$

   (b) Compute step size α_i;
5. $\alpha_i = \frac{\alpha_i^T x_i - b_i}{P_i^T a_i}$

Which is well defined with regard to (3) to (4).

(c) Compute the new approximation of the solution using.

6. $X_{i+H} = X_i - \alpha_i P_i$

   If $i = n$ stop; x_{n+1} solve the system (1).

   (d) Let $w_i \epsilon R^n$ be a vector arbitrary for the condition;
7. $w_i^T H_i a_i = 1$

   and to update the matrix H_1:
8. $H_{i+1} = H_i - H_i a_i w_i^T H_i$

The three eligible parameters of the ABS algorithm are the matrix H_1 and the systems of vectors z_i and w_i. By making suitable choice of these parameters one can create a new formulation.

C-APPROACH ORIENTED ABS ALGORITHM

```
//Program to solve LPP Problems using ABS Method.

#include<stdio.h>
#include<conio.h>
#include<math.h>

//Prototype of Multiply()
void multiply(float a[][10],float b[][10],float c[][10],int rl, int c l, int c2);

//Prototype of Transpose()
void trans(float a[][10],float t[][10],intr,int c);
```

```
//Prototype of ColArr()
void Colarr(float a[][10],float col[][10],intx,intr,int c);

//Prototype of Identity()
void identity(float idt[][10],int size);

int main()
{       float
        A[10][10],b[10][10],x[10][10],H[10][10],s[10][10],a[10][10],z[10][10],p[10][10],HT
        [10][10],ZT[10][10],Y[10][10],X[10][10],P[10][10];

        float
        Q,T,p1[10][10],PT[10][10],PT1[10][10],PT2[10][10],AT[10][10],BU[10][10],AU[10][10],;

        int r l, c l,i,j,kl,rank;

        //clrscr();

        printf("\n\n\t\t\tBasic ABS Algorithm forLPP");

        printf("\n\nInput Section : ");

        //Input of Matrix A
        printf("\n\nEnter Size of Matrix A : ");

        printf("\n\n\t\t\ Row : ");
        scanf("%d",&r 1)

        printf("\n\n\t\t\ Cols : ");
        scanf("%d",&c 1)
rank=(rl<cl)?rl:cl;


        printf("\n\nEnter Elements of Matrix A : ");
        for(i=0;i<rl;i++)
        {

                for(i=0;j<cl;j++)
                {
                        printf("\n\n\tElement [ %d ] [ %d ] : ",i+l,j+l);
                        scanf("%f',&A[i][j]);
                }
        }

        Input of Matrix b
        printf("\n\nEnter Elements of Matrix b : ");
        j=0;
```

```
for(i=0;i<rl;i++)
{

	printf("\n\n\tElement [ %d ] [ %d ] : ",i+l,j+l);
	scanf("%f',&b[i][0]);

}

If(rl>cl)
{
	trans(A,AT,rl,cl);
	multiply(AT,b,BU,cl,rl,l);
	//2on linea BU to b for updation
	for(k=0;k<rank;k++)
	{
		b[k][0]=BU[k][0];
	}
}
else
{
	Trans(A,AU,rl,cl);
	//2on linea AU to A for updation
	for(k=0;k<cl;k++)
	{
		for(j=0;j<rl:j++)
		{
			A[k][j]=AU[k][j];
		}
	}
}


//Initialization of Matrix x to zero.
For(i=0,i<rl;i++)
{
	X[i][0]=0;
}

//Generating Identity Matrix
Identity(H,rl);

//Computing Vector p and s
I=o

printf("\n\nOutput Section : ");

while(i<rank)
{
```

```
        colarr(A,a,i+l,rl,cl);          //gained matrix a

        multiply(H,a,s,rl,rl,l);        //gained matrix s

        trans(H,HT,rl,rl);              //gained matrix HT

        //Assigning a to z
        for(k=0;k<rl;k++)
        {
                Z[k][0]=a[k][0];
        }

        multiply(HT,a,p,rl,rl,l);       //gained matrix p (search direction)

        trans(z,ZT,rl,l);               //gained transpose of z (matrix ZT)

        multiply(ZT,s,Y,l,rl,l);        //gained matrix Y

        multiply(ZT,x,X,l,rl,l);

        T=X[0][0]–[b[i][0];

        if(s[i][0]==0&&T==0)
        {
                Continue:
        }
        else if(s[i][0]==0&&T!=0)
        {


        Printf("\n\nSystem is Incompatible…");
}
else
{
        If(y[0][0]!=0
        {
                //Updating x
                multiply(ZT,x,X,l,rl,l);
                multiply(ZT,p,P,l,rl,l);

                Q=(X[0][0]–b[i][0]/P[0][0];
                for(k=0;k<rl;k++)
                {
                        pl[k][0]=Q*p[k][0];
                }

                for(k=0;k<rl;k++)
```

```
                    {
                              x[k][0]=pl[k][0];
                    }

                    //Updating H
                    trans(p,PT,rl,l);
                    multiply(p,PT,PT1,lrl,l,rl);
                    multiply(,PT,p,PT2,l, rl,l);

                    for(k=0;k<rl;k++)
                    {
                              for[1=0;l<rl;l++)
                              {
                                        PT1[k][1]=PT2[0][0];
                              }
                    }

                    for(k=0;k<rl;k++)
                    {
                              for[1=0;l<rl;l++)
                              {
                                        H[k][l]–=PTl[k][l];
                              }
                    }
          }
}
//printing X
Printf("\n\nValue of x [%d} : ",i+l);
For(k=0;k<rl;k++)




          {
                    printf("%f ",x[k][0];
          }
          i++;
}

printf("\n\n Value of x : ");
for(i=0;i<rl;i++)
{
                    printf("%f ",x[i][0];
}

getch();
return();
}
```

```
//Definition of Multiply()
Void multiply(float a[][10],floatb[][10],floatc[][10],int rl, int cl, int c2)
{
        inti,j,k;
        for(i=0;j<c2;j++)
        {
                for(j=0;j<c2;j++)
                {
                        c[i][j]=0;
                }
        }

        for(i=0;i<rl;i++)
        {
                for(j=0;j<c2;j++)
                {
                        for(k=0;k<cl;k++)
                        {

                }

        }

}



//Definition of Transpose ()
void trans(float a[][10],float t[][10],intr,int c)
{

        inti,j;
        for (i=0;i<c;i++)
        {
                For (j=0;j<r;j++)
                {
                        t[i][j]=a[j][i];
                }
                printf("\n");
        }
}

//Definition of ColArr()
void colarr(float a[][10],float col[][10],intx,intr,int c)
{

        inti,j;
        for(i=0;i<r;i++)
        {
                col[i][0]=a[i][j];
        }
```

```
			printf("\n");
		}
}


//Definition of Identity()
void identity(float idt[][10],int size)
{

		inti,j;
		for(i=0;size;i++)
		{
			for(j=0;j<size;j++)
			{
				if(i==j)
				{
					idt[i][j]=l;
				}
				else
				{
					idt[i][j]=0;
				}
			}
		}
}
```

NUMERICAL EXAMPLE

Consider the Linear Fractional Programming Problem

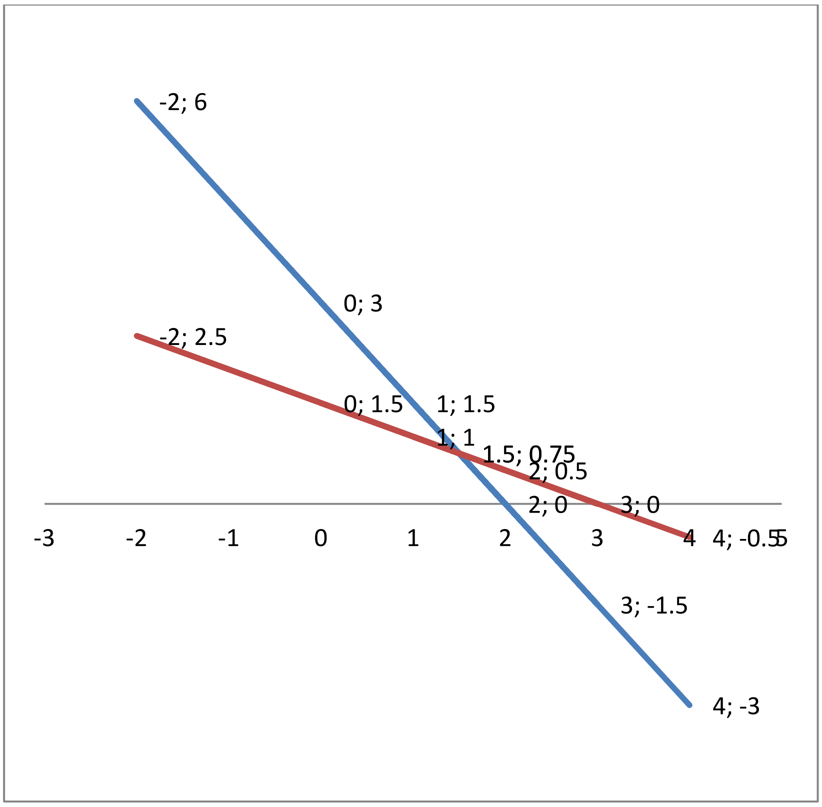
Maximize $Z = \frac{6x+5y}{2x+7}$

Subject to $x + 2y \leq 3$

$3x + 2y \leq 6$ ......(3)

and $x, y \geq 0$

Graphical Solution:



From the graphical analysis of the above problem ,we get three points such as (0,0); (2,0) and (1.5,0.75) and (0, 1.5).

Out of these three points the value of the objective function is maximum at (1.5,0.75) and its value is (51/40).

Ordinary Simplex Method for FPP:

Max. $Z = \frac{6x+5y}{2x+7}$

Subject to $x + 2y \leq 3$

$3x + 2y \leq 6$

$x, y \geq 0$

		C_j	-6	-5	0	0			
		d_j	2	0	0	0			
d_B	C_B	x_B	x	y	s_1	s_2	b	θ	
		s_1	1	2	1	0	3	3	$\rightarrow$out
		s_2	3	2	0	1	6	2	

$$
\begin{aligned}
Z^{(1)} &= C_B X_B + \alpha \\
&= -6x - 5y + 0 \\
&= -6(0) - 5(0) + 0 \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
Z^{(2)} &= d_B X_B + \beta \\
&= -2x + 7 = 0 + 7 \\
&= 7
\end{aligned}
$$

$$\therefore Z \;= \frac{Z^{(1)}}{Z^{(2)}} = \frac{0}{7} = 0$$

$C_j - Z_j^{(1)}$	-6	-5	0	0
$d_j - Z_j^{(2)}$	2	0	0	0
$\Delta_j = Z^2\left(C_j - Z_j^{(1)}\right) - Z^{(1)}\left(d_j - Z_j^{(2)}\right)$	-42 ↑ in	-35	0	0

		C_j	-6	-5	0	0			
		d_j	2	0	0	0			
d_B	C_B	x_B	x	y	s_1	s_2	b	θ	
2	-6	x_1	1	2	1	0	3	3/2	
0	0	s_2	0	-4	-3	1	-3	3/4	→out

$$
\begin{aligned}
Z^{(1)} &= -6x - 5y + 0 \;\; [x = 3, y = 0, \alpha = 0] \\
&= -6(3) \\
&= -18 \\
Z^{(2)} &= 2x + 7 \;\; [x = 3, y = 0, \beta = 7] \\
&= 2(3) + 7 \\
&= 13 \\
Z &= \frac{Z^{(1)}}{Z^{(2)}} = \frac{-18}{13}
\end{aligned}
$$

$C_j - Z_j^{(1)}$	0	7	6	0
$d_j - Z_j^{(2)}$	0	-4	-2	0
$\Delta_j = Z^2\left(C_j - Z_j^{(1)}\right) - Z^{(1)}\left(d_j - Z_j^{(2)}\right)$	0 ↑ in	19	42	0

		C_j	-6	-5	0	0		
		d_j	2	0	0	0		
d_B	C_B	x_B	x	y	s_1	s_2	b	θ
2	-6	x_1	1	0	$-1/2$	$1/2$	$3/2$	
0	-5	x_2	0	1	$3/4$	$-1/4$	$3/4$	

$$Z^{(1)} = -6x - 5y + \alpha \;\; [x = 3/2, y = 3/4, \alpha = 0]$$
$$= -6\left(\frac{3}{2}\right) - 5\left(\frac{3}{4}\right) = -51/4$$
$$Z^{(2)} = 2x + \beta \;\; [x = 3/2, y = 3/4, \beta = 7]$$
$$= 2(3/2) + 7$$

$$Z = \frac{Z^{(1)}}{Z^{(2)}} = \frac{\left(\frac{-51}{4}\right)}{10} = -\frac{-51}{40}$$

$C_j - Z_j^{(1)}$ 0 0 $-\frac{3}{4}$ $-7/4$

$d_j - Z_j^{(2)}$ 0 0 1 -1

$\Delta_j =$ 0 0 $\frac{21}{4}$ $-121/4$
$Z^2\left(C_j - Z_j^{(1)}\right)$
$-Z^{(1)}\left(d_j - Z_j^{(2)}\right)$

$x = 3/2, \qquad y = 3/4$

$$Max. Z = \frac{6x + 5y}{2x + 7}$$

$$= \frac{51}{40}$$

According to the method of Hasan, M.B. and Acharjee, S. the problem takes the form

Max. $F(y) = 6y_1 + 5y_2$

s.t. $13\, y_1 + 14\, y_2 \leq 3$

$33\, y_1 + 14\, y_2 \leq 6$

and $y_1, y_2 \geq 0$

optimal table for the above LPP is as follows :

		C_j	6	5	0	0
C_B	X_B	b	y_1	y_2	S_1	S_2
5	y_2	$\frac{3}{40}$	0	1	$\frac{33}{280}$	$-\frac{13}{280}$
6	y_1	$\frac{33}{220}$	1	0	$-\frac{1}{20}$	$\frac{1}{20}$
		(Z_j-C_j)	0	0	$\frac{81}{280}$	$\frac{19}{280}$

The optimal solution of LPP is

$y_1 = \frac{33}{220}$ and $y_2 = \frac{3}{40}$

Now to get the optimal solution of LFPP, we use the fact

$(x,y) = \frac{(y1\ y2)\beta}{1-d(y1\ y2)}$

$= (\frac{3}{2} \frac{3}{4})$

and $Z = \frac{51}{40}$

Solution by ABS algorithm

Max. $Z = \frac{6x+5y}{2x+7}$

Subject to $x + 2y \leq 3$

$3x + 2y \leq 6$

$x, y \geq 0$

now $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} A^T = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 3 \\ 6 \end{bmatrix} \mathrm{H} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Calculate the first decision parameter

$$Z_1^T H_1 a_1 = \begin{bmatrix}1 & 2\end{bmatrix} \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \begin{bmatrix}1\\2\end{bmatrix}$$

$$= \begin{bmatrix}1 & 2\end{bmatrix} \begin{bmatrix}1\\2\end{bmatrix} = 1 + 4 = 5 \neq 0$$

∴ **Then compute the search vector**

$$P_1 = H_1^T Z_1 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \begin{bmatrix}1\\2\end{bmatrix}$$

$$= \begin{bmatrix}1\\2\end{bmatrix}$$

Then compute the step size

$$x_2 = x_1 - \left[\frac{a_1^T x_1 - b_1}{a_1^T P_1}\right] P_1$$

$$= \begin{bmatrix}0\\0\end{bmatrix} - \left[\frac{\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} - 3}{\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}}\right] \begin{bmatrix}1\\2\end{bmatrix}$$

$$= \begin{bmatrix}0\\0\end{bmatrix} - \left[\frac{0-3}{5}\right] \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}3/5\\6/5\end{bmatrix}$$

Now for $i = 2$ solve the system again update H

i.e.

$$H_2 = H_1 - \left[\frac{P_1 P_1^T}{P_1^T P_1}\right]$$

$$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}1/5 & 2/5\\2/5 & 4/5\end{bmatrix}$$

$$= \begin{bmatrix}4/5 & -2/5\\-2/5 & 1/5\end{bmatrix}$$

Compute the next search vector

$$P_2 = H_2^T Z_2 = \begin{bmatrix}4/5 & -2/5\\-2/5 & 1/5\end{bmatrix} \begin{bmatrix}3\\2\end{bmatrix}$$

$$= \begin{bmatrix} 8/5 \\ -4/5 \end{bmatrix}$$

Compute the next step size

$$x_3 = x_2 - \left[\frac{a_2^T x_2 - b_2}{a_2^T P_2}\right] P_2$$

$$= \begin{bmatrix} 3/5 \\ 6/5 \end{bmatrix} - \left[\frac{\begin{bmatrix} 3 & 2 \end{bmatrix}\begin{bmatrix} 3/5 \\ 6/5 \end{bmatrix} - 6}{\begin{bmatrix} 3 & 2 \end{bmatrix}\begin{bmatrix} 8/5 \\ -4/5 \end{bmatrix}}\right] \begin{bmatrix} 8/5 \\ -4/5 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 3/4 \end{bmatrix}$$

After final iteration

$$x = 1.5$$

$$y = 0.75$$

and

$$Maximum Z = \frac{6x + 5y}{2x + 7}$$

$$= \frac{6\times 1.5 + 5\times 0.75}{2\times 1.5 + 7}$$

$$= 1.275 = 51/40$$

SOLUTION BY ABS THROUGH C

Input Section :

EnterSize of Matrix A:

Rows	:	2
Columns	:	2

Enter the elements of Matrix A

Element [1] [1]	:	1
Element [1] [2]	:	2
Element [2] [1]	:	3
Element [2] [2]	:	2

Enter the elements of Matrix b

Element [1] [1]	:	3
Element [2] [1]	:	6

Output Section:

Value of x [1]	:	0.6	1.2
Value of x [2]	:	1.5	0.75
Value of x	:	1.5	0.75

CONCLUSION

An example illustrates the whole procedure to solve a fractional programming problem by ABS algorithm with C- approach. The solution so obtained can be verified with graphical method, traditional simplex method and the method given by Hasan, M.B. and Acharjee, S. The feasible solution given by ABS method becomes an optimal solution provided that degeneracy has been treated properly.

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RELIABILITY ANALYSIS OF TWO UNIT STANDBY SYSTEM FOR HIGH PRESSURE DIE CASTING MACHINE

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ABSTRACT

The present paper is an attempt to find out the reliability of two die casting machine case, initially the primary unit is in working state and the second unit is at cold standby. This paper deals with a two unit redundant system in which one unit is operative and the other is cold standby, i.e. the standby unit is used to replace the operative failed unit instantaneously. Before the repair of the operative failed unit, it is sent for fault detection which takes a random amount of time. After the repair, the unit goes into inspection for deciding whether the repair is perfect or not. If the repair is found to be imperfect the unit is sent for post repair. The several reliability characteristics of interest such as mean sojourn time, MTSF obtained using Markov processes and regenerative point technique. The graphical study tells about reliability and profitability of the system.

Keyword: MTSF, Markov process, regenerative state, reliability, standby system

1. INTRODUCTION

Die casting machine is a mechanical device used for assembling of parts or some specific work in industry. It works automatically without any rest and more efficiently as compared to any other means of working. These machines are used in most of the industry applications like car part assembling, bicycle part assembling etc. Nowadays, die casting machines works for replacement of a large group of people, the high pressure die casting machine works very fast in comparison of people's work. Also the die casting machine is less costly in comparison of work done cost by people. The work will not suffer just like human problems. The die casting machines are used more frequently in industries so there a is need of analysis of robotic machine to improve the reliability of the system. As the system may be in failed state due to some problem in the machine, one solution is to use the standby unit. In this paper we explain this concept in detail.

In fact a large number of researchers in the field of reliability modelling including Nakagawa and Osaki (1975), Goel and Agnihotri (1992), Mokaddis and Labib (1997), Tuteja (2001), Sharma and Taneja (2011), Kumar and Bhatia (2011), Kumar and Rani (2013), V. Kumar and P. Bhatia with S. Ahmed (2014), etc. analyzed the one/ two unit redundant systems. Kumar and Vashistha (2001) explained the two unit redundant system with degradation and replacement of the faulty. Kumar and Bhatia (2011) discussed the behaviour of the single unit centrifuge system considering the concepts of inspections, halt of system, degradation, minor/major faults, neglected faults, online/offline maintenances, repairs of the faults. Kumar and Rani (2013) explained the cost benefit analysis for a redundant system. V. Kumar and P. Bhatia with S. Ahmed (2014) explained in very detail the profit analysis for a two unit standby centrifuge system having a single repairman. Jain (2014) explained the different failures in a repairable redundant system.

None of the researchers have analyzed so far taking real data for such die casting two-unit cold standby system with occurrence of various faults. To fill up this gap, the paper explored reliability analysis of a stochastic model for two-unit die casting system with failure, repair etc. and measures the affects in form of MTSF.

2. MODEL DESCRIPTION:

We are taking two die casting machines case, initially the primary unit is in working state and the second unit is at cold standby. There are some states which are called up states and some down states. The up states are those states in which at least one die casting machine is in operative mode either primary unit or secondary unit. The down states are states in which both machines are not in operative mode. The chapter deals with a two unit redundant system in which one unit is operative and the other is cold standby, i.e. the standby unit is used to replace the operative failed unit instantaneously. Before the repair of the operative failed unit, it is sent for fault detection which takes a random amount of time. After the repair, the unit goes into inspection for checking whether the repair is perfect or not. If the repair is found to be imperfect the unit is sent for post repair. The several reliability characteristics of interest such as mean sojourn time, MTSF have been calculated using Semi-Markov processes and regenerative point technique. The graphical study presents about reliability and profitability of the system.

2.1. Assumptions:

The main assumptions related to the system are:

a) The system will work as new after repairing.
b) The switching of machines is very fast as if system is not in stop state.
c) The repair team is totally watching the system carefully and will listen instantaneously.
d) The failure time distribution is exponential.
e) We consider stochastic modelling, so all possible random variables are considered, so all possible states of the system are considered.
f) The error in the machine will be self announcing.

2.2. Notations:

X_0	:	Priority unit is in operative
Y_0	:	Non-Priority Unit (cold standby unit)is operative
Y_{CS}	:	Non-Priority Unit (Earlier cold standby unit) is in standby mode
X_{FR}	:	Priority unit is failed and sent for repair
Y_{FR}	:	Non- Priority Unit (Earlier cold standby) is in failed state and under repair.

X_{FI}	:	Priority unit is failed and it is under inspection
Y_{FI}	:	Non Priority Unit is failed and it is under inspection
$H_{1(.)}$	:	c d f of repair time of priority unit at failed state
$H_{2(.)}$	:	c d f of time to repair of the Second unit at failed state
μ	:	constant rate for checking repaired unit to working unit
α_1	:	Failure time distribution for priority unit
α_2	:	Failure time distribution for non priority unit
p	:	Probability of repaired unit in working unit
~	:	Laplace Stieltjes Transform Symbol

2.3 The States of the system

The different states of the system having all the possibilities main unit is either in operative state, failed state, repairing state, inspection state, and similarly for second cold standby, the possibilities are operative state, failed state, repairing state, inspection state, waiting state are taken into account. Below are the states of the system as:

$S_0 = [X_0, Y_{CS}]$; $S_1 = [X_{FI}, Y_0]$; $S_2 = [X_{FR}, Y_0]$; $S_3 = [X_{FR}, Y_{FW}]$

$S_4 = [X_0, Y_{FI}]$; $S_5 = [A_0, Y_{FR}]$

3. THE MODEL

Here all the states during the system are as:
S_0, S_1, S_2, S_4, S_5 are the states when system is in up State. And S_3 is the state when system is in down State.
The Transition diagram is shown below:

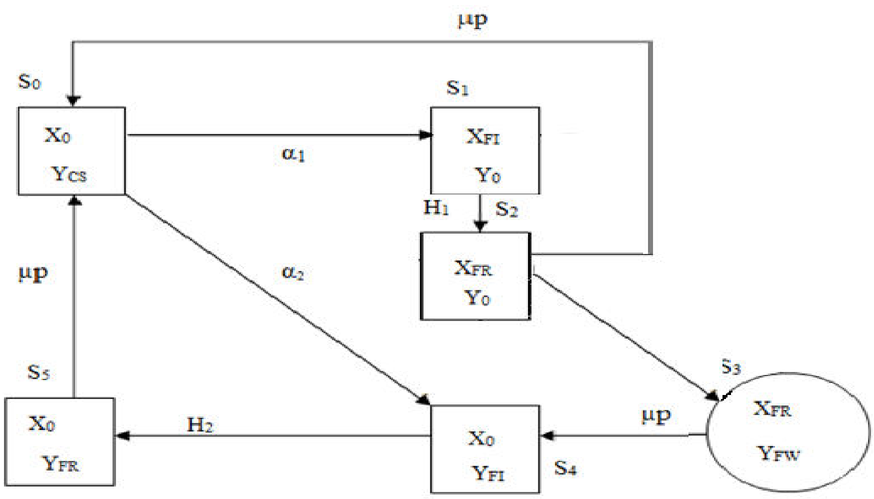

Fig 3.1: Transition of Different States for Two Units Cold Standby System

Up State **Failed state**

4. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Here$Q_{ij}(t)$ denotes the cdf (cumulative distribution function) of transition time from state Si to Sj in 0 to t .To determine the transition probabilities of states. LetT_0, T_1, T_2 ,....denotes the regenerative epochs. Then $\{ Xn, Tn\}$ constitute a space E, set of regenerative states and $Qij\ (t) = P[Xn + 1 = j, Tn + 1 - Tn \le t\ /Xn = i]$ is the semi Markov over E .The various transition probabilities are:

$Q_{01}(t) = \alpha_1\int_0^t e^{-(\alpha_1+\alpha_2)t}dt$ $\quad Q_{04}(t) = \alpha_2\int_0^t e^{-(\alpha_1+\alpha_2)t}dt$

$Q_{12}(t) = \int_0^t dH_1(t)e^{-\alpha_2 t}dt$ $\quad Q_{20}(t) = \mu p\int_0^t e^{-(\mu+\alpha_2)t}dt$

$Q_{23}(t) = \alpha_2\int_0^t e^{-(\alpha_1+\alpha_2)t}dt$ $\quad Q_{34}(t) = \mu p\int_0^t e^{-\mu t}dt$

$Q_{45}(t) = \int_0^t dH_2(t)e^{-\alpha_1 t}dt$ $\quad Q_{50}(t) = \mu p\int_0^t e^{-(\mu+\alpha_1)t}dt$

In order to calculate the c d f of the system, taking limit as t tends to infinity the probabilities of the transitions are found simply by calculations which are as below:

$P_{01} = \alpha_1/(\alpha_1+\alpha_2)$ $\quad P_{04} = \alpha_2/(\alpha_1+\alpha_2)$ $\quad P_{12} = \widetilde{H_1}(\alpha_2)$

$P_{20} = \mu p/\mu+\alpha_2$ $\quad P_{23} = \alpha_2/(\alpha_1+\alpha_2)$ $\quad P_{34} = p$

$P_{45} = \widetilde{H_2}(\alpha_1)$ $\quad P_{50} = \mu p/\mu+\alpha_1$

From the above equations, it is found that:

$P_{01}+P_{04}=1$ $\quad P_{12}=1$ $\quad P_{20}+P_{23}=1$

$P_{34}=1$ $\quad P_{45}=1$ $\quad P_{50}=1$

To calculate the sojourns time (the time that the system will spend in a particular state and then may transfer to another state) we proceed as below. The sojourns times for states S_0 to S_5 are Ψ_1, Ψ_2, Ψ_3, Ψ_4 and $\Psi_5$and are as:

$\Psi_0=1/(\alpha_1+\alpha_2)$ $\quad \Psi_1=(1-\widetilde{H}(\alpha_2))/\alpha_2$ $\quad \Psi_2=1/(\mu+\alpha_2)$

$\Psi_3=1/\mu$ $\quad \Psi_4=(1-\widetilde{H_2}(\alpha_1))/\alpha_1$ $\quad \Psi_5=1/(\mu+\alpha_1)$

Representing m_{ij} as the mean time elapsed in regenerative state by the system, from state T_i to T_j can be expressed in mathematical form as:

$\sum_j m_{ij} = \Psi_i$

Thus

$m_{01}+m_{04}=\Psi_0$ $\quad m_{12}=\Psi_1$ $\quad m_{20}+m_{23}=\Psi_2$

$m_{34}=\Psi_3$ $\quad m_{45}=\Psi_4$ $\quad m_{50}=\Psi_5$

5. THE MEAN TIME TO SYSTEM FAILURE (MTSF) AND OTHER PARAMETERS FOR THE SYSTEM

MTSF is the mean time between the failures of the system and it is also the time elapsed between the failed states of the system.

Mean time to system failure can be calculated from the transition probability and sojourn time as:

$MTSF = (\Psi_0+\Psi_1P_{01}+\Psi_2P_{01}P_{12}+\Psi_3P_{01}P_{12}P_{23})/(1-P_{01}P_{12}(P_{20}+P_{23}P_{34}P_{45}P_{50})$

Expected up time $A_0= N_1/D_1$
Downtime $A_{01}=N_2/D_1$
Busy time for inspection $B_I=N_3/D_1$
Busy time for repair time $B_R=N_4/D_1$
Busy time for Replacement $B_{RP}=N_5/D_1$

Where

$N_1= \Psi_0+P_{01}\Psi_1$
$N_2=P_{04}\Psi_4$
$N_3= (P_{04}+P_{01}P_{12}P_{23}P_{34}P_{45})\Psi_5+P_{01}P_{14}\Psi_4$
$N_4= P_{04}\Psi_4+(P_{04}+P_{01}P_{12}P_{23}P_{34}P_{45})P_{34}\Psi_4$
$N_5= P_{01}P_{12}P_{23}P_{34}\Psi_4+P_{04}P_{45}\Psi_5$
$D_1= \Psi_0+P_{01}(\Psi_1+P_{12}\Psi_2)+P_{04}(\Psi_4+P_{20}\Psi_0+P_{34}\Psi_4)$

6. The profit Analysis

For profit analysis of the system we should take care of all the factors in mind. After considering all the factors in mind, the profit will be:

Profit $P = C_0A_0-C_1A_{01}-C_2B_i-C_3B_r-C_4B_{rp}-C$
C_0= Revenue per unit for up state of the system
C_1= loss when system is down
C_2= Cost per unit for Inspection
C_3= Cost per unit for repair
C_4= Cost per unit for replacement
C= Cost of installation

The reliability of the system will be increased if the system will remain more in upstate and less in down state. Also less will be the error in system , the system will remain more reliable. In case, the system remains in down state, the repairman will remain busy, which will affect the reliability/availability of the system.

7. GRAPHICAL AND TABULAR REPRESENTATION

Solving the equations by taking repair rate λ_1 and λ_2; the cumulative distribution function can be solved as:

$M_i(t)=\lambda_i e^{-\lambda_i(t)}$ for i=1,2,…

So putting values of parameters, we get

$$\text{MTSF}=\frac{\frac{1}{\alpha_1+\alpha_2}\left(1+\frac{\alpha_1}{(\alpha_2+\lambda_1)}+\frac{\alpha_1\lambda_1}{(\alpha_2+\lambda_1)(\mu+\alpha_2)}+\frac{\alpha_1\alpha_2\lambda 1}{\mu(\alpha_2+\lambda 1)\,(\alpha_1+\alpha_2)}+\frac{\alpha_2}{\alpha_1+\lambda_2}\right)}{1-\frac{\alpha_1\lambda_1\mu p}{(\alpha_2+\lambda_1)(\alpha_1+\alpha_2)(\mu+\alpha_2)}-\frac{\alpha_2\lambda_2\mu p}{(\alpha_1+\lambda 2)(\alpha_1+\alpha_2)(\mu+\alpha_1)}}$$

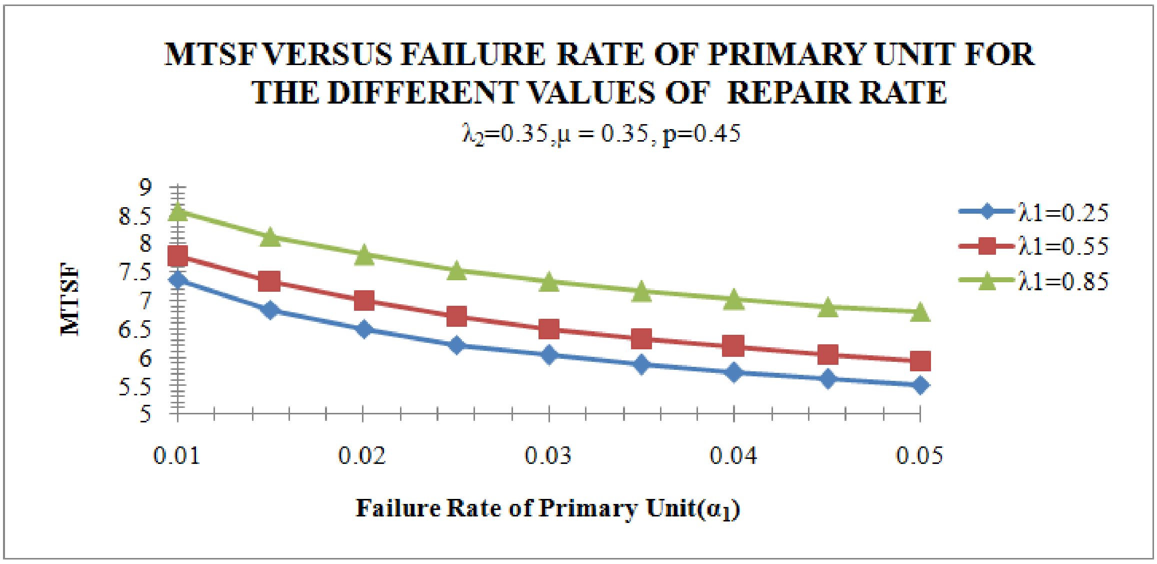

Figure 3.2: MTSF versus Failure rate of Primary Unit (For λ_1)

In the graph Fig. 3.2 MTSF is plotted with failure rate of primary unit and the three plots in graph are corresponding to the different values of repair rate of primary unit λ_1. Here ,The values of MTSF are corresponding to values of failure rate α_1(0.3 to 1), the value of λ_1 is taken as 0.25, 0.55, 0.85. And the other parameters are λ_2=0.35, α_2 = 0.25, μ =0.35, p=0.45.

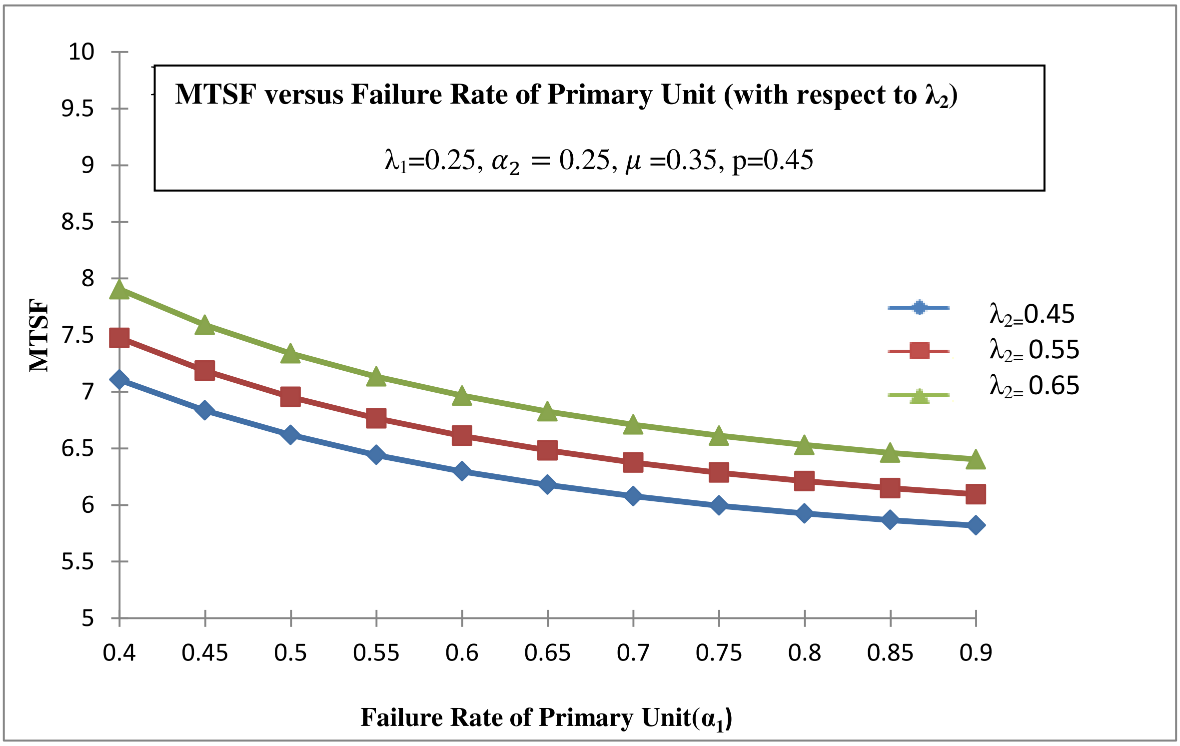

Figure 3.3: MTSF versus Failure Rate of Primary Unit (with respect to λ_2)

In the graph Fig. 3.3 MTSF is plotted with failure rate of primary unit and the three plots in graph are corresponding to the different values of repair rate of secondary unit λ_2. Here the values of MTSF are corresponding to values of failure rate α_1(0.3 to 1), the value of λ_2 is taken as 0.45, 0.55, 0.65. And the other parameters are λ_1=0.25, α_2 = 0.25, μ =0.35, p=0.45

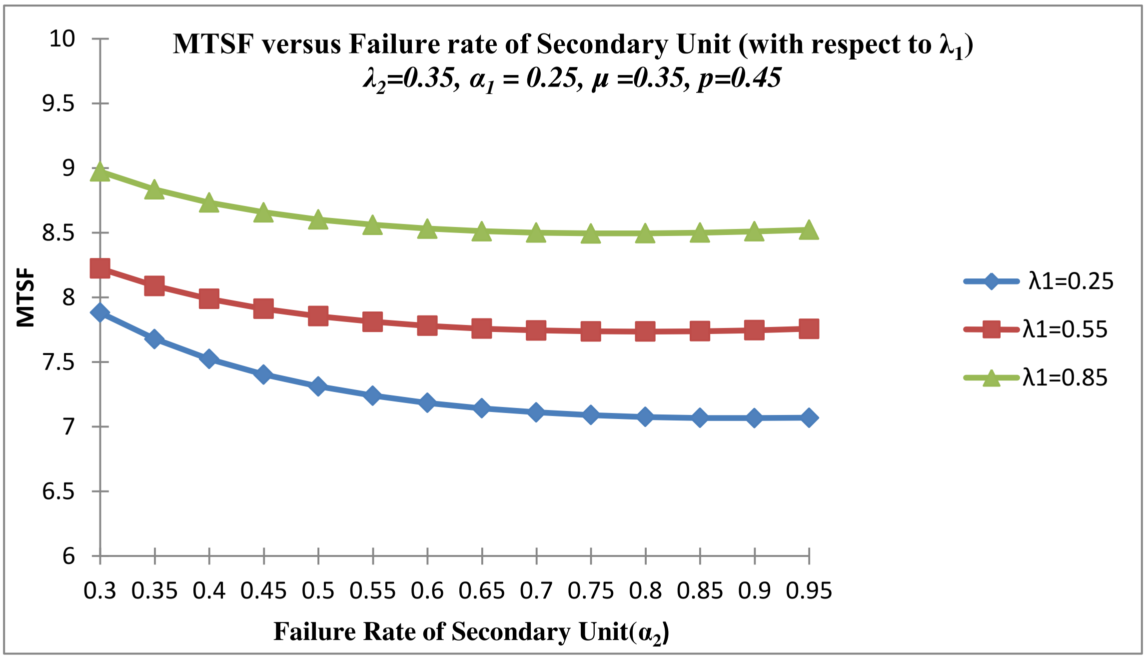

Figure 3.4: MTSF versus Failure rate of Secondary Unit (with respect to λ_1)

In the graph Fig. 3.4 MTSF is plotted with failure rate of secondary unit and the three plots in graph are corresponding to the different values of repair rate of secondary unit λ_2. Here the values of MTSF are corresponding to values of failure rate α_2(0.3 to 1), the value of λ_1 is taken as 0.25, 0.55, 0.85. And the other parameters are λ_2=0.35, α_1 = 0.25, μ =0.35, p=0.45.

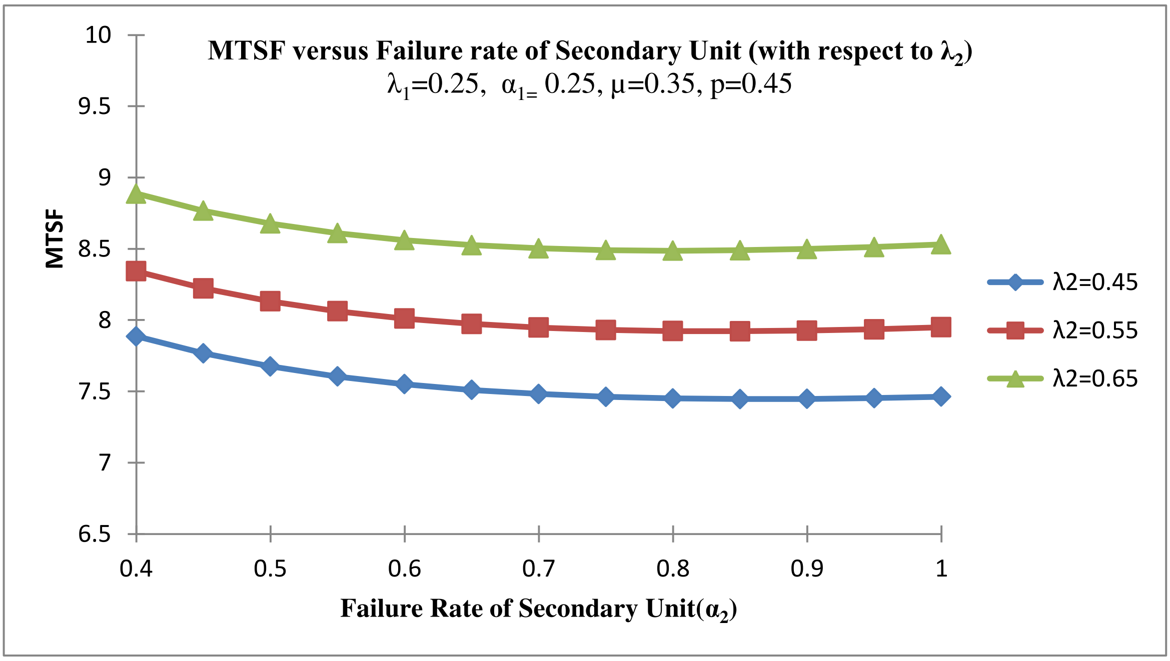

Figure 3.5: MTSF versus Failure rate of Secondary Unit (with respect to λ_2)

In the graph Fig. 3.5 MTSF is plotted with failure rate of secondary unit and the three plots in graph are corresponding to the different values of repair rate of secondary unit λ_2. Here the values of MTSF are corresponding to values of failure rate α_2(0.3 to 1), the value of λ_2 is taken as 0.45, 0.55, 0.65. And the other parameters are λ_1=0.25, $\alpha_1 = 0.25$, μ =0.35, p=0.45.

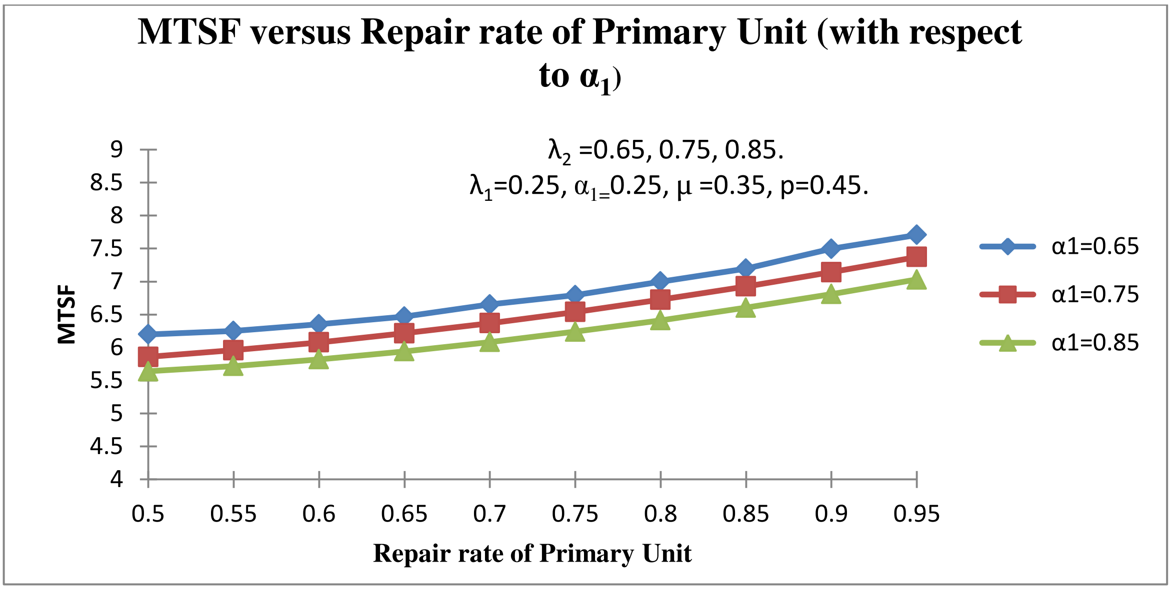

Fig. 3.6

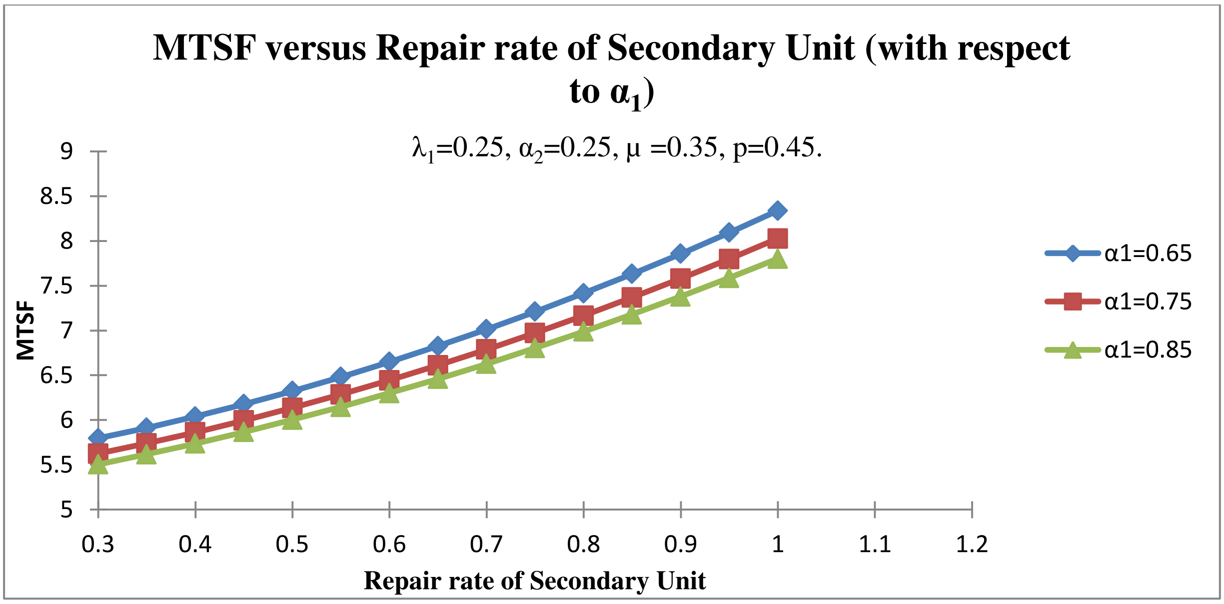

Figure 3.7: MTSF versus Repair rate of Secondary Unit (with respect to α_1)

In the graph Fig. 3.6 & 3.7 MTSF is plotted with repair rate of primary & secondary unit and the three plots in graph are corresponding to the different values of failure rate of primary unit α_1. Here the values of MTSF are corresponding to values of failure rate λ_2 (0.3 to 1), the value of α_1 is taken as 0.65, 0.75, 0.85. And the other parameters are λ_1=0.25, α_2=0.25, μ =0.35, p=0.45.

CONCLUSION:

MTSF is plotted with failure rate of primary unit, with increase in failure rate, from the observation and graph it is clear, MTSF is decreased, reliability of the system is decreased with increase in failure rate of primary unit.

MTSF is plotted with failure rate of secondary unit, with increase in failure rate MTSF is decreased, hence reliability of the system is decreased with increase in failure rate of secondary unit. When repair rate of primary/secondary unit is increased, then the MTSF is increased, hence, system is more reliable with more repair rate of primary/secondary unit.

MTSF is plotted with repair rate of primary unit, with increase in repair rate, MTSF is increased, hence reliability of the system is increased with increase in repair rate of primary unit.

MTSF is plotted with repair rate of secondary unit, with increase in repair rate, MTSF is increased, hence reliability of the system is increased with increase in repair rate of primary unit. When failure rate of primary/secondary unit is increased, then the MTSF is decreased, so system is less reliable with more failure rate of primary/secondary unit.

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ON DESIGNING A STOCHASTIC QUEUE MODEL FOR PASSPORT OFFICE SYSTEM

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1. ABSTRACT

During last two decades, the research analysts are making efforts in designing such queue models which are applicable for real world scenario. This study deals to frame a queue model which arises when people are being asked for issuing the passport to a person against his/her application getting through online or directly to the service counter. The applicant has to pass in phases through many service channels arranged in semi series in order to get clearance for the delivery of passport. In such situations the arrival pattern as well as the service pattern of customers are time independent i.e., the steady state is reached. The service is supposed to be completed only when the concerned applicant pass through all the phases and depart from the system for getting delivery of passport at home. A stochastic model is designed assuming arrival and service pattern both follow Poisson probability distribution. The designed model can also be applied in administrative setup, manufacturing concerns and other public life.

Key Words: Stochastic process, steady state, Poisson distribution, Phase service, Tandem queue etc.

2. INTRODUCTION

To stand in a line, for several hours in order to submit a passport application can be supposed as a physical punishment for an applicant. It is similar to going online in order to get an appointment with the office concern or the authority at a passport office can be imagined a torture through technology. This type of queue model depends on the nature of queue affected. Queue models help operation manager in making decisions which balance capacity cost via waiting time cost.

A number of theoretical models have been made to cover a lot of queuing situations. Simulation models avoid some of the pit falls of mathematical models.

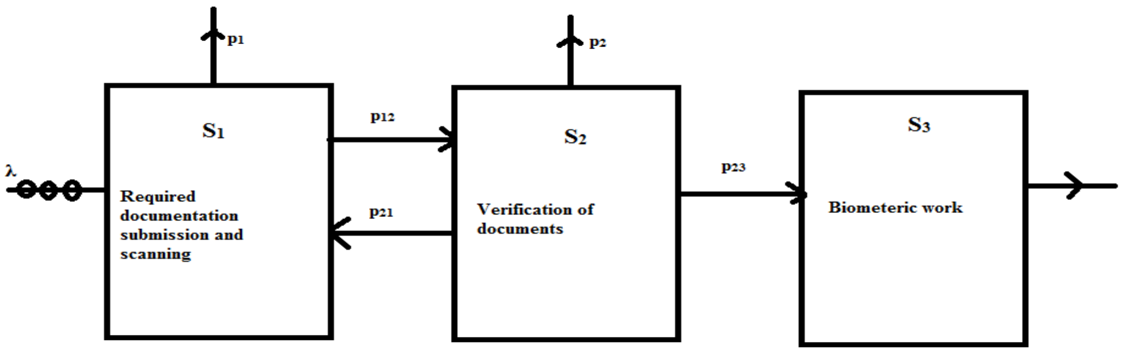
Jackson R.R.P (1954) first considered queues in series as a model in queuing system and studied the model in which services are made in phases. For deploying different strategies as optimizing service outlets structuring the service priorities and controlling the distribution of services and improving the physical design of distribution layout, Johnston Clark (2008) offers advice for delay queue applying his frame work to manage the resources service demands output etc. Recently Singh T.P etal (2011, 12) made the stochastic analysis of feedback queue model which was centrally linked to common channel and some generalized feedback queue system. Kumar S. &Taneja G.(2017) discussed a feedback queue model with provision of service by one or more out of three server . Recently, Maggu A. (2017) discussed passport office service system and made an attempt to frame the differential difference equations but failed to find the solution of the stated model but the author could not produce any result.

In the present study, we design a stochastic queue model for the passport public service system on the basis of work done by T.P. Singh & Arti (2012). We assume the arrival and service pattern both follow Poisson probability distribution and calling population as infinite, it is because the number of arrivals in any time interval is a Poisson random variable with a denumerable infinite sample space. The steady state analysis of the model has been made with the help of differential difference equations and finally the result has been summarized through tables and graphs. The results are well fitted in the current real world scenario.

3. MODEL DESCRIPTION

While issuing the passport to a person practically we find that the passport seva Kendra(PSK) consists of mainly three service channels arranged in Series(tandem) where passport making system is being performed in systematic manner. With passport application appointment in hand, applicant can visit the passport Seva Kendra (PSK).After security check the applicant enters into the preprocessing area where appointment slip and documents are checked. A token is issued and applicant proceeds to comfortable waiting lines until token number is appeared on electronic display. Then the applicant proceeds to stage one counter where demographic data is verified by citizen service executives. This counter equipped with documents scanning, biometric scanning, photograph & Fees. Uploading of the documents on site is also done here. If any document is found sort, applicant has to leave the system. The server issues token to the applicant and moves towards second counter (verification zone) where the verification of documents is done. Even if some lacuna remaining in the documents at this stage either the applicant is sent back to the first counter or leaves the system otherwise the applicant proceed for final phase (granting zone) of service where his application is checked for duplicity and other background information to take decision on granting of the passport to the applicant. Finally the applicant receives a letter confirming passport approval and exit from the system.

Fig.1:



4. MATHEMATICAL MODELING

Define $P_{n1,\,n2,\,n3}$ be the probability that there are n_1, n_2, n_3 units in the system in front of server S_1, S_2 and S_3. On the basis of probability consideration, the differential difference equation in the steady state can be expresses as:

$$(\lambda+\mu_1+\mu_2+\mu_3)P_{n_1,n_2,n_3} = \lambda P_{n_1-1,n_2,n_3} + \mu_1 p_1 P_{n_1+1,n_2,n_3} + \mu_1 p_{12} P_{n_1+1,n_2-1,n_3} + \mu_2 p_2 P_{n_1,n_2+1,n_3} + \mu_2 p_{21} P_{n_1-1,n_2+1,n_3} + \mu_2 p_{23} P_{n_1,n_2+1,n_3-1} + \mu_3 P_{n_1,n_2,n_3+1} \quad ;n_1,n_2,n_3>0 \quad (1)$$

$$(\lambda+\mu_1+\mu_2+\mu_3)P_{0,n_2,n_3} = \mu_1 p_1 P_{1,n_2,n_3} + \mu_1 p_{12} P_{1,n_2-1,n_3} + \mu_2 p_2 P_{0,n_2+1,n_3} + \mu_2 p_{23} P_{0,n_2+1,n_3-1} + \mu_3 P_{0,n_2,n_3+1} \quad ;n_1=0,n_2,n_3>0 \quad (2)$$

$$(\lambda+\mu_1+\mu_2+\mu_3)P_{n_1,0,n_3} = \lambda P_{n_1-1,0,n_3} + \mu_1 p_1 P_{n_1+1,0,n_3} + \mu_2 p_2 P_{n_1,1,n_3} + \mu_2 p_{21} P_{n_1-1,1,n_3} + \mu_2 p_{23} P_{n_1,1,n_3-1} + \mu_3 P_{n_1,0,n_3+1} \quad ;n2=0,n_1,n_3>0 \quad (3)$$

$$(\lambda+\mu_1+\mu_2+\mu_3)P_{n_1,n_2,0} = \lambda P_{n_1-1,n_2,0} + \mu_1 p_1 P_{n_1+1,n_2,0} + \mu_1 p_{12} P_{n_1+1,n_2-1,0} + \mu_2 p_2 P_{n_1,n_2+1,0} + \mu_2 p_{21} P_{n_1-1,n_2+1,0} + \mu_3 P_{n_1,n_2,1} \quad ;n_1,n_2,>0,n_3=0 \quad (4)$$

$$(\lambda+\mu_1+\mu_2+\mu_3)P_{0,0,n_3} = \mu_1 p_1 P_{1,0,n_3} + \mu_2 p_2 P_{0,1,n_3} + \mu_2 p_{23} P_{0,1,n_3-1} + \mu_3 P_{0,0,n_3+1} ;n_1,n_2=0,n_3>0 \quad (5)$$

$$(\lambda+\mu_1+\mu_2+\mu_3)P_{n_1,0,0} = \lambda P_{n_1-1,0,0} + \mu_1 p_1 P_{n_1+1,0,0} + \mu_2 p_2 P_{n1,1,0} + \mu_2 p_{21} P_{n_1-1,1,0} + \mu_3 P_{n_1,0,1} \quad ;n_1>0,n_2,n_3=0 \quad (6)$$

$$\left(\lambda+\mu_1+\mu_2+\mu_3\right)P_{0,n_2,0}=\mu_1 p_1 P_{1,n_2,0}+\mu_1 p_{12} P_{1,n_2-1,0}+\mu_2 p_2 P_{0,n_2+1,0}+$$
$$+\mu_3 P_{0,n_2,1} \qquad ;n_1,n_3=0,n_2>0 \tag{7}$$

$$\left(\lambda+\mu_1+\mu_2+\mu_3\right)P_{0,0,0}=\mu_1 p_1 P_{1,0,0}+\mu_2 p_2 P_{0,1,0}+\mu_3 P_{0,0,1} \qquad ;n_1,n_2,n_3=0 \tag{8}$$

with initial condition

$$P_{n_1,n_2,n_3}(0)=\begin{cases}1 & \left(n_1,n_2,n_3\geq 0\right)\\ 0 & \text{otherwise}\end{cases} \tag{9}$$

5. SOLUTION PROCESS:-

For Steady state solution of equation from (1) to (8), we apply Generating function technique as

$$G\left(x,y,z\right)=\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}P_{n_1,n_2,n_3}x^{n_1}y^{n_2}z^{n_3} \tag{10}$$

Also

$$G_{n_2,n_3}\left(x\right)=\sum_{n_1=0}^{\infty}P_{n_1,n_2,n_3}x^{n_1} \tag{11}$$

$$G_{n_3}\left(x,y\right)=\sum_{n_2=0}^{\infty}G_{n_2,n_3}\left(x\right)y^{n_2} \tag{12}$$

$$G\left(x,y,z\right)=\sum_{n_3=0}^{\infty}G_{n_3}\left(x,y\right)z^{n_3} \tag{13}$$

On Solving, we get the solution as:

$$G\left(x,y,z\right)=\frac{\begin{array}{c}\mu_1\left(1-\frac{p_1}{x}-\frac{p_{12}y}{x}\right)G\left(y,z\right)+\mu_2\left(1-\frac{p_2}{y}-\frac{p_{21}x}{y}-\frac{p_{23}z}{y}\right)G\left(x,z\right)\\ +\mu_3\left(1-\frac{1}{z}\right)G\left(x,y\right)\end{array}}{\left[\begin{array}{l}\lambda\left(1-x\right)+\mu_1\left(1-\frac{p_1}{x}-\frac{p_{12}y}{x}\right)+\\ \mu_2\left(1-\frac{p_2}{y}-\frac{p_{21}x}{y}-\frac{p_{23}z}{y}\right)+\mu_3\left(1-\frac{1}{z}\right)\end{array}\right]} \tag{14}$$

We define $G\left(y,z\right)=G_1,\quad G\left(x,z\right)=G_2,\quad G\left(x,y\right)=G_0$

$$G(x,y,z)=\frac{\mu_1\left(1-\frac{p_1}{x}-\frac{p_{12}y}{x}\right)G_1+\mu_2\left(1-\frac{p_2}{y}-\frac{p_{21}x}{y}-\frac{p_{23}z}{y}\right)G_2+\mu_3\left(1-\frac{1}{z}\right)G_0}{\left[\lambda(1-x)+\mu_1\left(1-\frac{p_1}{x}-\frac{p_{12}y}{x}\right)+\mu_2\left(1-\frac{p_2}{y}-\frac{p_{21}x}{y}-\frac{p_{23}z}{y}\right)+\mu_3\left(1-\frac{1}{z}\right)\right]} \quad (15)$$

Using L' Hospital rule, we get

1. When $x,y=1, \quad z\to 1$ i.e., $\left(\frac{0}{0}\right)$, we get

$$G(1,1,1)=\frac{\mu_1\left(1-\frac{p_1}{1}-\frac{p_{12}}{1}\right)G_1+\mu_2\left(1-\frac{p_2}{1}-\frac{p_{21}}{1}-\frac{p_{23}z}{1}\right)G_2+\mu_3\left(1-\frac{1}{z}\right)G_0}{\left[\lambda(1-1)+\mu_1\left(1-\frac{p_1}{1}-\frac{p_{12}}{1}\right)+\mu_2\left(1-\frac{p_2}{1}-\frac{p_{21}}{1}-\frac{p_{23}z}{1}\right)+\mu_3\left(1-\frac{1}{z}\right)\right]} \quad (16)$$

$$-\mu_2 p_{23} G_2+\mu G_0=-\mu_2 p_{23}+\mu_3 \quad (17)$$

2. When $x,z=1, \quad y\to 1$ i.e., $\left(\frac{0}{0}\right)$, we get

$$-\mu_1 p_{12} F_1+\mu_2 F_2=-\mu_1 p_{12}+\mu_2 \quad (18)$$

3. When $y,z=1, \quad x\to 1$ i.e., $\left(\frac{0}{0}\right)$, we get

$$\mu_1 G_1-\mu_2 p_{21} G_2=-\lambda+\mu_1-\mu_2 p_{21} \quad (19)$$

On solving, we get

$$G_0=1-\frac{\lambda(p_{12}p_{23})}{\mu_3(1-p_{12}p_{21})} \quad (20)$$

$$G_1=1-\frac{\lambda}{\mu_1(1-p_{12}p_{21})} \quad (21)$$

$$G_2=1-\frac{(\lambda p_{12})}{\mu_2(1-p_{12}p_{21})} \quad (22)$$

$$P_{n1,n2,n3}=\rho_1^{n1}\rho_2^{n2}\rho_3^{n3}(1-\rho_1)(1-\rho_2)(1-\rho_3)$$

Where $\rho_1=\dfrac{\lambda}{\mu_1(1-p_{12}p_{21})}$ (23)

$$\rho_2 = \frac{\lambda\,(p_{12})}{\mu_2\,(1-p_{12}p_{21})} \tag{24}$$

$$\rho_3 = \frac{(\lambda p_{12}p_{23})}{\mu_3\,(1-p_{12}p_{21})} \tag{25}$$

The solution of steady state exist if $\rho_1, \rho_2, \rho_3 < 1$

Now, Let

$$G(x,y,z) = \frac{f}{g} \tag{26}$$

Where,

$$f = \mu_1\left(1-\frac{p_1}{x}-\frac{p_{12}y}{x}\right)F_1 + \mu_2\left(1-\frac{p_2}{y}-\frac{p_{21}x}{y}-\frac{p_{23}z}{y}\right)F_2 + \mu_3\left(1-\frac{1}{z}\right)F \tag{27}$$

$$g = \lambda(1-x) + \mu_1\left(1-\frac{p_1}{x}-\frac{p_{12}y}{x}\right) + \mu_2\left(1-\frac{p_2}{y}-\frac{p_{21}x}{y}-\frac{p_{23}z}{y}\right) + \mu_3\left(1-\frac{1}{z}\right) \tag{28}$$

6. CALCULATING PARTIAL DERIVATIVES

at x=y=z=1 gives

$$\left(\frac{\partial f}{\partial x}\right)_{(1,1,1)} = \mu_1 G_1(1,1) - \mu_2 p_{21} G_2(1,1) \tag{29}$$

On putting the values, we get

$$\left(\frac{\partial f}{\partial x}\right)_{(1,1,1)} = \mu_1 - p_{21}\mu_2 - \lambda \tag{30}$$

$$\left(\frac{\partial^2 f}{\partial x^2}\right)_{(1,1,1)} = \frac{-2\mu_1(1-p_{12}p_{21}) + 2\lambda}{1-p_{12}p_{21}} \tag{31}$$

Similarly,

$$\left(\frac{\partial f}{\partial y}\right)_{(1,1,1)} = -p_{12}\mu_1 + \mu_2 \tag{32}$$

$$\left(\frac{\partial^2 f}{\partial y^2}\right)_{(1,1,1)} = \frac{-2\mu_2(1-p_{12}p_{21}) + 2\lambda p_{12}}{1-p_{12}p_{21}} \tag{33}$$

$$\left(\frac{\partial f}{\partial z}\right)_{(1,1,1)} = -\mu_2 p_{23} + \mu_3 \tag{34}$$

$$\left(\frac{\partial^2 f}{\partial z^2}\right)_{(1,1,1)} = \frac{-2\mu_3(1-p_{12}p_{21}) + 2\lambda p_{12}p_{23}}{1-p_{12}p_{21}} \tag{35}$$

Now, Similarly

$$g = \lambda(1-x) + \mu_1\left(1 - \frac{p_1}{x} - \frac{p_{12}y}{x}\right) + \mu_2\left(1 - \frac{p_2}{y} - \frac{p_{21}x}{y} - \frac{p_{23}z}{y}\right) + \mu_3\left(1 - \frac{1}{z}\right) \qquad (36)$$

$$\frac{\partial g}{\partial x} = -\lambda + \mu_1 - \mu_2 p_{21} \qquad (37)$$

$$\frac{\partial^2 g}{\partial x^2} = -2\mu_1 \qquad (38)$$

$$\frac{\partial g}{\partial y} = -p_{12}\mu_1 + \mu_2 \qquad (39)$$

$$\frac{\partial^2 g}{\partial y^2} = -2\mu_2 \qquad (40)$$

$$\frac{\partial g}{\partial z} = -p_{23}\mu_2 + \mu_3 \qquad (41)$$

$$\frac{\partial^2 g}{\partial z^2} = -2\mu_3 \qquad (42)$$

7. MEAN QUEUE LENGTH OF THE SYSTEM:

$$L = L_{q1} + L_{q2} + L_{q3} \qquad (43)$$

Where L_{q1}= Marginal mean queue length at first server

$$L_{q1} = \frac{\left(\frac{\partial g}{\partial x}\right)\left(\frac{\partial^2 f}{\partial x^2}\right) - \left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial^2 g}{\partial x^2}\right)}{2\left(\frac{\partial g}{\partial x}\right)^2} \qquad (44)$$

On Solving, we get

$$L_{q1} = \frac{\lambda}{-\lambda(1-p_{12}p_{21}) + \mu_1(1-p_{12}p_{21}) - p_{21}\mu_2(1-p_{12}p_{21})} \qquad (45)$$

$$L_{q2} = \frac{\left(\frac{\partial g}{\partial y}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial f}{\partial y}\right)\left(\frac{\partial^2 g}{\partial y^2}\right)}{2\left(\frac{\partial g}{\partial y}\right)^2} \qquad (46)$$

$$L_{q2} = \frac{\lambda p_{12}}{-p_{12}\mu_1\left(1-p_{12}p_{21}\right)+\mu_2\left(1-p_{12}p_{21}\right)} \tag{47}$$

$$L_{q3} = \frac{\left(\frac{\partial g}{\partial z}\right)\left(\frac{\partial^2 f}{\partial z^2}\right)-\left(\frac{\partial f}{\partial z}\right)\left(\frac{\partial^2 g}{\partial z^2}\right)}{2\left(\frac{\partial g}{\partial z}\right)^2} \tag{48}$$

$$L_{q3} = \frac{\lambda p_{12}p_{23}}{-\mu_2 p_{23}\left(1-p_{12}p_{21}\right)+\mu_3\left(1-p_{12}p_{21}\right)} \tag{49}$$

Let the mean queue length of the system be L

$$L = L_{q1} + L_{q2} + L_{q3} \tag{50}$$

$$L = \frac{\lambda}{-\lambda\left(1-p_{12}p_{21}\right)+\mu_1\left(1-p_{12}p_{21}\right)-p_{21}\mu_2\left(1-p_{12}p_{21}\right)} +$$

$$\frac{\lambda p_{12}}{-\mu_1 p_{12}\left(1-p_{12}p_{21}\right)+\mu_2\left(1-p_{12}p_{21}\right)} + \frac{\lambda p_{12}p_{23}}{-\mu_2 p_{23}\left(1-p_{12}p_{21}\right)+\mu_3\left(1-p_{12}p_{21}\right)}$$

7. NUMERICAL ANALYSIS OF THE MODEL:

Now, we discuss the behaviour analysis of mean queue parameter of the stated system on changing values of arrival parameter and service parameter in two different ways;

I. Behaviour of marginal mean queue lengths and the mean queue length of the entire system with respect to arrival rate λ is depicted in table 1.

Table 1: Mean queue length of the entire system with respect to variable arrival parameter λ keeping other parametric values constant.

λ	$\mu_1 = 4, \mu_2 = 5, \mu_3 = 6, p_1 = 0.2, p_{12} = 0.8, p_2 = 0.1, p_{21} = 0.15, p_{23} = 0.75$			
	L_{q1}	L_{q2}	L_{q3}	L
1	0.505	0.505	0.303	1.313
1.5	0.974	0.757	0.454	2.185
2	1.818	1.01	0.606	3.434
2.2	2.38	1.11	0.666	4.156
2.4	3.20	1.21	0.727	5.137
2.6	4.54	1.31	0.787	6.637
2.8	7.07	1.41	0.848	9.328
3	13.63	1.51	0.909	16.049
3.2	72.72	1.61	0.969	75.299

Graphical study has also been made for marginal mean queue lengths and mean queue length of the entire system as shown in Fig. 2 to 5.

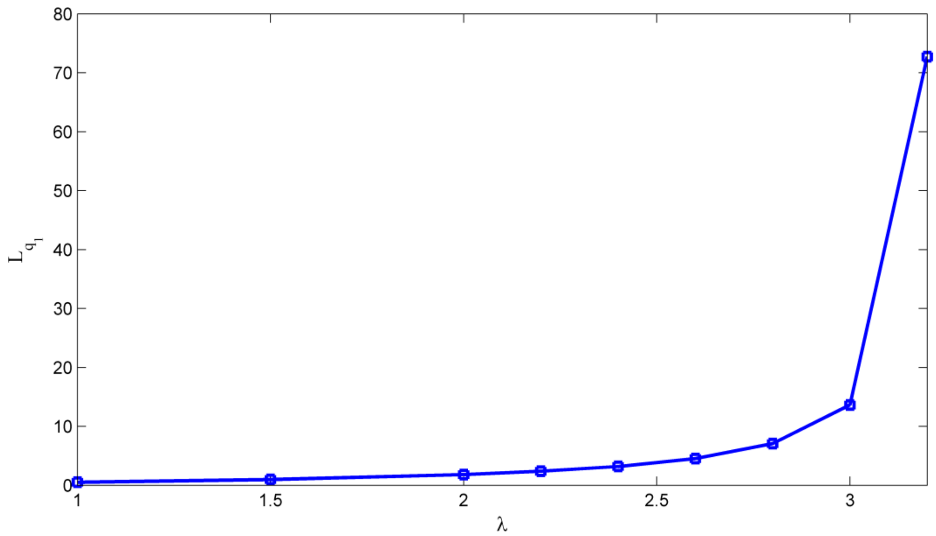


Fig. 2: L_{q1} vs λ

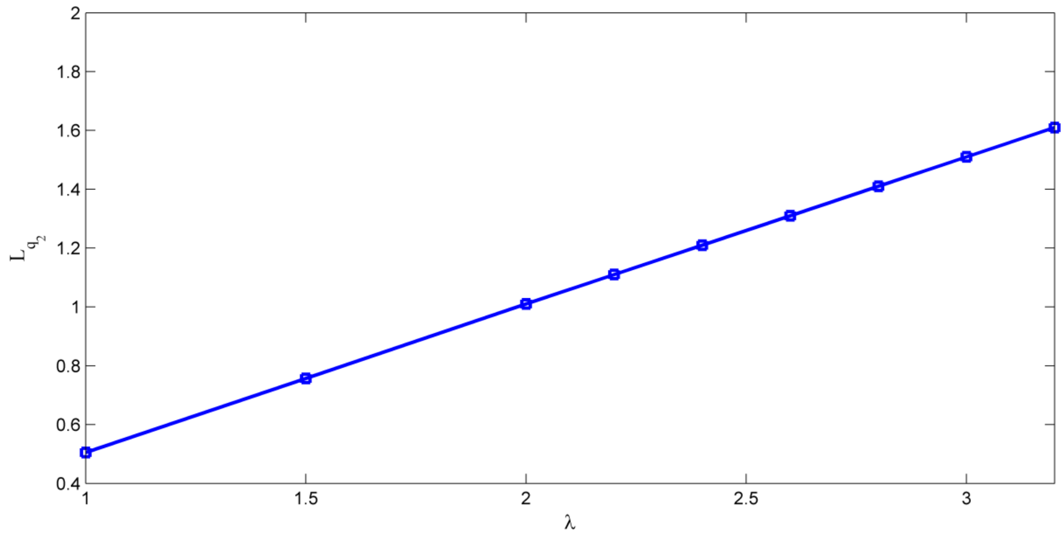


Fig. 3: L_{q2} vs λ

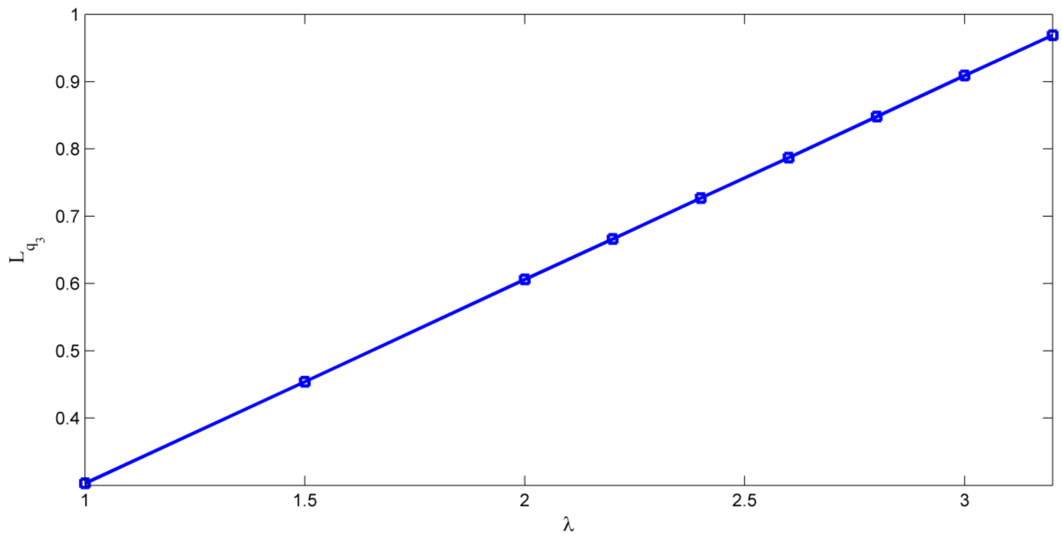


Fig. 4: L_{q3} vs λ

8. RESULTS & DISCUSSIONS WITH GRAPHICAL ANALYSIS:

From Table 1 & Fig. 2 to 5, we observe the followings:

(i) Marginal queue lengths and mean queue length of the entire system increases with increase in the mean arrival rate λ.

(ii) Gradual increase in the mean queue length of the entire system is observed for λ up to 2.8. A sudden increase can be noticed for the values of λ beyond 2.8.

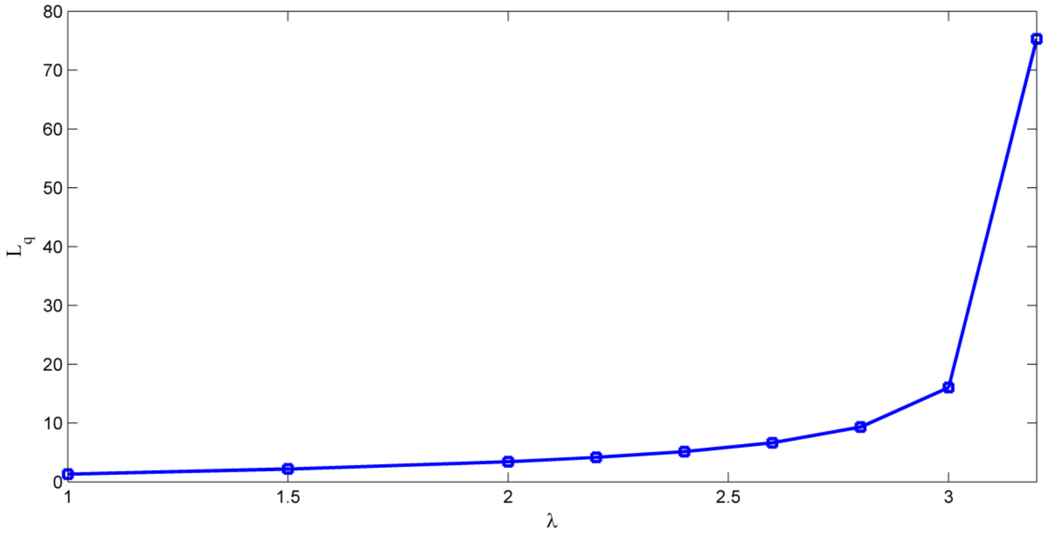


Fig. 5: L vs λ

8.1 Behaviour of the mean queue length for the entire system with respect to service rate μ_1 is depicted in Table 2 and Fig. 6

μ_1	$\lambda=1,\mu_2=5,\mu_3=6,p_1=0.2,p_{12}=0.8,$ $p_2=0.1,p_{21}=0.15,p_{23}=0.75$			
	L_{q1}	L_{q2}	L_{q3}	L
4	0.505	0.505	0.303	1.313
4.2	0.463	0.554	0.303	1.32
4.4	0.428	0.614	0.303	1.345
4.6	0.398	0.688	0.303	1.389
4.8	0.372	0.783	0.303	1.458
5	0.349	0.909	0.303	1.561
5.2	0.329	1.08	0.303	1.712
5.4	0.311	1.33	0.303	1.944

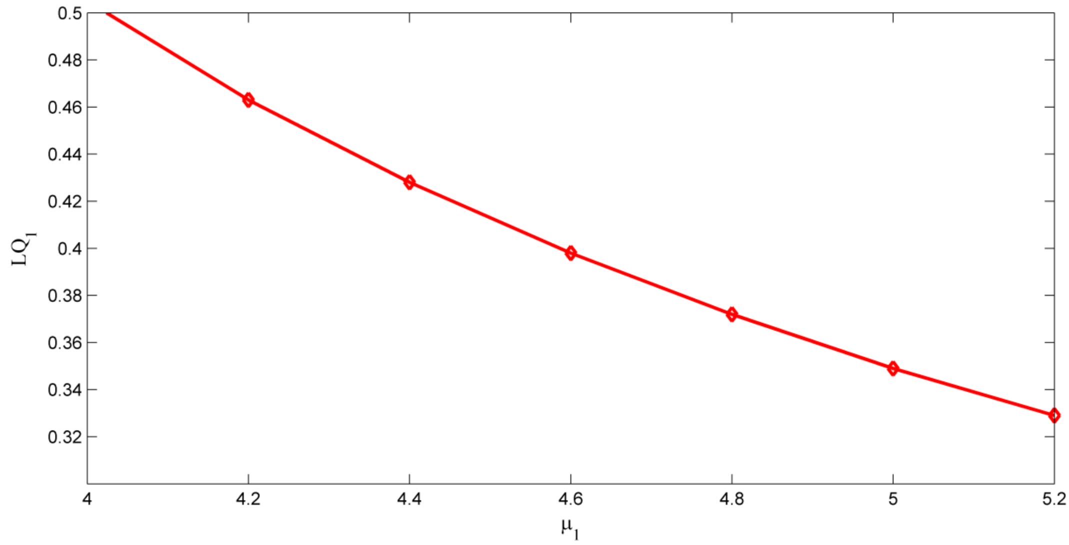


Fig. 6: L_{q1} vs μ_1

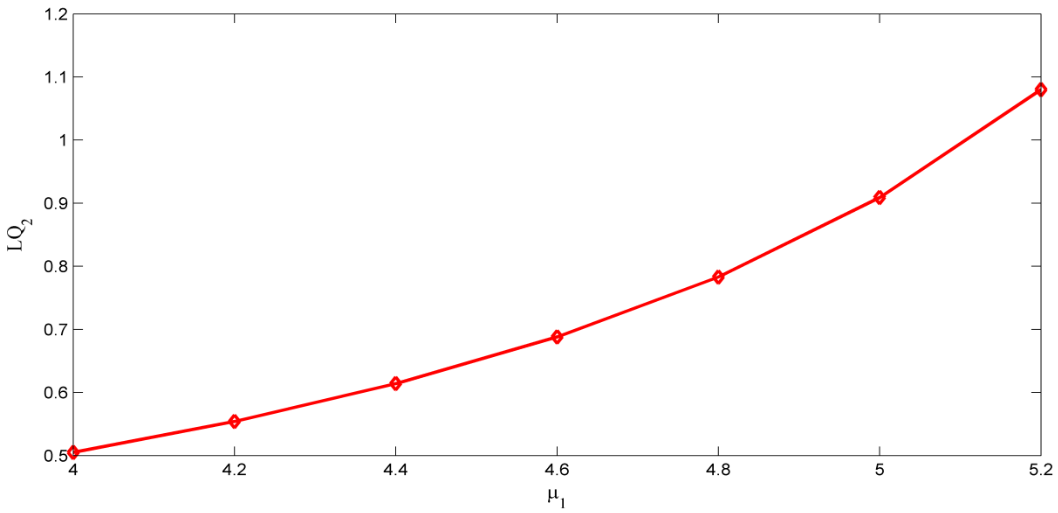


Fig. 7: L_{q2} vs μ_1

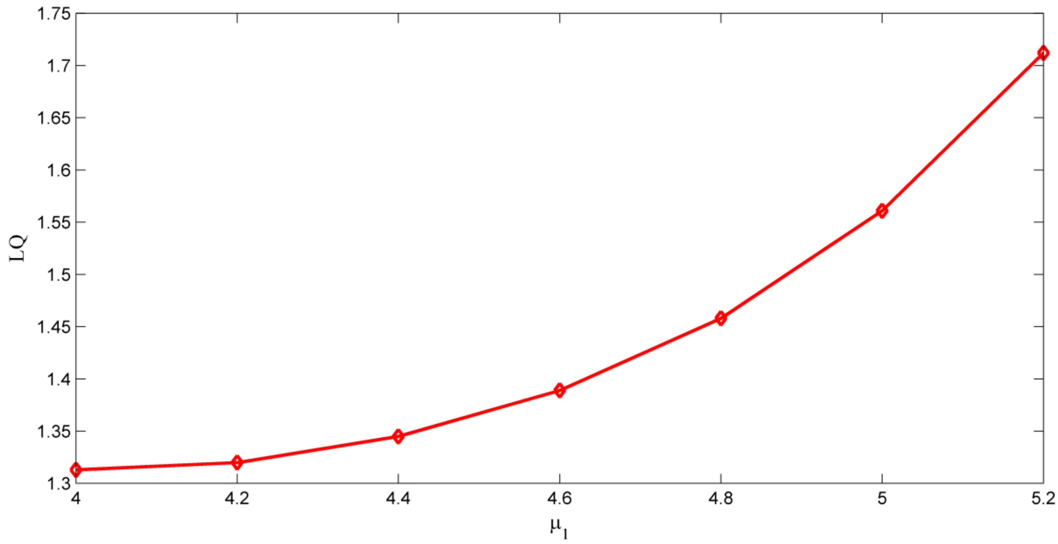


Fig. 8: L vs μ_1

From Table 2 and fig. 6 to 8, we may observe that initially the mean queue length for the entire system increases gradually with increase in the value of μ_1 and at $\mu_1=5$, it increases rapidly with increase in the value of μ_1.

μ_1	L
5	1.561
5.2	1.712
5.4	1.944

9. LIMITATIONS OF THE MODEL

In this stochastic model, only the applicants with fresh passport or renewal of passport are considered but diplomatic passport, official passport, emergency passport or medical cases have been left for further research.

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DUAL FACTOR ROLE IN MODELING OF DATA ENVELOPMENT ANALYSIS

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1. ABSTRACT

This paper develops a methodological framework for modelling and efficiency analysis by the use of Data Envelopment Analysis (DEA) technique. DEA finds the weights of inputs and outputs of the Decision-making unit(DMU) under consideration so that it gives an efficiency index by taking ratio of weighted sum of outputs to the weighted sum of inputs. An input is a quantity which is inversely affecting to the efficiency index and an output is a quantity which is directly affecting the efficiency index. The input and output weights are modelled so that they satisfy the non-negativity constraint. This efficiency index is used for discriminating among efficient and inefficient DMUs. Thus, tradition DEA requires discrete set of inputs and outputs. However, there may be cases where a parameter cannot be clearly categorized as input or output if initially it cannot be stated whether it will directly or inversely affect the efficiency index. Such parameters are termed as dual role factors. This paper tries to attempt a modelling approach towards this problem and proposes a methodological framework towards solving it.

Keywords: Data Envelopment Analysis, Dual role factor, DMU, efficiency index, weights of inputs and outputs

2. INTRODUCTION

Data Envelopment Analysis (DEA) is one of the mathematical techniques used widely across the globe in public as well as private sectors. Being developed by Charnes, Cooper and Rohes(CCR) in 1978 as basic model, it has now been researched widely to include different case scenarios. It is basically a non-parametric method, as it does not require the functional form the variables involved. It considers the similar type of objects having some processing or input features and resulting in some form of outputs. These objects under consideration are termed as Decision Making Units(DMUs) and these features have specific properties to term them as input or output. Thus, evaluating efficiency under traditional DEA model requires separate set of inputs and outputs for calculations. The efficiency scores determine the better DMU from the peer group and thus give scope for improvement of non-efficient DMUs [1][15].
There are number of fields where these models can be applied successfully in determining the benchmark performer among the peer group, including Schools, Govt. Institutions, Bank branches, Technologies, Energy Efficiency, Hospitals and many other fields [2][3][9][18][19][20][21][22]. In real life cases, there may be scenario, where a particular feature of DMU cannot be clearly segregated as input or output. It means that, it is difficult rather impossible to find effect of one attribute of objects under consideration as input or output into efficiency measurement using simple models [4][5][6].

The problem of dual role is well identified in the literature in different set of applications and to find its weight in DEA model is given under different assumption. For example [1] had proposed a new chance-constrained data envelopment analysis (CCDEA) approach is to assist the decision makers to determine the most appropriate third-party reverse logistics (3PL) providers in the presence of both dual-role factors and stochastic data. [16] and [17] has applied DEA with multiple dual-role factors for selecting third-party reverse logistics. [6] presents a scheme for classifying the Data Envelopment Analysis (DEA) literature. The taxonomy allows one to distinguish articles on the basis of the data source used (D) if any, the type of envelopment (E) invoked, the approach to analysis (A) used, and the nature (N) of the paper. Each of the above attributes D, E, A and N (DEAN) are further subdivided to obtain a detailed description of each article comprising this rather wide-ranging field of knowledge. There are many other literature work which deals with problem of dual role factor and takes into account the weights of multiplier for it [7][8][10]. [11] has used two stage DEA models whereas [12] has used possibility set concept to deal with it. Even in uncertainties also dual factor is modelled in [13][14].

However, these methods defined in various literature are very complicated and specific to their own domain. In this paper, it is being tried to devise a simple methodology to model the dual role factor which is applicable in different domains and scenarios.

3. METHODS AND EXPERIMENTAL DETAILS

The basic model for DEA is Constant Return to Scale model being given in Banker, Charnes and Cooper in 1978, known as CCR model. There are models for formulating self-efficiency in case of discrete set of multiple inputs and multiple outputs.

3.1 CCR Model

Suppose there are n numbers of DMUs, each having m number of inputs and s number of outputs. Let index for DMU is j, for input is i, and for output is r. Thus we have j = 1, 2,3,..n DMUs each using i=1,2,..m inputs x_{ij} and delivering r=1,2,..s outputs y_{rj}.

The efficiency score of a DMU say k, under consideration is calculated by taking ratio of weighted output to weighted input of DMU k, subject to the constraint that values of optimal weights for inputs and outputs for DMU k should be such that it keeps Efficiency score of all of the DMUs within definition of efficiency i.e. between 0(minimum) to 1(maximum).

Mathematically, the formulation is

$$\max \quad \frac{\sum_{r=1}^{s} \mu_{rk} y_{rk}}{\sum_{i=1}^{m} \vartheta_{ik} x_{ik}}$$

$$\text{s.t.} \quad \frac{\sum_{r=1}^{s} \mu_{rk} y_{rj}}{\sum_{i=1}^{m} \vartheta_{ik} x_{ij}} \le 1 \qquad \text{for } j = 1,2,..,n$$

$$\mu_{rk} \ge 0 \qquad \text{for } r = 1,2,..,s$$

$$\vartheta_{ik} \ge 0 \qquad \text{for } r = 1,2,..,s$$

The above formulation is fractional programming problem which is converted to equivalent linear for using Charnes Cooper method as

$$\text{put } u_{rk} = t\,\mu_{rk}$$

$$\text{put } v_{rk} = t\,\vartheta_{rk}$$

$$\text{where } t = \left[\sum_{i=1}^{m} \vartheta_{ik} x_{ik}\right]^{-1}$$

Thus the fractional programming problem can be written in linear form as

$$\max \quad \sum_{r=1}^{s} u_{rk} y_{rk}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_{ik}x_{ik} = 1$$

$$\sum_{r=1}^{s} u_{rk}y_{rj} - \sum_{i=1}^{m} v_{ik}x_{ij} \le 0 \qquad for\ j = 1,2,..,n$$

$$u_{rk} \ge 0 \qquad for\ r = 1,2,..,s$$

$$v_{ik} \ge 0 \qquad for\ r = 1,2,..,s$$

3.2 Proposed Methodology:

The dual role factor in the basic DEA is modelled as follows.

Step 1: Using the basic CCR model, find the efficiency of each of the DMU without considering the dual role factor. List the efficiency of each of the DMU.

Step 2: Now consider each of the DMU with dual role factor as input and find the modified efficiency scores for each of the DMU. List the efficiency of each of the DMU.

Step 3: Now consider each of the DMU with dual role factor as output and find the modified efficiency scores for each of the DMU. List the efficiency of each of the DMU.

Step 4: Compare the efficiency scores of those obtained from the three models and analyze the result.

Step 5: Find the optimal range for efficiency scores for each of the DMU.

Step 6: Use the efficiency score range and model dual role for getting the final efficiency score using modified model as:

$$\max \theta_k^* = \sum_{r=1}^{s} u_{rk}y_{rk} + w_{1k}y_{(s+1)k} - w_{2k}y_{(s+1)k}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_{ik}x_{ik} = 1$$

$$min\,\theta_k \le \theta_k^* \le max\theta_k$$

$$\sum_{r=1}^{s} u_{rk}y_{rj} - \sum_{i=1}^{m} v_{ik}x_{ij} \le 0 \qquad for\ j = 1,2,..,n$$

$$u_{rk} \ge 0 \qquad for\ r = 1,2,..,s$$

$$v_{ik} \ge 0 \qquad for\ r = 1,2,..,s$$

Where $min\,\theta_k \le \theta_k^* \le max\theta_k$ defines the optimal range of efficiency score of DMU k under consideration. This minimal and maximum values are obtained from step 5.

3.3 Data:

The data for analysis is taken from one of the prior arts. Clear inputs are Direct Expenditure and Personnel costs. Clear outputs are Cost savings. One of the attributes “donation for tax benefit” cannot be clearly defined as it is cost to the institutes or it is benefit to the institutes. Thus, on first hand it cannot be taken as input or output clearly for efficiency calculations. The numerical values against each of the inputs and outputs is given in table1. There are 10 Institutes under consideration.

DMU	INPUT		OUTPUT	Dual role factor
	I1	I2	O1	
DMU1	2600	186	117	312
DMU2	4180	417	213.2	543.4
DMU3	1620	224	97.2	243
DMU4	3850	325	250.2	616
DMU5	6900	640	372.6	3312
DMU6	2841	465	127.8	681
DMU7	3087	250	145.1	710
DMU8	3420	620	191.5	615
DMU9	1500	94	87	375
DMU10	8100	510	502.2	3726

Table 1: Numerical example of 10 DMU with the presence of dual role factor

4. RESULTS AND DISCUSSION:

Using step 1 as given in methodology, efficiency scores and weight table are obtained. Here basic DEA model is used and dual role factor is not taken under consideration as input or output for calculation purpose.

Model 1: Efficiency scores and weight table without considering dual role factor

DMU	EFFICIENCY SCORES
DMU1	0.712093
DMU2	0.784846
DMU3	0.923261
DMU4	1
DMU5	0.830935
DMU6	0.692202
DMU7	0.728632
DMU8	0.866206
DMU9	0.939907
DMU10	1

Table 2: Efficiency scores without considering dual role

DMU	v1	v2	u1
DMU1	3.24E-04	8.47E-04	6.09E-03
DMU2	2.39E-04	0	3.68E-03
DMU3	6.17E-04	0	9.50E-03
DMU4	2.60E-04	0	4.00E-03
DMU5	1.45E-04	0	2.23E-03
DMU6	3.52E-04	0	5.42E-03
DMU7	2.67E-04	2.67E-04	5.02E-03
DMU8	2.92E-04	0	4.50E-03
DMU9	0	1.06E-02	1.08E-02
DMU10	0	1.96E-03	1.99E-03

Table 3 : Weight table without considering dual role factor

factor

We can see from table 2 that DMU4 and DMU 10 are efficient as having score 1 and others are inefficient. Table 3 detailed out the v1, v2 and u1 as weights vectors of two inputs and one output for 10 DMUs. Next, we run model taking dual factor as input and again run the model for all 10 DMUs. The results are summarized in table 4 and table 5.

Model 2: Efficiency scores and weight table considering dual role factor as input

DMU	EFFICIENCY SCORES
DMU1	0.923261
DMU2	0.965964
DMU3	0.984812
DMU4	1
DMU5	0.830935
DMU6	0.692202
DMU7	0.728632
DMU8	0.861621
DMU9	1
DMU10	1

Table 4: Efficiency scores considering dual role factor as input

DMU	v1	v2	w	u1
DMU1	0	0	3.21E-03	7.89E-03
DMU2	0	0	1.84E-03	4.53E-03
DMU3	0	0	4.12E-03	1.01E-02
DMU4	2.60E-04	0	0.00E+00	4.00E-03
DMU5	1.45E-04	0	0.00E+00	2.23E-03
DMU6	3.52E-04	0	0	5.42E-03
DMU7	2.67E-04	6.99E-04	0	5.02E-03
DMU8	2.92E-04	0	0	4.50E-03
DMU9	0	9.82E-03	2.05E-04	1.15E-02
DMU10	0.00E+00	1.70E-03	3.55E-05	1.99E-03

Table 5 : Weight table considering dual role factor as input

Table 5 detailed out the v1, v2 and u1 as weights vectors of two inputs and one output and was weight for dual role factor when taken as input for 10 DMUs. We can see from table 4 that now DMU 9 also becomes efficient along with DMU4 and DMU having score 1. This is important observation as generally we may think that, if no. of input increases, then efficiency should decrease. Next, we run model taking dual factor as output and again run the model for all 10 DMUs. The results are summarized in table 6 and table 7.

Model 3: Efficiency scores and weight table considering dual role as output

DMU	EFFICIENCY SCORES
DMU1	0.712093
DMU2	0.785508
DMU3	0.923602
DMU4	1
DMU5	1
DMU6	0.711486
DMU7	0.740365
DMU8	0.867896
DMU9	0.939907
DMU10	1

Table 6: Efficiency scores considering dual role as output

	V1	V2	U1	W
DMU1	3.24E-04	8.47E-04	6.09E-03	0.00E+00
DMU2	2.39E-04	0	3.59E-03	3.58E-05
DMU3	6.17E-04	0	9.27E-03	9.23E-05
DMU4	2.60E-04	0	4.00E-03	0.00E+00
DMU5	1.45E-04	0	5.89E-04	2.36E-04
DMU6	3.52E-04	0	5.29E-03	5.26E-05
DMU7	3.24E-04	0.00E+00	4.87E-03	4.84E-05
DMU8	2.92E-04	0	4.39E-03	4.37E-05
DMU9	0.00E+00	1.06E-02	1.08E-02	0.00E+00
DMU10	0	1.80E-04	0	2.68E-04

Table 7 : Weight table considering dual role as output

Table 7 detailed out the v1, v2 and u1 as weights vectors of two inputs and one output and w as dual role factor as output for 10 DMUs. We can see from table 6 that now DMU 5 became efficient along with DMU 4 and DMU 10. Next , using these three model run, we obtain the optimum efficiency scores for each of the DMUs. Note that DMU 4 and DMU 10 are efficient in all the three cases thus they may be termed as always efficient DMUs. Table 8 gives the comparative analysis for efficiency indices obtained from previous three models and table 9 defines the optimal range of efficiency scores for each of the 10 DMUs.

	EFFICIENCY INDEX		
DMU	**MODEL I**	**MODEL II(DUAL NPUT)**	**MODEL III(DUAL OUTPUT)**
DMU1	0.712093	0.923261	0.712093
DMU2	0.784846	0.965964	0.785508
DMU3	0.923261	0.984812	0.923602
DMU4	1	1	1
DMU5	0.830935	0.830935	1

DMU6	0.692202	0.692202	0.711486
DMU7	0.728632	0.728632	0.740365
DMU8	0.866206	0.861621	0.867896
DMU9	0.939907	1	0.939907
DMU10	1	1	1

Table 8: Comparative table for efficiency scores

Range for Efficiency scores:

Efficiency score	**Lower limit**	**Upper limit**
θ_1	0.712093	0.923261
θ_2	0.784846	0.965964
θ_3	0.923261	0.984812
θ_4	1	1
θ_5	0.830935	1
θ_6	0.692202	0.711486
θ_7	0.728632	0.740365
θ_8	0.861621	0.867896
θ_9	0.939907	1
θ_{10}	1	1

Table 9: Range for Efficiency scores

Now we run our modified model considering Dual role factor under optimum range of efficiency scores for each of the DMUs The results are summarized in table 10 and table 11.

Efficiency score and weight table using modified model for inefficient DMUs:

DMU	EFFICIENCY SCORES
DMU1	0.801871
DMU2	0.785508
DMU3	0.923602
DMU5	1
DMU6	0.711486
DMU7	0.740365
DMU8	0.867896
DMU9	1

Table 10: Efficiency scores using modified model

	u1	u2	v1	v2
DMU1	8.55E-03	6.36E-04	0	5.38E-03
DMU2	3.59E-03	3.58E-05	2.39E-04	0.00E+00
DMU3	9.27E-03	9.23E-05	6.17E-04	0.00E+00
DMU5	0.00E+00	3.02E-04	1.45E-04	0.00E+00
DMU6	5.29E-03	5.26E-05	3.52E-04	0.00E+00
DMU7	4.87E-03	4.84E-05	3.24E-04	0.00E+00
DMU8	4.39E-03	4.37E-05	2.92E-04	0.00E+00
DMU9	1.25E-02	2.22E-04	0.00E+00	1.06E-02

Table 11: Weight table using modified model

Using our modified model, we can see that among inefficient units, DMU 5 and DMU 9 also become Efficient.

5. CONCLUSIONS

In this study we have simplified the modelling of dual role factor using simple steps. Once we obtain the range for optimal efficiency scores for each of the unit under consideration, we can then find final efficiency score using modified model for inefficient units. We can then define role of dual factor on case to case basis as well. Here in this case study we find that The DMU 4 and 10 are efficient with all the three models. Efficiency of 4 DMUs increases when we take dual role factor as input. Efficiency of other 4 DMUs increases when we take dual role factor as output. Thus, we find the optimal range of efficiency score for each of the DMU which are inefficient and find the modified efficiency scores. From the weight table, we conclude that given dual role factor can be taken as output for the case study. The model derived in the paper is generic in nature since the optimum range of efficiency scores can be obtained in any case where we have a factor which plays dual role. However, we can further investigate and validate the model on different data domains. For the future, cross efficiency scores can be obtained from modified model to get peer evaluation and raking of the DMU. Further deviation models can be run to further investigate the results.

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ON (∈ VQ)-FUZZY TRIGROUP

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ABSTRACT

In this paper, we introduce the concept of fuzzy singleton to tri-group, and uses it to define (∈ vq)- fuzzy tri-group and discuss its properties. We investigate whether or not the fuzzy point of a tri-group will belong to or quasi coincident with its fuzzy set if the constituent fuzzy points of the constituting subgroups both belong to or quasi coincident with their respective fuzzy sets, and vice versa. We also prove that a fuzzy tri-subset μ is an (∈ vq)-fuzzy sub-tri-group of the tri-group G if its constituent fuzzy subsets are (∈ vq)-fuzzy subgroups of their respective subgroups among others.

Key Words: Tri-groups, fuzzy sub tri-groups, fuzzy singleton on tri-group, (∈vq)- fuzzy subgroups, (∈vq)- fuzzy tri-group

1. INTRODUCTION

Fuzzy set was introduced by Zadeh [15] in 1965. Rosenfeld [9] introduced the notion of fuzzy subgroups in 1971. Ming and Ming [8] in 1980 gave a condition for fuzzy subset of a set to be a fuzzy point, and used the idea to introduce and characterize the notions of quasi coincidence of a fuzzy point with a fuzzy set. Bhakat and Das [2,3] used these notions by Ming and Ming to introduce and characterize another class of fuzzy subgroup known as (∈ vq)- fuzzy subgroups. This concept has been further developed by other researchers. Recent contributions in this direction include those of Yuan et al [13,14].

The notion of bi-group was first introduced by P.L.Maggu [5] in 1994. This idea was extended in 1997 by Vasantha and Meiyappan [11]. These authors gave modifications of some results earlier proved by Maggu. Among these results was the characterization theorems for sub-bi-group. Meiyappan [12] introduced and characterized fuzzy sub-bi-group of a bi-group in 1998.

We introduced Tri-group and fuzzy sub tri-group [10] in 2013. In this paper, using these mentioned notions and with emphases on the elements that are three in G_1, G_2 and G_3 of the tri-group G, we define the notion of (∈, ∈ vq) - fuzzy sub tri-groups as an extension of the notion of (∈, ∈ vq)- fuzzy subgroups and discuss its properties.

2. PRELIMINARY RESULTS

Definition: 2.1

Let $(G_1,+)$, $(G_2, \bullet)$ and $(G_3, *)$ be three groups. Then $G = G_1 \cup G_2 \cup G_3$ is called tri group.

A subset H (≠ G) of a tri group G is called a sub – tri group, if H itself is a tri group under '+' , '•', and '∗' operations is said to be G.

Example: 2.2

If $G_1 = R$(where R is a set of all real numbers), $G_2 = R - \{0\}$and

$G_3 = Q$(Q is a set of all rational numbers), then $(G_1,+)$, $(G_2, \bullet)$ and $(G_3, *)$ are three groups ('+' is defined by addition, '•' is defined by multiplication and '*' is defined by $\frac{ab}{2}$). Hence, $G = G_1 \cup G_2 \cup G_3 = R$ is a tri group.

Definition 2.3([11])

A subset H(≠ 0) of a bi-group (G,+, ·) is called a sub tri-group of G if H itself is a tri-group under the operations of " + " , " · " and "*" defined on G.

Theorem 2.4([11])

Let (G,+, ·,*) be a tri-group. If the subset H ≠ 0 of a trigroup G is a sub Tri-group of G, then (H,+), (H, ·) and (H, *) are generally not groups.

Definition 2.5([15])

Let G be a non empty set. A mapping $\mu : G \rightarrow [0, 1]$ is called a fuzzy subset of G.

Definition 2.6([15])

Let μ be a fuzzy set in a set G. Then, the level subset μ_t is defined as:

$\mu_t = \{x \in G : \mu(x) \geq t\}$ for $t \in [0, 1]$.

Definition 2.7([9])

Let μ be a fuzzy set in a group G. Then, μ is said to be a fuzzy subgroup of G, if the following hold:

(i) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ $\forall$ x, y ∈ G;

(ii) $\mu(x-1) = \mu(x)$ $\forall$ x ∈ G.

Definition 2.8 ([8])

A fuzzy subset μ of a group G of the form

$$\mu(y) = \begin{cases} t\ (\neq 0) & \text{if } y = x, \\ 0 & \text{if } y \neq x \end{cases}$$

is said to be a fuzzy point with support x and value t and is denoted by x_t.

Definition: 2.9 [10]

Let μ_1 be a fuzzy subset of a set X_1 , μ_2 be a fuzzy subset of a set X_2 and μ_3 be a fuzzy subset of a set X_3, then the fuzzy union of the sets μ_1, μ_2 and μ_3 is defined as a function.

$\mu_1 \cup \mu_2 \cup \mu_3 : X_1 \cup X_2 \cup X_3 \rightarrow [0,1]$ given by

$$(\mu_1 \cup \mu_2 \cup \mu_3)(x) = \begin{cases} \max(\mu_1(x), \mu_2(x), \mu_3(x)) & \text{if } x \in X_1 \cap X_2 \cap X_3 \\ \max(\mu_1(x), \mu_2(x)) & \text{if } x \in X_1 \cap X_2 \,\&\, x \notin X_3 \\ \max(\mu_2(x), \mu_3(x)) & \text{if } x \in X_2 \cap X_3 \,\&\, x \notin X_1 \\ \max(\mu_3(x), \mu_1(x)) & \text{if } x \in X_3 \cap X_1 \,\&\, x \notin X_2 \\ \mu_1(x) & \text{if } x \in X_1,\, x \notin X_2 \,\&\, x \notin X_3 \\ \mu_2(x & \text{if } x \in X_2,\, x \notin X_1 \,\&\, x \notin X_3 \\ \mu_3(x) & \text{if } x \in X_3,\, x \notin X_1 \,\&\, x \notin X_2 \end{cases}$$

Definition: 2.10 [10]

Let $G = (G_1 \cup G_2 \cup G_3, +, \bullet, *)$ be a tri group. Then $\mu : G \rightarrow [0, 1]$ is said to be a fuzzy sub-tri group of the tri group G if there exists three fuzzy subsets μ_1 of G_1, μ_2 of G_2 and μ_3 of G_3 such that

i. $(\mu_1, +)$ is a fuzzy subgroup of $(G_1, +)$
ii. $(\mu_2, \bullet)$ is a fuzzy subgroup of $(G_2, \bullet)$
iii. $(\mu_3, *)$ is a fuzzy subgroup of $(G_3, *)$
iv. $\mu = (\mu_1 \cup \mu_2 \cup \mu_3)$.

Example: 2.11

Consider the tri group G = {±i, ±0, ±1, ±2, ±3, ±4, ……..} under the operation '+', '•' and '∗' where G_1 = {0, ±1, ±2, ±3,………………},G_2 = {±i, ±1} and G_3 = {1, 2, 4}.

Define $\mu: G \rightarrow [0, 1]$ by

$$\mu(x) = \begin{cases} \frac{1}{3} & \text{if } x = i, -i \\ 1 & \text{if } x \in \{0, \pm 2, \pm 4, \ldots\} \\ \frac{1}{2} & \text{if } x \in \{\pm 1, \pm 3, \pm 5 \ldots\} \end{cases}$$

We can find

Define $\mu_1: G_1 \rightarrow [0,1]$ by

$$\mu_1(x) = \begin{cases} 1 & \text{if } x \in \{0, \pm 2, \pm 4, \ldots\ldots\} \\ \frac{1}{2} & \text{if } x \in \{\pm 1, \pm 3, \pm 5, \ldots\ldots\ldots\} \end{cases}$$

Define $\mu_2: G_2 \rightarrow [0,1]$ by

$$\mu_2(x) = \begin{cases} \frac{1}{4} & \text{if } x = \pm i \\ \frac{1}{2} & \text{if } x = \pm 1 \end{cases}$$

Define $\mu_3: G_3 \rightarrow [0,1]$ by

$$\mu_3(x) = \begin{cases} \frac{1}{5} & \text{if } x = 4 \\ \frac{1}{2} & \text{if } x = 2 \\ \frac{1}{6} & \text{if } x = 1 \end{cases}$$

Hence there exists two fuzzy subgroups $\mu_1$of G_1, μ_2 of G_2 and μ_3 of G_3 such that $\mu = (\mu_1 \cup \mu_2 \cup \mu_3)$.

Definition 2.12([8])

A fuzzy point x_t is said to belong to (resp. be quasi-coincident with) a fuzzy set μ , written as $x_t \in \mu$ (resp. $x_t\ q\ \mu$) if $\mu(x) \geq t$(resp. $\mu(x) + t > 1$). ”$x_t \in \mu$ or $x_t\ q\ \mu$” will be denoted by $x_t \in vq\ \mu$).

Definition 2.13([2,3])

A fuzzy subset μ of G is said to be an ($\in$ vq)- fuzzy subgroup of G if for every x, y $\in$ G and t, r $\in$ (0, 1]:

(i) $x_t \in \mu$, $y_r \in \mu \Rightarrow (xy)M(t, r) \in vq\ \mu$

(ii) $x_t \in \mu \Rightarrow (x^{-1})_t \in vq\ \mu$.

Theorem 2.14([3])

(i) A necessary and sufficient condition for a fuzzy subset μ of a group G to be an ($\in$, $\in$ vq)-fuzzy subgroup of G is that $\mu(xy-1) \geq M(\mu(x), \mu(y), 0.5)$ for every x, y $\in$ G.

(ii). Let μ be a fuzzy subgroup of G. Then $\mu_t = \{x \in G : \mu(x) \geq t\}$ is a fuzzy subgroup of G for every $0 \leq t \leq 0.5$. Conversely, if μ is a fuzzy subset of G such that μt is a subgroup of G for every $t \in (0, 0.5]$, then μ is an fuzzy $(\in,\in vq)$-fuzzy subgroup of G.

Definition 2.15([3])

Let X be a non empty set. The subset $\mu_t = \{x \in X : \mu(x) \geq t\}$ or $\{\mu(x)+t > 1\}= \{x \in X : x_t \in vq\ \mu\}$ is called $(\in vq)$- level subset of X determined by μ and t.

Theorem 2.16([3])

A fuzzy subset μ of G is a fuzzy subgroup of G if and only if μ_t is a subgroup for all $t \in (0, 1]$.

3. MAIN RESULTS

Definition 3.1

Let $G = G_1 \cup G_2 \cup G_3$ be a tri-group. Let $\mu = \mu_1 \cup \mu_2 \cup \mu_3$ be a fuzzy tri-group. A fuzzy subset $\mu = \mu_1 \cup \mu_2 \cup \mu_3$ of the form:

$$\mu(x) = \begin{cases} M(t, s, r) \neq 0 & \text{if } x = y \in G, \\ 0 & \text{if } x \neq y \end{cases}$$

where $t, s, r \in [0, 1]$ such that

$$\mu_1(x) = \begin{cases} t \neq 0 & \text{if } x = y \in G_1, \\ 0 & \text{if } x \neq y \end{cases}$$

$$\mu_2(x) = \begin{cases} s \neq 0 & \text{if } x = y \in G_1, \\ 0 & \text{if } x \neq y \end{cases}$$

and

$$\mu_3(x) = \begin{cases} r \neq 0 & \text{if } x = y \in G_1, \\ 0 & \text{if } x \neq y \end{cases}$$

is said to be a fuzzy point of the tri-group G with support x and value M(t, s, r) and is denoted by $x_{M(t,s,r)}$.

Theorem 3.2

Let $x_{M(t,s,r)}$ be a fuzzy point of the bi-group $G = G_1 \cup G_2 \cup G_3$. Then:

(i) $x_{M(t,s,r)} = x_t \Leftrightarrow x \in G_1 \cap G_2^c \cap G_3^c$ or $t > s,r$

(ii) $x_{M(t,s)} = x_s \Leftrightarrow x \in \cap G_1^c \cap G_2 \cap G_3^c$ or $s > t,r$

(iii) $x_{M(t,s)} = x_r \Leftrightarrow x \in \cap G_1^c \cap G_2^c \cap G_3$ or $r < t, s$

$\forall\ t, s, r \in [0, 1]$, where x_t, x_s and x_r are fuzzy points of the groups G_1, G_2 and G_3 respectively.

Proof

(i) Suppose $x_{M(t,s,r)} = x_t$. Then $M(t, s, r) = t \Rightarrow t > s, r$. And $t > s, r \Rightarrow 0 \leq s < t$, $0 \leq r < t$. Hence, if $s = 0$ & $r = 0$ then $x \in G_1 \cap G_2^c \cap G_3^c$.

Conversely, suppose $x \in G_1 \cap G_2^c \cap G_3^c$, then $x \in G_1$ and $x \notin G_2 \& G_3$, which implies that $x_s = 0$ & $x_r = 0$.

Therefore $x_{M(t,s,r)} = x_t$. Also, if $t > 0$, $x_{M(t,s,r)} = x_t$.

Hence, the proof. (ii) & (iii) Similar to that of (i).

Definition : 3.3

A fuzzy point $x_{M(t,s,r)}$ of the tri-group $G = G_1 \cup G_2 \cup G_3$, is said to belong to (resp. be quasi coincident with) a fuzzy subset $\mu = (\mu_1 \cup \mu_2 \cup \mu_3)$ of G, written as $x_{M(t,s,r)} \in \mu$[resp. $x_{M(t,s,r)} q\mu$] if $\mu(x) \geq M(t, s)$(resp. $\mu(x) + M(t, s, r) > 1$). $x_{M(t,s,r)} \in \mu$ or $x_{M(t,s,r)}\, q\mu$ will be denoted by $x_{M(t,s,r)} \in vq\mu$.

Theorem : 3.4

Let $G = G_1 \cup G_2 \cup G_3$ be a tri-group. Let μ_1, μ_2 and μ_3 be fuzzy subsets of G_1, G_2 and G_3 respectively. Suppose that x_t, x_s and x_r are fuzzy points of the groups G_1, G_2 and G_3 repectively such that $x_t \in vq\mu_1$, $x_s \in vq\mu_2$ and $x_r \in vq\mu_3$, then $x_{M(t,s,r)} \in vq\mu$ where $x_{M(t,s,r)}$ is a fuzzy point of the tri-group G, and $\mu : G \to [0, 1]$ is such that $\mu = (\mu_1 \cup \mu_2 \cup \mu_3)$.

Proof

Suppose that $x_t \in vq\mu_1$, $x_s \in vq\mu_2$ and $x_r \in vq\mu_3$,

then we have that

$\mu_1(x) \geq t$ or $\mu_1(x) + t > 1$,

$\mu_2(x) \geq s$ or $\mu_2(x) + s > 1$.

and $\mu_3(x) \geq r$ or $\mu_3(x) + r > 1$.

$\mu_1(x) \geq t$, $\mu_2(x) \geq s$ and $\mu_3(x) \geq r \Rightarrow Max[\mu_1(x), \mu_2(x), \mu_3(x)] \geq M(t, s, r)$. This means that

$(\mu_1 \cup \mu_2 \cup \mu_3)(x) \geq M(t, s, r)$ since $x \in G_1 \cap G_2 \cap G_3$

That is $\mu(x) \geq M(t, s, r)$ (1)

Similarly, $\mu_1(x) + t > 1$, $\mu_2(x) + s > 1$ and $\mu_3(x) + r > 1$

imply that

$\mu_1(x) + t + \mu_2(x) + s + \mu_3(x) + r > 3$

$\Rightarrow 3Max[\mu_1(x), \mu_2(x), \mu_3(x)] + 3M[t, s, r] > 3$

$\Rightarrow Max[\mu_1(x), \mu_2(x), \mu_3(x)] + M[t, s, r] > 1$

$\Rightarrow (\mu_1 \cup \mu_2 \cup \mu_3)(x) + M(t, s, r) > 1$

$\Rightarrow \mu(x) + M(t, s, r) > 1$ (2)

Combining (1) and (2), it follows that:

$\mu(x) \geq M(t, s, r)$ or $\mu(x) + M(t, s, r) > 1$

which shows that $x_{M(t,s,r)} \in vq\mu$

hence, the proof.

Theorem 3.5

Let $G = G_1 \cup G_2 \cup G_3$ be a tri-group. $\mu = \mu_1 \cup \mu_2 \cup \mu_3$ a fuzzy subset of G, where μ_1, μ_2& μ_3 are fuzzy subsets of G_1, G_2 and G_3 respectively. Suppose that $x_{M(t,s,r)}$ is a fuzzy point of the tri-group G then $x_{M(t,s,r)} \in vq\mu$ does not imply that $x_t \in vq\mu_1$, $x_s \in vq\mu_2$ and $x_r \in vq\mu_3$, where x_t , x_s and x_r are fuzzy points of the groups G_1, G_2 and G_3, respectively.

Proof

Suppose that $x_{M(t,s,r)} \in vq\mu$, then

$\mu(x) \geq M(t, s, r)$ or $\mu(x) + M(t, s, r) > 1$

By definition, this implies that

$(\mu_1 \cup \mu_2 \cup \mu_3)(x) \geq M(t, s, r)$ or $(\mu_1 \cup \mu_2 \cup \mu_3)(x) + M(t, s, r) > 1$

$\Rightarrow \text{Max}[\mu_1(x), \mu_2(x), \mu_3(x)] \geq M(t, s, r)$ or $\text{Max}[\mu_1(x), \mu_2(x), \mu_3(x)]] + M(t, s, r) > 1$

Now, suppose that $t > s, r$, so that $M(t, s, r) = t$, we then have that

$\text{Max}[\mu_1(x), \mu_2(x), \mu_3(x)] \geq t$ or $(\text{Max}[\mu_1(x), \mu_2(x), \mu_3(x)] + t) > 1$

which means that $x_t \in vq\text{Max}[\mu_1, \mu_2, \mu_3]$, and by extended implication, we have that $x_s \in vq\text{Max}[\mu_1, \mu_2, \mu_3]$ and $x_r \in vq\text{Max}[\mu_1, \mu_2, \mu_3]$.

If we assume that $\text{Max}[\mu_1, \mu_2, \mu_3] = \mu_1$, then we have that

$x_t \in vq\mu_1$, $x_s \in vq\mu_1$ and $x_r \in vq\mu_1$,

and since $0 \leq r < s < t < 1$, we now need to show that at least $x_s \in vq\mu_2$ since by assumption, $\mu_1 > \mu_2 > \mu_3$. To this end, let the fuzzy subset μ_3 and the fuzzy singleton x_r be defined in such a way that $\mu_3 < r < 0.5$, then it becomes a straight forward matter to see that $x_s \bar{\in} vq\mu_2$. Even though, $x_{M(t,s,r)} \in vq\mu$ still holds. And the result follows accordingly.

Corollary 3.6

Let $G = G_1 \cup G_2 \cup G_3$ be a tri-group. $\mu = \mu_1 \cup \mu_2 \cup \mu_3$ a fuzzy subset of G, where μ_1, μ_2 and μ_3 are fuzzy subsets of G_1, G_2 and G_3 respectively. Suppose that $x_{M(t,s,r)}$ is a fuzzy point of the tri-group G then $x_{M(t,s,r)} \in vq\mu$ imply that $x_t \in vq\mu_1$, $x_s \in vq\mu_2$ and $x_r \in vq\mu_3$, if and only if $0.5 < \min[t, s, r] \leq \min[\mu_1(x), \mu_2(x), \mu_2(x)] < 1$ where x_t, x_s and x_r are fuzzy points of the groups G_1 , G_2 and G_3 respectively.

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ROUTING PROTOCOLS IN ADHOC NETWORK CONSIDERING VARYING NUMBER OF NODES AND PACKET SIZE

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ABSTRACT

Wireless network is said to be Adhoc when it is decentralized in nature that is it is not dependent on the pre-existing infrastructure.Routing Protocols in the Computer Network specifies the communication among various routers that enble them to select routes between the nodes on a Network.The data transmits in the form of Packets from one Router to the other.In this paper,we have studied the effect of changing the number of nodes and data packet size keeping the simulation time and maximum packets in the queue constant on the performance of various Routing Protocols in the Mobile Adhoc Network.The simulation work has been carried out on Network Simulator 2.35 and the Routing Protocols (Table-Driven and On-Demand) have been simulated with TCP as a Traffic Agent.

Keywords: *Simulation,DSDV,AODV,Throughput, Packet Loss, Delay,Jitter*

1. INTRODUCTION

An Adhoc Network is an autonomous System of Mobile Hosts(also known as Routers) connected by Wireless links that does not rely on the pre-existing infrastructure such as Routers or Access Points in a Network.In Mobile Adhoc Networks(MANETs),the Network Topology may dynamically change in an unpredictable manner since Nodes are free to move.Adhoc Networks work as peer-to-peer multi-hop mobile wireless networks where information packets are transmitted in a store-and-forward manner from a source to an arbitrary destination via intermediate nodes.The nodes move and thus the resulting change in the Network Topology is updated to the other nodes.It is not necessary that the nodes in this Network are within Radio Range of each other.This type of Network is very useful in Emergency Search and Rescue operations,meetings or conventions in which persons wish to quickly share the information.Routing in such Networks depend on many factors such as topology,selection of Routers as well as the characteristics that could find the path quickly and efficiently.Routing protocols are meant to select the routes between the various nodes but they face some challenges such as Mobility,Bandwidth,Scalability,Security so there is a need to study the different performance metrics that affect these protocols.The Adhoc Routing Protocols can be divided into two categories namely

Table-Driven and **On-Demand.**

In **Table-driven(Proactive)** routing protocols, each node maintains a routing table and continuously maintains up-to-date routes to every other node in the network by exchanging the information among all the nodes in the network. With **On-demand(Reactive)** routing protocols, if a source Node requires a route to the destination and it does not have the route to destination, so it initiates a route discovery process which goes from one node to the other until it reaches to the destination or an intermediate node that has a route to the destination. Mixture of both reactive routing protocol and proactive routing protocol yields a better solution known as **Hybrid** Routing Protocol.

Data in the network is transmitted in the form of packets and thus Transmission Control Protocol(TCP) acts as a Traffic Agent that provides end to end delivery of data over unreliable network in the form of packets.

2. RELATED WORK

Various researchers have done a lot of work to analyse the performance of Routing Protocols considering different parameters.In **2008**, Performance Evaluation of Ad Hoc Routing Protocols Using NS2 Simulation was done by **Samyak Shah and Khandre.**In **2009**, **Md. Rahman and Islam** measured the performance of various Routing Protocols in Ad-hoc Network.In **2010**,**Vijay Kumar and Reddy** surveyed Current Research Work on Routing Protocols for MANET.In **2011**,**Ismail and Hassan** studied the Effects of Packet Size on AODV Routing Protocol Implementation in Homogeneous and Heterogeneous MANET. In **2014**,**Gautam and Gulbir** compared TCP Performance over Routing Protocols for Mobile Ad Hoc Networks. In **2014**,**Hemant Rai** did a Comparison of Performance Metrics for various Routing Protocols in MANET. In **2014**, **Arwinder Singh and Amit Kumar** studied a Comparative Performance of Different TCP Variants over Routing Protocols in MANET using NS2.In **2017, Kling** studied the impact of transport protocol, packet size and connection type on the round trip time.We have extended the work of above Researchers to calculate the efficiency of the Routing Protocols(**Table-Driven** and **On-Demand**) by varying the Number of Nodes and Data Packet Size.

3. ROUTING PROTOCOLS

In this Section, we study different routing protocols in MANET.
In **DSDV (Destination Sequence Distance Vector)**, each mobile node in the network keeps a routing table. Each of the routing table includes all available destinations and the number of hops to reach that destination. Each entry in the routing table has a sequence number. If a Link is present then sequence number will be even otherwise odd number will be used. This number is generated by the destination

and the sender node should have to send out the next update with this number. In DSDV protocol, each mobile node in the network will send its routing table to its current neighbours. This is possible either by broadcasting or by multicasting.

In **AODV (Adhoc On-demand Distance Vector)**, it is an On-Demand routing Protocol. Each Node maintains only the next hop information of the route to destination. Destination sequence number is used to check the freshness of the route to destination. Periodic use of Beacons i.e. Hello packets used to check the presence of the neighbour. Each node uses a sequence number which is increased whenever the node observe a change in neighbour topology . Each node maintains a routing table and the information is stored as **<Destination IP address, Destination Sequence number, Next hop address, hop count to destination>.**

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4. SIMULATION ENVIRONMENT

The Research work has been carried out on different Routing Protocols using the following softwares:

I. Network Simulator 2.35

II. NS2 Scenarios Generator(NSG2.1)

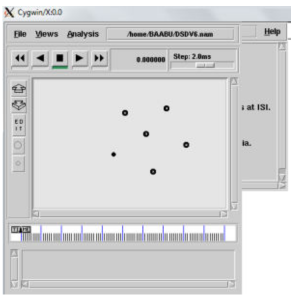
III. NS2-Visual Trace Analyser-0.2.72

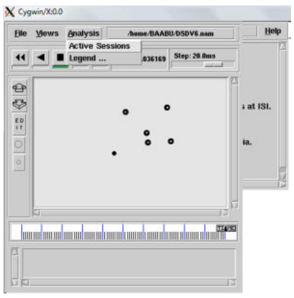
The performance of the Routing Protocols(**Table-Driven & On-Demand)** in Adhoc Network considering TCP as a Traffic-Agent has been simulated by increasing the number of Nodes and the Data packet Size in the Network.The Simulation Time and the maximum packets in the Queue has been kept constant.The Average Throughput, Delay, Packet Delivery Ratio and Jitter have been analysed for each protocol by changing the number of nodes amd the packet size.Following are the Experimental Results and finally the conclusion is made on the basis of these results in the upcoming section:

For N=6,18,24 and 36 screenshots for Network Animator are given below:

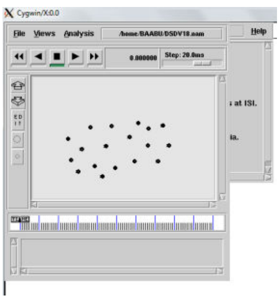
BEFORE SIMULATION **AFTER SIMULATION**

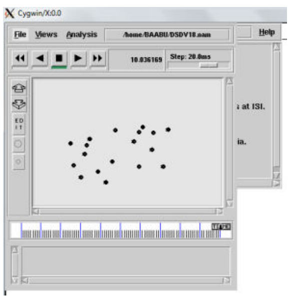
N=6



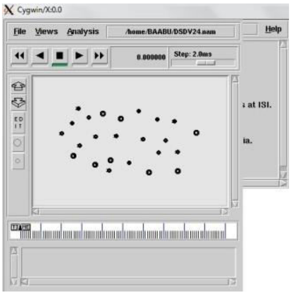


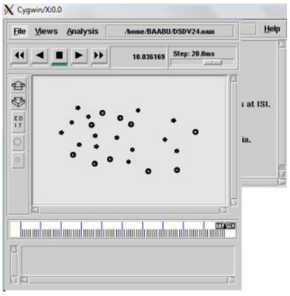
N=18



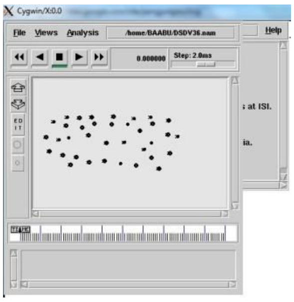


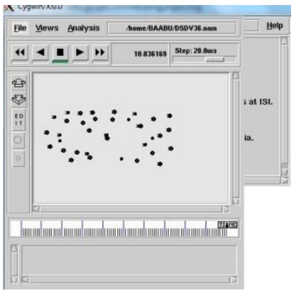
N=24

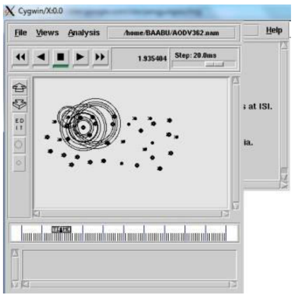




N=36





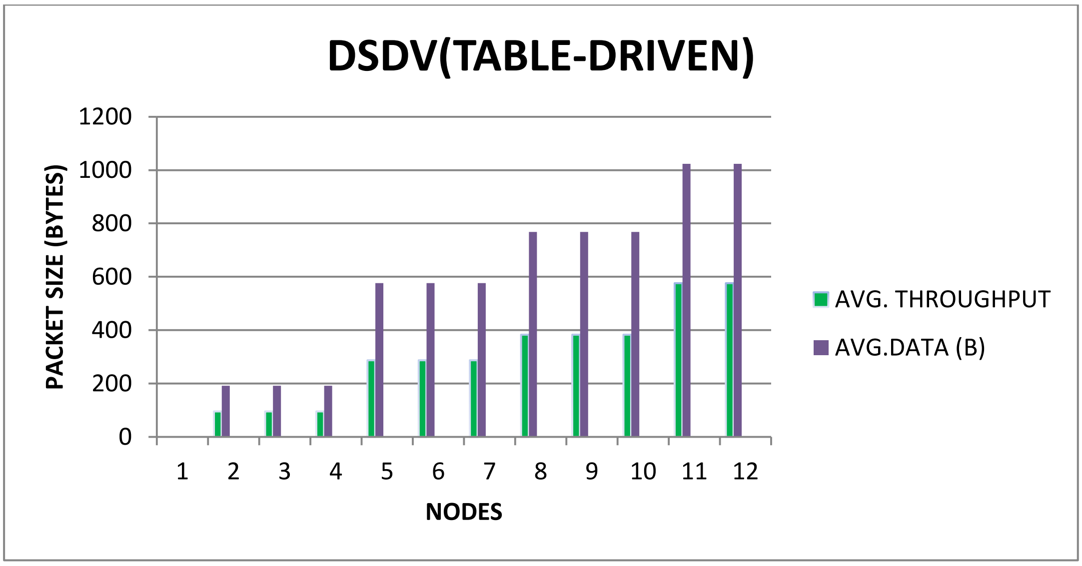


5. EXPERIMENTAL RESULTS AND ANALYSIS

The main objective of this paper is to compare the performance of DSDV and AODV Protocols by increasing the number of nodes and the packet size using following metrices:

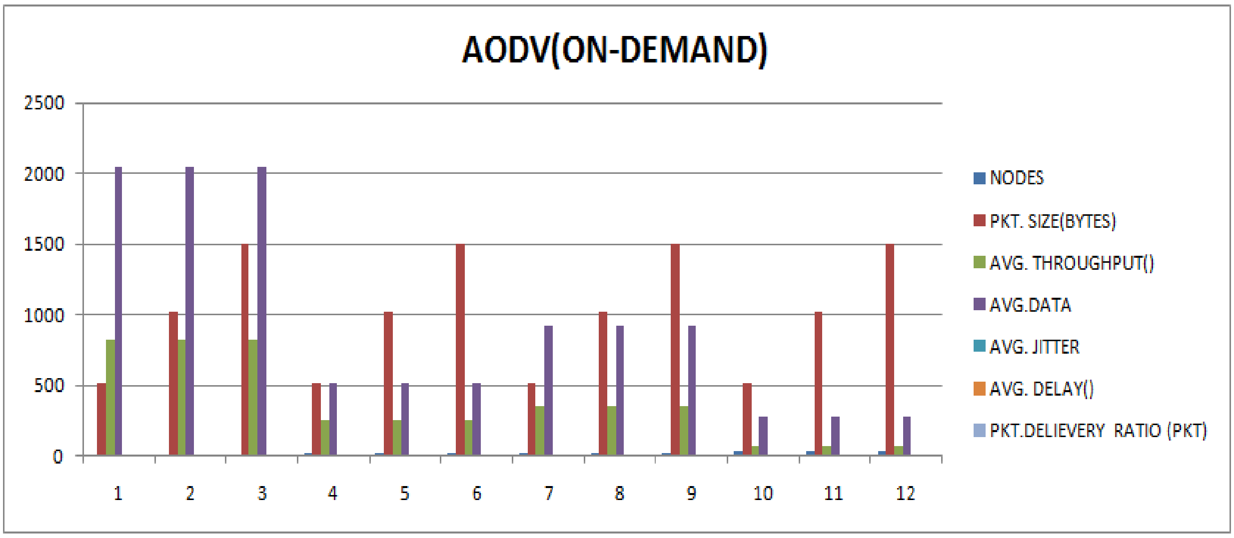
DSDV(TABLE-DRIVEN)						
NODES	**PKT. SIZE(BYTES)**	**AVG. THROUGHPUT (B/s)**	**AVG.DATA (B)**	**AVG. JITTER (s)**	**AVG. DELAY (s)**	**PKT.DELIVERY RATIO**
6	512	96	192	0	0.001	1
	1024	96	192	0	0.001	1
	1500	96	192	0	0.001	1
18	512	288	576	0	0.001	1
	1024	288	576	0	0.001	1
	1500	288	576	0	0.001	1
24	512	384	768	0	0.001	1
	1024	384	768	0	0.001	1

	1500	384	768	0	0.001	1
36	512	576	1024	0	0.001	1
	1024	576	1024	0	0.001	1
	1500	576	1024	0	0.001	1



AODV(ON-DEMAND)						
NODES	**PKT. SIZE(BYTES)**	**AVG. THROUGHPUT (B/s)**	**AVG.DATA (B)**	**AVG. JITTER (s)**	**AVG. DELAY (s)**	**PKT.DELIVERY RATIO**
6	512	820	2048	0.01	0.16	1
	1024	820	2048	0.01	0.16	1
	1500	820	2048	0.01	0.16	1
18	512	260	520	0.04	0.05	1
	1024	260	520	0.04	0.05	1
	1500	260	520	0.04	0.05	1
24	512	360	920	0.03	0.09	0.78
	1024	360	920	0.03	0.09	0.78

	1500	360	920	0.03	0.09	0.78
36	512	80	280	0.04	0.04	0.57
	1024	80	280	0.04	0.04	0.57
	1500	80	280	0.04	0.04	0.57



- **Throughput:**Throughput of the routing protocol means that in certain time the total size of useful packets received at all the destination nodes.
- **Average Data:** Rate of Successful Data transferred.
- **Average Jitter:** Variance in time delay between Data Packets over a Network.
- **Average Delay:**It defines how long it takes for a bit of data to travel across the network from one node to the endpoint.
- **Packet Delivery ratio :**The ratio of the data packets delivered to the destinations to those generated by the Constant Bit Rate (CBR) source is known as packet delivery fraction.

6. CONCLUSION AND FUTURE SCOPE

N=Increasing	**METRICES**	**DSDV PROTOCOL**	**AODV PROTOCOL**
	Avg. Throughput	Increase	Decrease
	Avg. Data	Increase	Decrease
Pkt. Size=Increasing	Avg. Jitter	Constant	Increase
	Avg. Delay	Constant	Decrease
	Pkt. Delivery Ratio	Constant	Decrease

From above table and analysis we conclude that:

- Increase of packet size does not have any effect on Throughput,Avg. Delay and Packet Delivery ratio in DSDV and AODV.
- With increase in Number of Nodes, the Average Data and hence the throughput of the Network with DSDV increases whereas AODV gives the highest throughput value if nodes are less and decreases with increase in number of nodes.
- Packet delivery ratio is constant in DSDV and decreases in AODV because of increase in traffic due to the heavy packet size.
- Average Jitter and Delay in the network through DSDV and AODV both are negligible.
- Hence,we conclude that the values of different performance metrics observed through AODV Protocol is better then DSDV Protocol.

There are some more routing protocols like DSR,TORA,ZRP and many more can be compared using the same analysis as above.Also these parameters can be simulated using Fuzzy for enhancing the efficiency and better throughput.

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FORECASTING HIV PREVALENCE IN INDIA AND STATES USING ONLINE SEARCH TRAFFIC DATA

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1. INTRODUCTION

Big data is generating a lot of hype in every industry including healthcare. In 2001 Doug Laney, now at Gartner coined the term "the 3 V's" to define big data–Volume, Velocity, and Variety. Other analysts have argued that this is too simplistic, and there are more things to think about when defining big data. They suggest more V's such as Variability and Veracity, and even a C for Complexity.

Google Trends is an online search tool that allows the user to see how often specific keywords, subjects and phrases have been queried over a specific period of time.

Google Trends works by analyzing a portion of Google searches to compute how many searches have been done for the terms entered, relative to the total number of searches done on Google over the same time. It's possible to query up to five words or topics simultaneously in Google Trends. Results are displayed in a graph that Google calls a "Search Volume Index" graph. Data in the graph can be exported into a .csv file, which can be opened in Excel and other spread sheet applications.

Numerous studies on the subject have revealed that Google Trend's data are related to public health data factually [1-17]. The topics that have been covered are Ebola, Measles, the Bud-Bug epidemic, tuberculosis, seasonal disease like influenza illness (the flu). Efforts have also been made to relate Google data with official health data. A study of forecasting AIDS prevalence in the United States using online search traffic data [18] aimed at introducing the novel approach in forecasting AIDS Prevalence in the US using data from Google Trends.

Developing country like India where the infections like HIV is still misunderstood and people hesitate to talk on this topic candidly, and not even visit to the doctor directly; rather they feel much comfortable in searching it on internet. The anonymity provided by the internet allows people to search for information online. Internet has brought stupendous speed, luxury, comfort, convenience, high productivity and eventually a great deal of profitability. Goole trend data has also become indispensable in every mode of life.

Google trend data is boon for the human kind. Data can be retrieved in real time and thus allow the now casting of human behaviour based on Internet data. Google Trends is being used to study health phenomena in a variety of topic domains in myriad ways. However, there is limited knowledge about its potential uses and limitations. We therefore took the initiative to emphasize the importance of Google trend data. The aim of this study is to provide a novel method of forecasting HIV prevalence in India and sates using online search traffic data from Google Trends on HIV.

2. RESEARCH METHODOLOGY

State wise data were downloaded from Google trends in .csv format and are normalized over the selected time-frame as follows: " Search results are proportionate to the time and location of a query: Each data point is divided by the total searches of the geography and time range it represents, to compare relative popularity. Otherwise places with the most search volume would always be ranked highest. The resulting numbers are then scaled on range of 0-100 based on a topic's proportion to all searches on all topics. Different regions that show the same number of searches for a term will not always have the same total search volumes."[19].

Over the past two decades, the HIV surveillance has expanded: the geographical unit of data generation, analysis and use for planning through HIV surveillance has shifted from the national to the State and district level [20]. Data on HIV Prevalence (2004-2017) has been fetched from National AIDS Control Organisation HIV Sentinel surveillance 2016-17 among different states and sentinel surveillance sites i.e. Ante Natal Clinic (ANC), Men who have sex with men (MSM), Female Sex workers (FSW) and Injecting Drug Users (IDU) .

Data Fetch from Google trend and from NACO has been downloaded in Microsoft Excel and after that correlation coefficients were calculated. To determine which regression model is best among Linear, Logarithmic, Inverse, Quadratic, Cubic, Compound, Power, S, Growth and exponential, a separate model is produced for each and on the basis of R^2 , which is the measure of goodness of fit of a model, the regression model is finalised for predictions.

The value of R^2 is between 0 and 100%. R^2 = 0% indicates that the model explains none of the variability of the response data around its mean. R^2 = 100% indicates that the model explains all the variability of the response data around its mean.

3. RESULTS

In the beginning, total evaluation of the online interest towards HIV in India is performed, followed by the exploring of the correlations between HIV prevalence and Google Trends data in India and each state individually. In totality forecasting models for HIV prevalence in India are estimated at both national and state level so as to enhance the importance of the tool in health assessment in India.

HIV online interest in INDIA

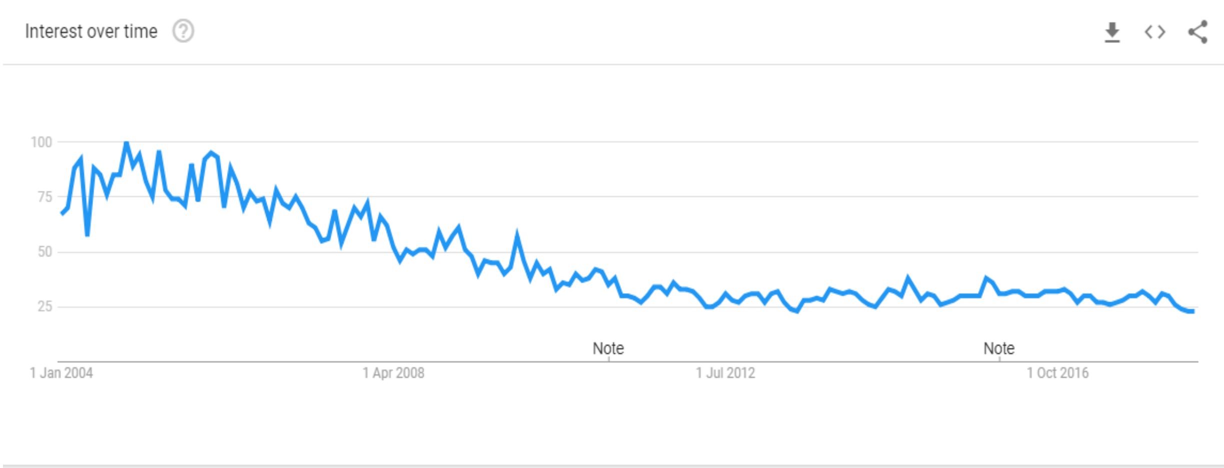


Figure 1

Figure 1 depicts the changes in the online interest in India which gradually decreases with passing years from January 2004-January 2017.

4. HIV PREVALENCE VS. GOOGLE TRENDS

In order to scrutinize the accountability of forecasting HIV prevalence in India, the relationships between online search traffic data from Google and official health data on HIV prevalence are at first examined, by calculating the respective correlations at both national and state level (Table 1). Depending upon the significance of the correlations, the possibility of forecasting HIV prevalence in India has been examined.

Statistically significant correlations are observed between online search traffic data and official health data. For HIV Prevalence, where the data was sufficient, all correlations are statistically significant. Therefore it is evident that online behaviour towards HIV follows that of HIV prevalence. Thus the states that exhibit statistically significant correlations are further selected for the forecasting of HIV in India. After exploring Correlation coefficients, it has been observed that data of India and six states in ANC, Five states in FSW, three states in MSM and one state in IDU was found sufficient for regression analysis.

Table-1 : Pearson correlation coefficients between HIV prevalence and HIV Google search data from 2004-2017

Sate	ANC	FSW	MSM	IDU
Andhra Pradesh	0.927978	0.823908087		
Assam				0.730662
Delhi		0.872858	0.71295	
Gujarat		0.932375		
Karnataka	0.977422	0.833871		
Kerala	0.774871		0.790552	
Maharashtra	0.989117	0.876407		
Tamil Nadu	0.902172		0.878099	
West Bengal	0.734577			
India	0.961385	0.944789	0.851352	0.732921

5. FORECASTING HIV PREVALENCE IN INDIA

The next step is to examine the relationships between Google data and HIV data and estimate the forecasting models. A separate model is produced for each dependent variable which exhibit that relationship is Cubic and of the form $y=a+b\,x+cx^2+dx^3$ (y-axis-dependent variable).Where y denotes the HIV prevalence. x (x-axis-independent variable) denotes the respective Google trend data, ‘a’, ‘b’, ‘c’ and ‘d’ are constants. Further to enhance the strength of the estimated models, the R2 is selected, as it is the statistical measure by which the variable variation is explained. R-squared is always between 0 and 100%.

In India the estimated models for HIV prevalence among different sites have an R^2 of 0.984 for ANC sites and 0.947 for FSW sites, which shows that the relationship between HIV prevalence and Google trend data is well described using the estimated equations and that HIV prevalence can be predicted based on online search traffic data from Google. Furthermore, most states models among different sites show high R^2, which is evident of the significance of the estimated forecasting models of HIV prevalence in India.

6. DISCUSSION

The HIV epidemic attacks the body's immune system, specifically the CD4 (T cell) which helps the immune system to fight off infections. HIV is a critical health issue that need to be seriously analysed expediently. Google trend data plays a profound role in dealing with sensitive issue like HIV especially in developing countries like India where people still stammer to talk on this topic. In this study we provide a novel approach of monitoring online search traffic data retrieved from Google trends in order to develop forecasting models for HIV Prevalence in India. Study clearly exhibit that there is a significant correlations with official data on HIV prevalence and Google trend.

Outcome of our study support the research over the last decade presenting that factual relationship immensely exist between Google trends data and Official health data [1, 3, 12-14, 21-27]. Therefore, the forecasting of HIV prevalence is feasible, as the estimated models for numerous States are robust despite the limitation of data being available for only 13 years.

Table 2, 3, 4 and 5 consists of the coefficients and the R^2 for the estimated forecasting cubic model of the form $y=a+bx+cx^2+dx^3$ for states that exhibit high significance in all four categories, i.e. HIV prevalence among ANC sites, HIV Prevalence in FSW sites, HIV prevalence in MSM sites and HIV Prevalence in IDU sites.

Figure 2-20 depicts the cubic model fit and the observed data for the states, all India and for the different surveillance sites. It is evident from the graphs the cubic model follows the observed data well, within the observed time period.

Table 2:HIV Prevalence in ANC

States	R^2	Sig.	Constant	b1	b2	b3
Andhra Pradesh	.923	.003	-.729	.176	-.005	5.352E-5
Karnataka	.971	.000	-.266	.136	-.005	7.215E-5
Kerala	.755	.055	-.089	.027	.000	-2.260E-5
Maharashtra	.988	.000	.038	.040	-.001	5.393E-6
Tamil Nadu	.818	.027	.151	.024	-.001	9.218E-6
West Bengal	.870	.012	.397	-.096	.010	.000
India	.984	.000	.001	.056	-.002	3.520E-5

Table 3:HIV prevalence in FSW

States	R^2	Sig.	Constant	b1	b2	b3
Andhra Pradesh	.894	.056	-24.432	4.328	-.170	.002
Delhi	.768	.176	1.213	-.026	.006	-9.447E-5
Gujarat	.882	.066	-.688	.588	-.008	1.762E-5
Karnataka	.735	.213	-7.660	2.997	-.188	.004
Maharashtra	.783	.159	9.827	-1.407	.099	-.001
India	.947	.020	-2.922	.860	-.034	.000

Table 4:Hiv Prevalence in MSM

States	R^2	Sig.	Constant	b1	b2	b3
Delhi	.565	.418	-2.782	.999	-.020	.000
Kerala	.942	.023	-2.339	.761	-.054	.001
Tamil Nadu	.946	.021	-3.370	.832	-0.25	.000
India	.929	.031	-3594	1.179	-.043	.001

Table 5:Hiv prevalence in IDU

States	R^2	Sig.	Constant	b1	b2	b3
Assam	.680	.275	2.127	-.943	.257	-.013
India	.835	.108	.689	1.131	-.052	.001

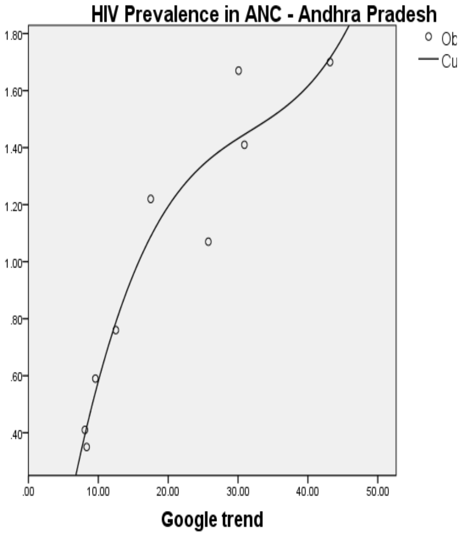
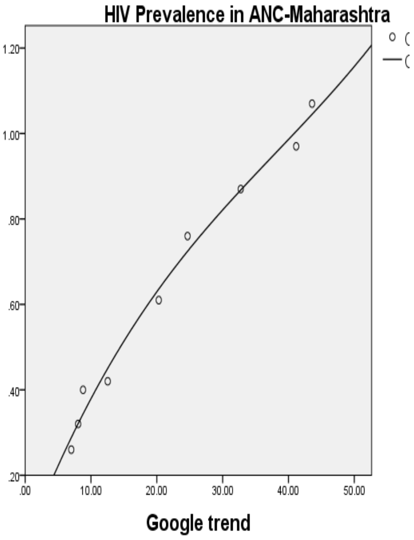


Figure 2



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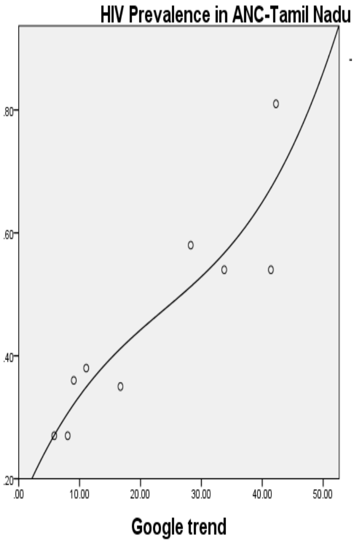

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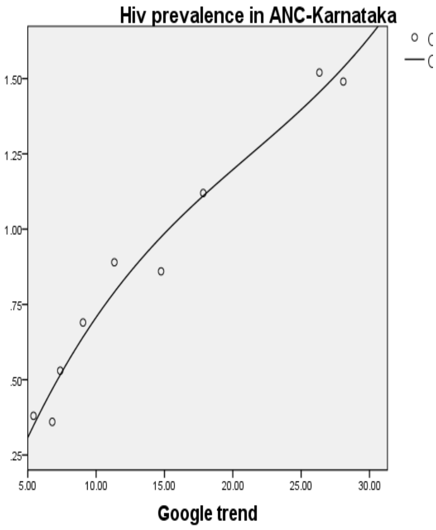

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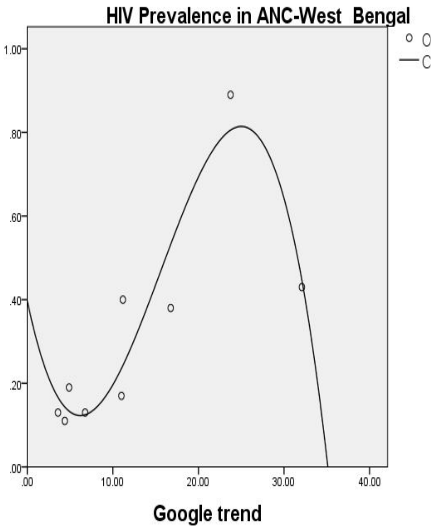

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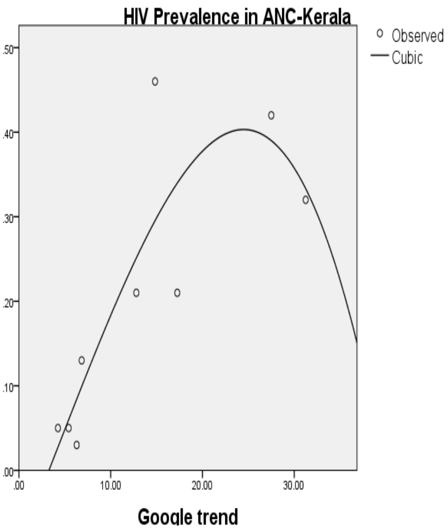

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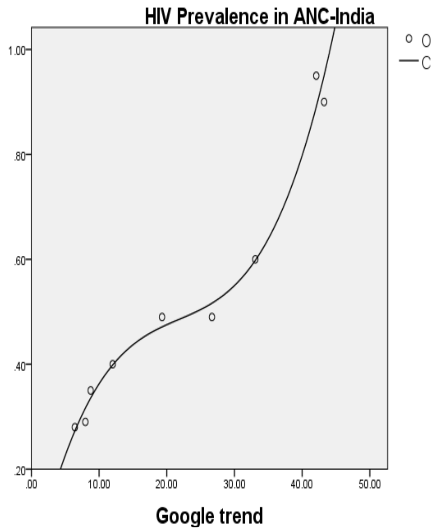

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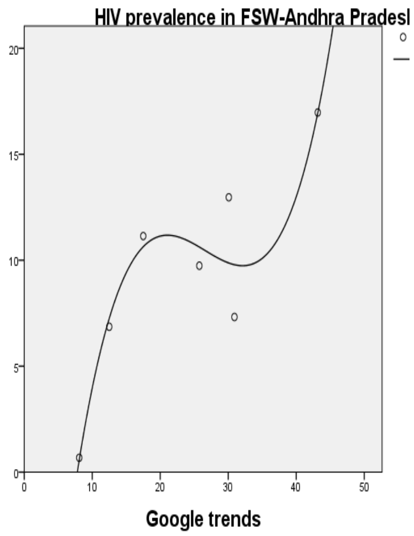

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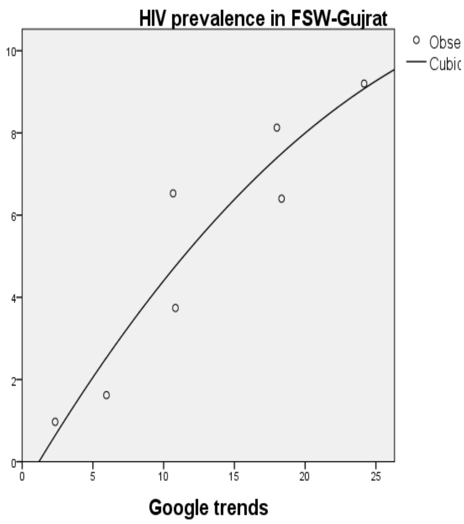

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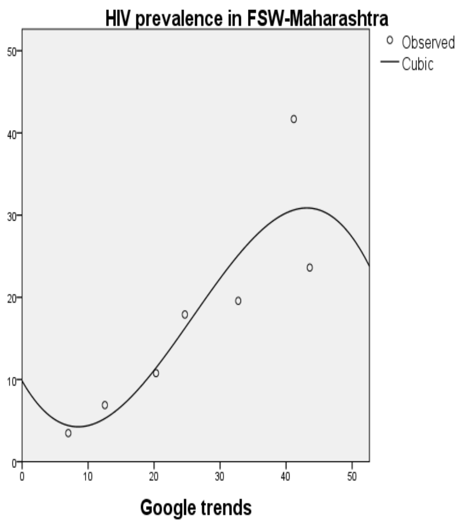

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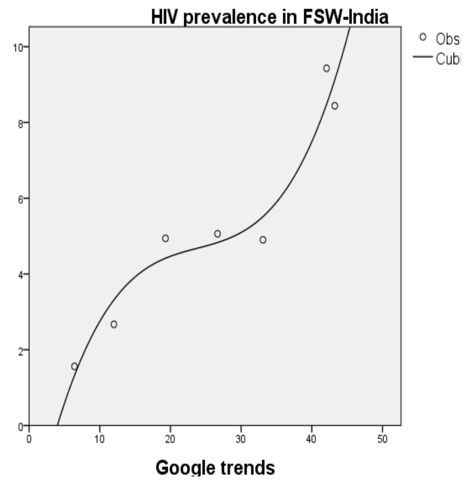

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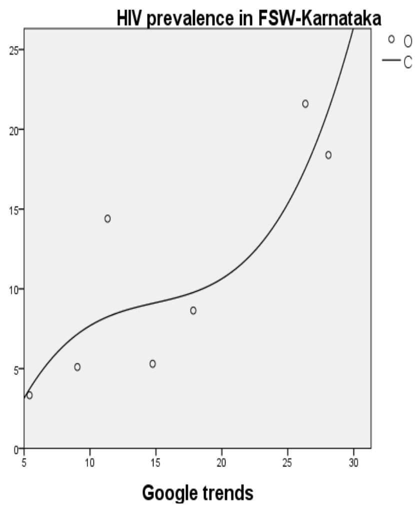

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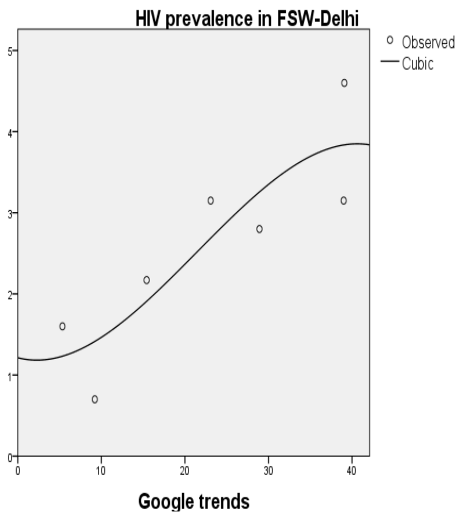

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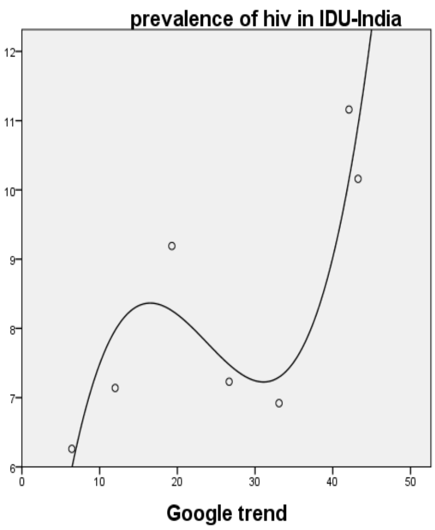

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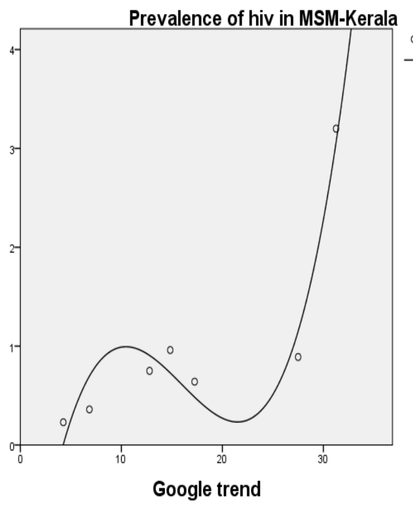

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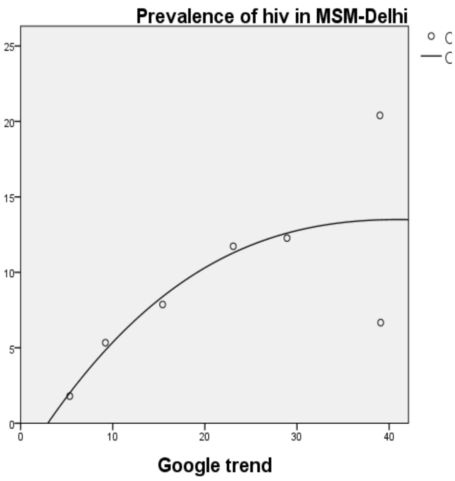

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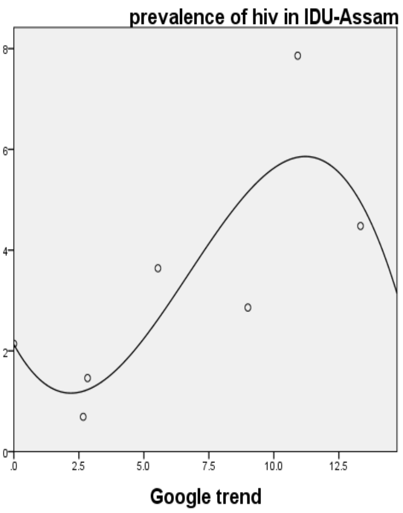

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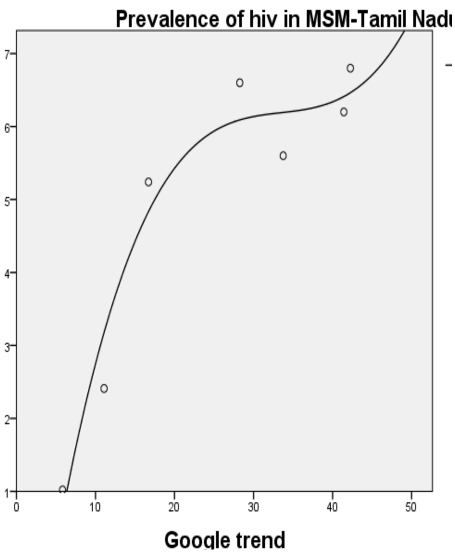

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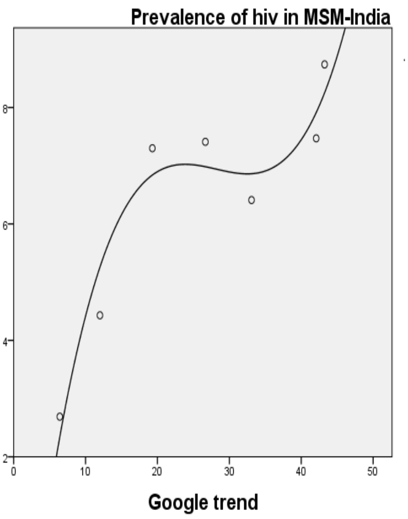

Figure 20

Moreover, this study has some constraints. Outcome of estimated forecasting model is based on only 13 years data, thus the tenacity of the models will increase when more years or smaller interval data are made officially available. Simultaneously we cannot argue that each hit on HIV related Keyword relates to HIV case and visa-versa, as hits can also be ascribed to general or academic interest, due to an event, incident or distinguished figure that announces something related to the disease. Significance of the data will be augmented when different people of different states will take keen interest in online search.

Overall the study exhibit that Google trend data is of immense importance and can be used when some kind of discrepancy occur. There are some kinds of researches which require colossal amount of money and due to this reason research remain uncompleted, in these types of researches Google trend data is of tremendous importance, as real time data can be retrieved any time and from anywhere.

7. CONCLUSION

It is crystal clear through this study that Google trend data is a big bolster in analysis, monitoring and forecasting of several health topics. Study exhibit that there is a significant correlation between Google trend data and official health data on HIV (2004-2017) and high significance of the estimated forecasting models in Indian states, support previous work on the subject suggesting that Google Trends' data have been shown to be factually related to health data and that they can help with the analysis, monitoring, and forecasting of several health topics. This study, however, also addresses a more important issue; that of Finance.

Hence, surveillance of the keenness towards states with increased rate of HIV prevalence is intrinsic so that endeavour should be made by health officials to make necessary information available on the internet always. People should be made more aware by organizing events which facilitates free blood test and information about the infection. Moreover, application of this method can be used in future research in different countries and regions, simultaneously data fetched by other online sources should be taken into consideration.

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TWO DIMENSIONAL FOURIER-LAPLACE TRANSFORMS AND TESTING FUNCTION SPACES

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ABSTRACT :

In mathematical analysis and applications, multidimensional transforms are used to analyze the frequency content of signals in a domain of two or more dimensions. One of the more popular multidimensional transforms is the Fourier transform, which converts a signal from a time/space domain representation to a frequency domain representation. The two dimensional Laplace transformation is useful in the solution of non-homogeneous partial differential equations.

The proposed paper provides the generalization of two dimensional Fourier-Laplace transform in the distributional sense. Different types of testing function spaces by using Gelfand-Shilov technique and some topological properties are defined.

Keywords: Fourier Transform, Laplace Transform, Two Dimensional Fourier-Laplace Transform, Generalized function.

1. INTRODUCTION

The integral transforms provide both theory and applications for science and engineering. Integral transform methods are at the heart of engineering curriculum. In spite all that things their uses are still predominant in advanced study and research. So, these integral transforms are attracting to all researchers, since their fabulous work and applications in almost all fields. Characteristically, one uses the integral transformation as mathematical or physical tool to alter the problems into one that can be solved. Integral transform method providing unifying mathematical approach to the study of electrical, network, devices for energy conversion and control, antennas and other component of electrical system [1]. Also they are widely applied to solve several different type of differential and integro-differential equations [2]. In the literature there are several multidimensional integral transforms instead of one dimensional integral transform the two dimensional integral transforms have several applications.

The two dimensional Fourier Transform is used in the field of exploration geophysics, image filtering in the frequency domain. So far we have focused pretty much exclusively on the application of Fourier analysis to time-series, which by definition are one-dimensional. However, Fourier techniques are equally applicable to spatial data and here they can be applied in more than one dimension. Two-dimensional Fourier transforms are used extensively in the processing of potential field data (gravity and magnetics), are a useful tool for looking at topography/bathymetry or any variable that we might plot on a map, and are also used in reflection seismology to look at record sections in which one variable is time and the other is spatial location.

In addition, the theory of the Laplace transformation introduces a large number of additional rules and methods that are important in the analysis of problems in the wide variety of branches of engineering and physics. One dimensional wave equation involving special functions and boundary value problems are solved by using Two-dimensional Laplace transforms [3], [4], [5].

In the present paper, Two dimensional Fourier-Laplace transform is extended in the distributional sense. The plan of the paper is as follows:- The definitions are given in section 2. In section 3, twelve testing function space are defined by Gelfand-Shilov technique. Section 4 describes the distributional generalized two dimensional Fourier-Laplace transforms (2DFLT). In section 5 some topological properties are discussed. Lastly we conclude the paper.

The notations and terminology are as per Zemanian [7], [8].

2. DEFINITIONS

The Two Dimensional Laplace transform with the parameters $p,\ v,$ of function $f\left(x,y\right)$ denoted by $L\left[f\left(x,y\right)\right]=F\left(p,v\right)$ and is given by,

$$L\left[f\left(x,y\right)\right]=F\left(p,v\right)=\int_0^\infty\int_0^\infty e^{-px-vy}f\left(x,y\right)dxdy \tag{2.1}$$

The Two Dimensional Fourier transform with the parameters $s,\ u,$ of function $f\left(t,z\right)$ denoted by $F\left[f\left(t,z\right)\right]=F\left(s,u\right)$ and is given by,

$$F\left[f\left(t,\mathrm{z}\right)\right]=F\left(s,u\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-i(st+uz)}f\left(t,z\right)dtdz \tag{2.2}$$

The Two Dimensional Fourier-Laplace transform with parameters $s,\ u,\ p,\ v,$ of $f\left(t,z,x,y\right)$ is defined as,

$$FL\left\{f\left(t,z,x,y\right)\right\}=F\left(s,u,p,v\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^\infty\int_0^\infty f\left(t,z,x,y\right)e^{-i\{(st+uz)-i(px+vy)\}}dtdzdxdy \tag{2.3}$$

where the kernel $K\left(s,u,p,v\right)=e^{-i\{(st+uz)-i(px+vy)\}}$

3. VARIOUS TESTING FUNCTION SPACES:-

3.1. The Space $FL_{a,b,c,d,\alpha}$ (S_α -type space):

Let I be the open set in $R_+\times R_+$ and E_+ denotes the class of infinitely differentiable function defined on I , the space $FL_{a,b,c,d,\alpha}$ is given by

$$FL_{a,b,c,d,\alpha}=\Big\{\phi:\phi\in E_+\,/\,\gamma_{a,b,c,d,k,r,q,m,l,n}\left[\phi\left(t,z,x,y\right)\right]$$
$$=\sup_{I_1}\left|t^k z^r K_{a,b}\left(x\right)R_{c,d}\left(y\right)D_t^l D_x^q D_z^n D_y^m\left[\phi\left(t,z,x,y\right)\right]\right|\le C_{l,q,n,m}A^k k^{k\alpha}B^r r^{r\alpha}\Big\}$$

Where, the constants A,B and $C_{l,q,n,m}$ depend on the testing function ϕ .

3.2. The Space $FL_{a,b,c,d}^{\beta}$ (S^{β} -type space):

This space is given by,

$$FL_{a,b,c,d}^{\beta} = \{\phi : \phi \in E_{+} / \sigma_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$= \sup_{I_1}\left|t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m \left[\phi(t,z,x,y)\right]\right| \le C_{k,r,q,m} G^l l^{l\beta} H^n n^{n\beta}\Bigg\}$$

Where, the constants G, H and $C_{k,r,q,m}$ depend on the testing function ϕ.

3.3. The Space $FL_{a,b,c,d,\alpha}^{\beta}$ (S_{α}^{β} -type space):

This space is formed by combining the conditions (3.1) and (3.2)

$$FL_{a,b,c,d,\alpha}^{\beta} = \{\phi : \phi \in E_{+} / \rho_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$= \sup_{I_1}\left|t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m \left[\phi(t,z,x,y)\right]\right| \le C A^k k^{k\alpha} B^r r^{r\alpha} G^l l^{l\beta} H^n n^{n\beta}\Bigg\}$$

where, the constants A, B, G, H and C depend on the testing function ϕ.

3.4. The space $FL_{a,b,c,d,\gamma}$ (S_{γ} -type space):

This space is given by,

$$FL_{a,b,c,d,\gamma} = \{\phi : \phi \in E_{+} / \xi_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$= \sup_{I_1}\left|t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m \left[\phi(t,z,x,y)\right]\right| \le C_{k,r,l,n} A^q q^{q\gamma} B^m m^{m\gamma}\Bigg\}$$

where, the constants A, B and $C_{k,r,l,n}$ depend on the testing function ϕ.

3.5. The Space $FL_{a,b,c,d,\alpha,g,h}$:

It is defined as,

$$FL_{a,b,c,d,\alpha,g,h} = \{\phi : \phi \in E_{+} / \gamma'_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$= \sup_{I_1}\left|t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m \left[\phi(t,z,x,y)\right]\right| \le C_{l,q,n,m,\delta,\mu}(g+\delta)^k k^{k\alpha}(h+\mu)^r r^{r\alpha}\Bigg\}$$

For any $\delta > 0$, $\mu > 0$ and g, h are the constants depending on the function ϕ.

3.6. The space $FL_{a,b,c,d}^{\beta,i,j}$:

It is defined as,

$$FL_{a,b,c,d}^{\beta,i,j} = \{\phi : \phi \in E_+ / \sigma'_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$= \sup_{I_1}\left|t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m\left[\phi(t,z,x,y)\right]\right| \le C_{k,r,q,m,\varepsilon,\eta}(i+\varepsilon)^l l^{l\beta}(j+\eta)^n n^{n\beta}\Big\}$$

For any $\varepsilon > 0,\ \eta > 0$ and i, j are the constants depending on the function ϕ.

3.7. The Space $FL_{a,b,c,d,\alpha,g,h}^{\beta,i,j}$:

This space is defined by combining (3.5) and (3.6) as,

$$FL_{a,b,c,d,\alpha,g,h}^{\beta,i,j} = \{\phi : \phi \in E_+ / \rho'_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$= \sup_{I_1}\left|t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m\left[\phi(t,z,x,y)\right]\right|$$

$$\le C_{\delta,\mu,\varepsilon,\eta}(g+\delta)^k k^{k\alpha}(h+\mu)^r r^{r\alpha}(i+\varepsilon)^l l^{l\beta}(j+\eta)^n n^{n\beta}\Big\}$$

For any $\delta > 0,\ \mu > 0\ \varepsilon > 0,\ \eta > 0$ and for given g, h, i, j are the constants depending on the function ϕ.

3.8. The Space $FL_{a,b,c,d,\gamma,e,f}$:

This space is given by,

$$FL_{a,b,c,d,\gamma,e,f} = \{\phi : \phi \in E_+ / \xi'_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$= \sup_{I_1}\left|t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m\left[\phi(t,z,x,y)\right]\right| \le C_{k,r,l,n,\tau,\omega}(e+\tau)^q q^{q\gamma}(f+\omega)^m m^{m\gamma}\Big\}$$

For any $\tau > 0,\ \omega > 0$ and e, f are the constants depending on the function ϕ.

3.9. The Space $F^{\nu}L_{a,b,c,d,\alpha}$:

It is given by,

$$F^{\nu}L_{a,b,c,d,\alpha} = \{\phi : \phi \in E_- / \zeta_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$= \sup_{\substack{-\infty < t,z < 0 \\ 0 < x,y < \infty}}\left|(-t)^k(-z)^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m\left[\phi(t,z,x,y)\right]\right| \le C_{l,q,n,m} A^k k^{k\alpha} B^r r^{r\alpha}\Big\}$$

The smooth function $\phi(t,z,x,y)$ defined on I_2 is in $F^{\nu}L_{a,b,c,d,\alpha}$ if $\phi^{\nu}(t,z,x,y) = \phi(-t,-z,x,y)$ is in $FL_{a,b,c,d,\alpha}$

3.10. The Space $F^{\nu}L^{\beta}_{a,b,c,d}$:

We define this space as,

$$F^{\nu}L^{\beta}_{a,b,c,d}=\{\phi:\phi\in E_{-}/\lambda_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$=\sup_{\substack{-\infty<t,z<0\\0<x,y<\infty}}\left|(-t)^{k}(-z)^{r}K_{a,b}(x)R_{c,d}(y)D_t^lD_x^qD_z^nD_y^m\left[\phi(t,z,x,y)\right]\right|\le C_{k,r,q,m}G^l l^{l\beta}H^n n^{n\beta}\Bigg\}$$

Here $\phi^{\nu}(t,z,x,y)=\phi(-t,-z,x,y)$ is in $F^{\nu}L^{\beta}_{a,b,c,d}$

3.11. The Space $F^{\nu}L^{\beta}_{a,b,c,d,\alpha}$:

Combining the conditions (3.9) and (3.10) we get

$$F^{\nu}L^{\beta}_{a,b,c,d,\alpha}=\{\phi:\phi\in E_{-}/\kappa_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$=\sup_{\substack{-\infty<t,z<0\\0<x,y<\infty}}\left|(-t)^{k}(-z)^{r}K_{a,b}(x)R_{c,d}(y)D_t^lD_x^qD_z^nD_y^m\left[\phi(t,z,x,y)\right]\right|\le CA^k k^{k\alpha}B^r r^{r\alpha}G^l l^{l\beta}H^n n^{n\beta}\Bigg\}$$

where the constants A,B,G,H and C depend on the testing function ϕ.

3.12. The Space $F^{\nu}L_{a,b,c,d,\gamma}$:

It is given by,

$$F^{\nu}L_{a,b,c,d,\gamma}=\{\phi:\phi\in E_{-}/\psi_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$=\sup_{\substack{0<t,z<\infty\\-\infty<x,y<0}}\left|t^k z^r K_{a,b}(-x)R_{c,d}(-y)D_t^lD_x^qD_z^nD_y^m\left[\phi(t,z,x,y)\right]\right|\le C_{k,r,l,n}A^q q^{q\gamma}B^m m^{m\gamma}\Bigg\}$$

where the constants A,B and $C_{k,r,l,n}$ depend on the testing function ϕ.

The spaces introduced from (3.1) to (3.12) be considered equipped with their natural Hausdorff locally convex topologies to be denoted respectively by
$T_{a,b,c,d,\alpha}$, $T^{\beta}_{a,b,c,d}$, $T^{\beta}_{a,b,c,d,\alpha}$, $T_{a,b,c,d,\gamma}$, $T_{a,b,c,d,\alpha,g,h}$, $T^{\beta,i,j}_{a,b,c,d}$, $T^{\beta,i,j}_{a,b,c,d,\alpha,g,h}$, $T_{a,b,c,d,\gamma,e,f}$, $T^{\nu}_{a,b,c,d,\alpha}$, $T^{\nu,\beta}_{a,b,c,d}$, $T^{\nu,\beta}_{a,b,c,d,\alpha}$ and $T^{\nu}_{a,b,c,d,\gamma}$.

These topologies are respectively generated by the total families of semi-norms.

$\{\gamma_{a,b,c,d,k,r,q,m,l,n}\}$, $\{\sigma_{a,b,c,d,k,r,q,m,l,n}\}$, $\{\rho_{a,b,c,d,k,r,q,m,l,n}\}$, $\{\xi_{a,b,c,d,k,r,q,m,l,n}\}$, $\{\gamma'_{a,b,c,d,k,r,q,m,l,n}\}$, $\{\sigma'_{a,b,c,d,k,r,q,m,l,n}\}$, $\{\rho'_{a,b,c,d,k,r,q,m,l,n}\}$, $\{\xi'_{a,b,c,d,k,r,q,m,l,n}\}$, $\{\zeta_{a,b,c,d,k,r,q,m,l,n}\}$, $\{\lambda_{a,b,c,d,k,r,q,m,l,n}\}$, $\{\kappa_{a,b,c,d,k,r,q,m,l,n}\}$, $\{\psi_{a,b,c,d,k,r,q,m,l,n}\}$.

4. DISTRIBUTIONAL GENERALIZED TWO DIMENSIONAL FOURIER-LAPLACE TRANSFORM (2DFLT):-

For $f(t,z,x,y)\in FL^{*}_{a,b,c,d,\alpha}$, where $FL^{*}_{a,b,c,d,\alpha}$ is the dual space of $FL_{a,b,c,d,\alpha}$. It contains all distributions of compact support. The distributional two dimensional Fourier-Laplace transform is a function of $f(t,z,x,y)$ is defined as,

$$FL\{f(t,z,x,y)\}=F(s,u,p,v)=\langle f(t,z,x,y),\phi(t,z,x,y,s,u,p,v)\rangle, \tag{4.1}$$

Where $\phi(t,z,x,y,s,u,p,v)=e^{-i\{(st+uz)-i(px+vy)\}}$ and for each fixed $t(0<t<\infty)$, $z(0<z<\infty)$, $x(0<x<\infty)$ and $y(0<y<\infty)$. Also $s>0,\ u>0,\ p>0$ and $v>0$. The right hand side of (4.1) has a sense as an application of $f(t,z,x,y)\in FL^{*}_{a,b,c,d,\alpha}$ to $\phi(t,z,x,y,s,u,p,v)\in FL_{a,b,c,d,\alpha}$.

5. TOPOLOGICAL PROPERTIES:-

5.1. Theorem: For a real numbers $a_1, a_2, b_1, b_2, c_1, c_2, d_1$ and d_2 such that $a_1\le a_2$, $b_2\le b_1$, $c_1\le c_2$ and $d_2\le d_1$ then $2DFL_{a_2,b_2,c_2,d_2,\alpha}\subset 2DFL_{a_1,b_1,c_1,d_1,\alpha}$ and the induced topology on $2DFL_{a_2,b_2,c_2,d_2,\alpha}$ is weaker than the original topology that is $T_{a_1,b_1,c_1,d_1,\alpha}/2DFL_{a_2,b_2,c_2,d_2,\alpha}\subset T_{a_2,b_2,c_2,d_2,\alpha}$

Proof:-Consider,

$$\gamma_{a_1,b_1,c_1,d_1,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]=\sup_{I_1}\left|t^k z^r K_{a_1,b_1}(x)R_{c_1,d_1}(y)D_t^l D_x^q D_z^n D_y^m\left[\phi(t,z,x,y)\right]\right|$$

$$\le\sup_{I_1}\left|t^k z^r K_{a_2,b_2}(x)R_{c_2,d_2}(y)D_t^l D_x^q D_z^n D_y^m\left[\phi(t,z,x,y)\right]\right|$$

$$=\gamma_{a_2,b_2,c_2,d_2,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

Hence $2DFL_{a_2,b_2,c_2,d_2,\alpha}\subset 2DFL_{a_1,b_1,c_1,d_1,\alpha}$ if $a_1\le a_2$, $b_2\le b_1$, $c_1\le c_2$ and $d_2\le d_1$.

Second part of the proof is simple and hence omitted. This completes the proof.

5.2. Theorem: If $\alpha_1<\alpha_2$ and $\beta_1<\beta_2$ then $FL^{\beta_1}_{a,b,c,d,\alpha_1}\subset FL^{\beta_2}_{a,b,c,d,\alpha_2}$ and the topology of $FL^{\beta_1}_{a,b,c,d,\alpha_1}$ is equivalent to the topology induced on $FL^{\beta_1}_{a,b,c,d,\alpha_1}$ by $FL^{\beta_2}_{a,b,c,d,\alpha_2}$.

Proof:- Let $\phi\in FL^{\beta_1}_{a,b,c,d,\alpha_1}$ therefore

$$\rho_{a,b,c,d,k,r,q,m,l,n}(\phi) = \sup_{I_1} \left| t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m \left[\phi(t,z,x,y) \right] \right|$$

$$\leq C A^k k^{k\alpha_1} . B^r r^{r\alpha_1} . G^l l^{l\beta_1} . H^n n^{n\beta_1}$$

$$\leq C A^k k^{k\alpha_2} . B^r r^{r\alpha_2} . G^l l^{l\beta_2} . H^n n^{n\beta_2} \text{ where } k,r,q,m,l,n = 0,1,2,3,.........$$

Hence $\phi \in FL^{\beta_2}_{a,b,c,d,\alpha_2}$.

Consequently, $FL^{\beta_1}_{a,b,c,d,\alpha_1} \subset FL^{\beta_2}_{a,b,c,d,\alpha_2}$. The topology of $FL^{\beta_1}_{a,b,c,d,\alpha_1}$ is equivalent to the topology $T^{\beta_2}_{a,b,c,d,\alpha_2} / FL^{\beta_2}_{a,b,c,d,\alpha_2}$. It is clear from the definition of the topologies of these spaces.

5.3. Theorem: $FL_{a,b,c,d} = \bigcup_{\alpha_i,\beta_i=1}^{\infty} FL^{\beta_i}_{a,b,c,d,\alpha_i}$ and if the space $FL_{a,b,c,d}$ is equipped with the strict $FL_{a,b,c,d}$ inductive limit topology defined by the injection maps from $FL^{\beta_i}_{a,b,c,d,\alpha_i}$ to $FL_{a,b,c,d}$ then the sequence $\{\phi_n\}$ in $FL_{a,b,c,d}$ converges to zero iff $\{\phi_n\}$ is contained in some $FL^{\beta_m}_{a,b,c,d,\alpha_m}$ and converges to zero.

Proof: Once we show that $FL_{a,b,c,d} = \bigcup_{\alpha_i,\beta_i=1}^{\infty} FL^{\beta_i}_{a,b,c,d,\alpha_i}$.

Clearly $\bigcup_{\alpha_i,\beta_i=1}^{\infty} FL_{a,b,c,d,\alpha_i,\beta_i} \subset FL_{a,b,c,d}$.

For proving other inclusion, let $\phi \in FL_{a,b,c,d}$ then

$\rho_{a,b,c,d,k,r,q,m,l,n}(\phi) = \sup_{I_1} \left| t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m \left[\phi(t,z,x,y) \right] \right|$, is bounded by some number. We can choose the integers α_m and β_m such that

$$\rho_{a,b,c,d,k,r,q,m,l,n}(\phi) \leq C A^k k^{k\alpha_m} . B^r r^{r\alpha_m} . G^l l^{l\beta_m} . H^n n^{n\beta_m}$$

$\therefore \phi \in FL^{\beta_m}_{a,b,c,d,\alpha_m}$, for some integers α_m and β_m

Hence $FL_{a,b,c,d} \subset \bigcup_{\alpha_i,\beta_i=1}^{\infty} FL^{\beta_i}_{a,b,c,d,\alpha_i}$

Thus, $FL_{a,b,c,d} = \bigcup_{\alpha_i,\beta_i=1}^{\infty} FL^{\beta_i}_{a,b,c,d,\alpha_i}$

5.4. Definition:

Let $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ and $\{d_n\}$ be monotonic sequence, converging to $w+$, $z-$, $\theta+$, $\pi-$ respectively.

Now we define countable union space. $FL\{w,z,\theta,\pi,\alpha\} = \bigcup_{n=1}^{\infty} FL_{a_n,b_n,c_n,d_n,\alpha}$

(5.4) Concerning this space we prove next theorem.

5.5. Theorem: The space $FL\{w,z,\theta,\pi,\alpha\}$ is independent of the choice of the sequence $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ and $\{d_n\}$. If $FL\{w,z,\theta,\pi,\alpha\}$ is equipped with the strict inductive limit topology defined by $FL_{a_n,b_n,c_n,d_n,\alpha}$ then the sequence $\{\phi_n\}$ in $FL\{w,z,\theta,\pi,\alpha\}$ converges to zero iff $\{\phi_n\}$ belongs to some $FL_{a_n,b_n,c_n,d_n,\alpha}$ and converges to zero in that space. Moreover $FL\{w,z,\theta,\pi,\alpha\}$ is complete.

Proof: Proof is easy and hence omitted.

6. CONCLUSIONS

The proposed paper provides the generalization of two dimensional Fourier-Laplace transform in the distributional sense. Different types of testing function spaces by using Gelfand-Shilov technique and some topological properties are defined.

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UPPER AND LOWER *b*-*I*-CONTINUOUS MULTIFUNCTIONS

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1. ABSTRACT

In this paper, we have introduced and study two new class of functions called upper (lower) *b*-*I*-continuous multi-functions in ideal topological space.

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Key words and phrases. *b*-*I*-open sets, upper *b*-*I*-continuity, lower *b*-*I*-continuity.

1. INTRODUCTION

The subject of ideals in topological spaces has been introduced and studied by Kuratowski [6] and Vaidyanathasamy [9]. An ideal I on a topological space $(X;\tau)$ is a nonempty collection of subsets of X which satisfies (i) $A\in I$ and $B \subset A$ implies $B \in I$ and (ii) $A \in I$ and $B \in I$ implies $A \cup B\in I$. Given a topological space $(X; \tau)$ with an ideal I on X and if $P(X)$is the set of all subsets of X, a set operator $(:)*$: $P(X) \to P(X)$, called the local function [9] of A with respect to $\tau$and I, is defined as follows: For $A \subset X$, $A*(\tau; I) = \{x \in X | U \cap A = \in I$ *forevery neighbourhood* Uofx$\}$.A Kuratowski closure operator Cl$*(:)$ for a topology $\tau*(\tau; I)$ called the $*$-topology, finer than $\tau$is defined by Cl$*(A) = A \cup A*(\tau; I)$ when there is no chance of confusion, $A*(I)$ is denoted by $A*$. If I is an ideal on X, then$(X; \tau; I)$ is called an ideal topological space. By a space, we always meana topological space $(X; \tau)$ with no separation properties assumed. The aim of this paper is to give two new class of functions called upper (lower) *b*-*I*continuous multi-functions in ideal topological space. Some characterizations and several basic properties of this class of functions are obtained.

2. PRELIMINARIES

Let $(X; \tau)$ be a space and A is a subset of X. We denote the closure of A and the interior of A by Cl(A) and In t(A) respectively. A subset A of X is said to be *b*-open [2] $A \subset$Cl($int(A)$) $\cup$Int(Cl(A)). A subset S of an ideal topological space $(X; \tau; I)$ is said to be *b*-*I*-open [5] if $S \subset$Int(Cl$*(S)$)$\cup$Cl$*$(Int(S)). The complement of a *b*-*I*-open set is called *b*-*I*-closed [5]. The intersection of all *b*-*I*-closed sets containing S is called the *b*-*I*-closure [5] of S and is denoted by *b*-*I* Cl(S). The *b*-*I*-Interior [5] of S is defined by the union of all *b*-*I*-open sets contained in S and is denoted by *b*-*I* Int(S). The family of all *b*-*I*-open(resp. *b*-*I*-closed) sets of $(X; \tau; I)$ is denoted by $BIO(X)$ (resp. $BIC(X)$).The family of all *b*-*I*-open (resp. *b*-*I*-closed) sets of $(X; \tau; I)$ containing apoint $x \in X$ is denoted by $BIO(X; x)$ (resp. $BIC(X; x)$). A subset U of$X$ is called a *b*-*I*-neighbourhood of a point $x \in X$ if there exists a *b*-*I*-openset V of $(X; \tau; I)$ such that $x \in V \subset U$. By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y , and always assume that $F(x) = \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, following [3],we shall denote the upper and lower inverse of a set B of Y by $F+(B)$ and $F-(B)$, respectively, that is, $F+(B) = \{x \in X : F(x) \subset B\}$ and $F-(B) =\{x \in X : F(x) \cap B \neq \emptyset\}$. In particular, $F-(Y) = \{x \in X : y \in F(x)\}$ for each point $y \in Y$ and for each $A \subset X$, $F(A) = \cup x \in A\ F(x)$. Then F is said to be surjection if $F(X) = Y$.

3. UPPER AND LOWER *B*-*I*-CONTINUOUS MULTIFUNCTIONS

Definition 3.1. *A multifunction* $F : (X; \tau; I) \to (Y; \tau)$ *is said to be:*

(i) *upper b-I-continuous at a point* $x \in X$ *if for each open set* V *of* Y *such that* $F(x) \subset V$ *, there exists* $U \in BIO(X; x)$ *such that* $F(x) \subset V$ *;*

(ii) *lower b-I-continuous at a point* $x \in X$ *if for each open set* V *of* Y *such that* $F(x) \cap V = \emptyset$*, there exists* $U \in BIO(X; x)$ *such that* $F(U) \cap V \neq \emptyset$ *for every* $u \in U$*;*

(iii) *upper(lower) b-I-continuous if F has this property at each point of X.*

Example 3.2. *Let* $X = \{a; b; c\}$, _ $= \{\emptyset, \{b\}, \{a; c\}, X\}$ *and* $I = \{\emptyset; \{b\}\}$.*Then the identity function* $f : (X; \tau; I) \to (X; \tau)$ *is upper b-I-continuous.*

Example 3.3. *Let* $X = \{a; b; c\}$. $\sigma = \{\emptyset, \{b\}, \{a; c\}, X\}$, $\sigma = \{\emptyset, \{a\},X\}$ *and* $I = \{\emptyset; \{a\}\}$. *Then the identity function* $f : (X; \sigma; I) \to (X; \sigma)$ *notupper-b-I-continuous.*

Proposition 3.4. *Let* $F : (X; \tau; I) \to (Y; \sigma)$ *be a multifunction. Then,*

(i) *If* $I = \emptyset$*, then F is upper-b-continuous;*

(ii) *If* $I = P(X)$*, then F is upper continuous;*

(iii) *If* $I = N$*, then F is upper-b-continuous (N is the ideal of all nowheredense sets).*

Proof. Follows from Proposition 2 of [5]

Theorem 3.5. *The following statements are equivalent for a multifunction* $F : (X; \sigma; I) \to (Y; \sigma)$*:*

(a) *F is upper b-I-continuous;*

(b) $F+(V) \in BIO(X)$ *for any open set V of Y ;*

(c) $F-(V) \in BIC(X)$ *for any closed set V of Y ;*

(d) *b-I* $\mathrm{Cl}(F-(\mathrm{Cl}(B)) \subset F-(\mathrm{Cl}(B))$ *for each* $*$*-dense-in-itself subset Bof Y ;*τ

(e) *for each point x of X and each neighbourhood V of* $F(x)$, $F+(V)$ *isa b-I-neighbourhood of x;*

(f) *for each point x of X and each neighbourhood V of* $F(x)$*, there existsa b-I-neighbourhood U of x such that* $F(U) \subset V$ *;*

(g) $\mathrm{Cl}*(\mathrm{Int}(F-(B))) \cap \mathrm{Int}(\mathrm{Cl}*(F-(B))) \subset F-(\mathrm{Cl}(B))$ *for every subsetB of Y ;*

(h) $F+(\mathrm{Int}(B)) \subset \mathrm{Int}(\mathrm{Cl}*(F+(B))) \cup \mathrm{Cl}*(\mathrm{Int}(F+(B)))$ *for every subsetB of Y .*

Proof. (a)$\Rightarrow$(b): Let V be any open subset of Y and $x \in F+(V)$. Thereexists $U \in BIO(X; x)$ such that $F(U) \subset V$. herefore, we obtain $x \in U \subset \mathrm{Cl}*(\mathrm{Int}(U)) \cup \mathrm{Int}(\mathrm{Cl}*(U)) \subset \mathrm{Cl}*(\mathrm{Int}(F+(V))) \cup \mathrm{Int}(\mathrm{Cl}*(F+(V)))$. We have, $F+(V) \subset \mathrm{Cl}*(\mathrm{Int}(F+(V))) \cup \mathrm{Int}(\mathrm{Cl}*(F+(V)))$ and hence $F+(V) \in BIO(X)$.

(b)$\Leftrightarrow$(c): This follows immediately from the fact that $F+(Y - B) = X - F-(B)$ for every subset B of Y .

(c)$\Rightarrow$(d): For any subset B of Y , $\mathrm{Cl}(B)$ is closed in Y and $F-(\mathrm{Cl}(B)) \in BIC(X)$. Therefore, we obtain *b-I* $\mathrm{Cl}(F-(\mathrm{Cl}(B))) \subset F-(\mathrm{Cl}(B))$.

(d)$\Rightarrow$(c): Let V be any closed set of Y . Then, we have *b-I* $\mathrm{Cl}(F-(V)) \subset F-(V)$ and hence $F-(V) \in BIC(X)$.

(b)⇒(e): Let $x \in X$ and V be a neighbourhood of $F(x)$. Then thereexists an open set G of Y such that $F(x) \subset G \subset V$. Therefore, we obtain$x \in F+(G) \subset F+(V)$. Since, $F+(G) \in BIO(X)$, $F+(V)$ is a b-Ineighbourhoodof x.
(e)⇒(f): Let $x \in X$ and V be a neighbourhood of $F(x)$. Put $U = F+(V)$,then U is a b-I-neighbourhood of x and $F(U) \subset V$.
(f)⇒(a): Let $x \in X$ and B be any open set Y such that $F(x) \subset V$. Then$V$ is a neighbourhood U of x such that $F(U) \subset V$. Therefore, there exists$A \in BIO(X)$ such that $x \in A \subset U$, hence $F(A) \subset V$.(c)⇒(g): For any subset B of Y , $\mathrm{Cl}(B)$ is closed in Y and by (c), $F-(\mathrm{Cl}(B)) \in BIC(X)$. This means that $F-(\mathrm{Cl}(B)) \supset \mathrm{Int}(\mathrm{Cl}*(F-(\mathrm{Cl}(B)))) \cap \mathrm{Cl}*(\mathrm{Int}(F-(\mathrm{Cl}(B)))) \supset \mathrm{Int}(\mathrm{Cl}*(F-(B))) \cap \mathrm{Cl}*(\mathrm{Int}(F-(B)))$.
(g)⇒(h): Clear.
(h)⇒(b): Let V be any open set of Y . Then, by using (h) we have $F+(V) \in BIO(X)$. τ

Theorem 3.6. *The following are equivalent for a multifunction* $F : (X; \tau; I) \to (Y; \tau)$*:*
(a) *F is lower b-I-continuous;*
(b) $F-(V) \in BIO(X)$ *for any open set V of Y ;*
(c) $F+(V) \in BIC(X)$ *for any closed set V of Y ;*
(d) b-$I\,\mathrm{Cl}(F+(B)) \subset F+(\mathrm{Cl}(B))$ *for any subset B of Y ;*
(e) $F(b$-$I\,\mathrm{Cl}(A)) \subset \mathrm{Cl}(F(A))$ *for any subset A of X;*
(f) $\mathrm{Cl}*(\mathrm{Int}(F+(B))) \cap \mathrm{Int}*(\mathrm{Cl}*(F+(B))) \subset F+(\mathrm{Cl}(B))$ *for every subsetB of Y ;*
(g) $F-(\mathrm{Int}(B)) \subset \mathrm{Int}(\mathrm{Cl}*(F-(B))) \cup \mathrm{Cl}*(\mathrm{Int}(F-(B)))$ *for every subsetB of Y .*

Proof. The proof is similar to that of Theorem 3.5 and is thus omitted. _
Theorem 3.7. *If* $F : (X; \tau; I) \to (Y;\sigma)$ *is upper b-I-continuous (lowerb-I-continuous) and* $F(X)$ *is endowed with subspace topology, then* $F :(X; \tau; I) \to F(X)$ *is upper b-I-continuous (lower b-I-continuous).*
Proof. Since $F : (X; \tau; I) \to (Y; \tau)$ is upper b-I-continuous (lower b-I-continuous)for every open set V of Y , $F+(V \cap F(X)) = F+(V) \cap F+(F(X))= F+(V)$ $(F-(V \cap F(X))) = F-(V) \cap F-(F(X)) = F-(V)$ is b-I-open. Hence $F : (X; \tau; I) \to F(X)$ is upper b-I-continuous (lower b-I-continuous).
Remark 3.8. *The following example shows that the composition of twoupper b-I-continuous function is need not be upper b-I-continuous.*
Example 3.9. *Let* $X = \{a; b; c\}$, $\_ = \{\emptyset, \{a\}, \{b\}, \{a; b\}, X\}$, $\_ = \{\emptyset,\{a\}, \{b; c\}, X\}$, $= \{\emptyset, \{c\}, X\}$ *and* $I = \{\emptyset; \{a\}\}$. *Clearly, the identity functions* $f : (X; \tau; I) \to (X; \tau; I)$ *and* $g : (X; \tau; I) \to (X;)$ *are upper-b-I- continuous but their composition is not upper-b-I-continuous.*
Theorem 3.10. *Let* $\{F\_ : (X; \tau; I) \to (X\_; \sigma);\alpha\_ \in \Delta\}$ *be a family of multi functions and let* $F : (X; \_; I) \to \Pi\_ \in \Delta X\_$ *be a multifunction de_nedby* $F(x) = (F\_(x))$. *Then F is upper b-I-continuous if and only if each*
$F\_ : X \to X\_$ *is upper b-I-continuous.*
Proof. Let G_0 be an open set of X_0 . Then $(P\_0 \circ F)+(G\_0) = F+(P+\_0(G\_0))= F+(G\_0 \times \Pi\_\neq\_0 X\_)$. Since F is upper b-I-continuous, $F+(G_0 \times \Pi_\neq_0X_)$is b-I-open in X. Thus $P\_0 = F = F\_0$ is upper b-I-continuous. Here $P_$denotes the projection of X onto _-coordinate space $X\_$. Conversely, supposethat each $F\_: X \to X\_$ is upper b-I-continuous. To show that themultifunction F is upper b-I-continuous, in view of Theorem 3.5, it is sufficientto show that $F+(V)$ is b-I-open for each open set V in the productspace $\Pi\_ \in \Delta X\_$. Let $U\_ \times \Pi\_\neq\_X\_$ be a subbasic open set in $\Pi\_ \in \Delta X\_$. Then $F+(U\_ \times$

Π_≠_X_) = F+(P+_ (U_)) = F+_ (U_) is *b-I*-open set in *X*and consequently *F* is upper *b-I*-ontinuous. _For a multifunction *F* : (*X*; _) → (*Y*; _), the graph multifunction *GF* : *X* →*X* × *Y* is defined as follows: *GF* (*x*) = {*x*} × *F*(*x*) for every *x* ∈*X*.

Lemma 3.11. *For a multifunction F* : (*X*; _) → (*Y*; _), *the following hold:*
(i) *GF*+(*A* × *B*) = *A* ∩ *F*+(*B*) *and*
(ii) *GF*□(*A* × *B*) = *A* ∩ *F*−(*B*) *for any subsets A* ⊂*X and B* ⊂*Y* .

Theorem 3.12. *A multifunction F* : (*X*; _; *I*) → (*Y*; _) *is lower b-I-continuous(upper b-I-continuous) if and only if GF* : (*X*; _; *I*) → (*X*×*Y*; _×_) *is lowerb-I-continuous (upper b-I-continuous).*

Proof. Case (i): (Necessity) Suppose that *F* is lower *b-I*-continuous. Let*x* ∈*X* and *W* be any open set of *X* × *Y* such that *x* ∈*GF*□(*W*). Since*W* ∩ ({*x*} ∩ *F*(*x*)) ≠ ∅, there exists *y* ∈*F*(*x*) such that (*x*; *y*) ∈*W* andhence (*x*; *y*) ∈
U × *V* ⊂*W* for some open sets *U* ⊂*X* and *V* ⊂*Y* . Since*F*(*x*) ∩ *V*≠ ∅, there exists *G* ∈*BIO*(*X*; *x*) such that *G* ⊂*F*−(*V*). ByLemma 3.11, we have *U* ∩ *G* ⊂*U* ∩ *F*−(*V*) = *GF*□(*U* × *V*) ⊂*GF*□(*W*).Moreover, *x* ∈*U* ∩ *G* ∈*BIO*(*X*) and hence *GF* is lower *b-I*-continuous.(Sufficiency) Suppose that *GF* is lower *b-I*-continuous. Let *x* ∈*X* and *V*be an open set of *Y* such that *x* ∈*F*−(*V*). Then *X* × *V* is open in *X* × *Y*and *GF* (*x*) ∩ (*X* × *V*) = ({*x*} × *F*(*x*)) ∩ (*X* × *V*) = {*x*} × (*F*(*x*) ∩ *V*)≠ ∅.Since *GF* is a lower *b-I*-continuous, there exists *U* ∈*BIO*(*X*; *x*) such that*U* ⊂*GF*□(*X* × *V*). By Lemma 3.11, we obtain *U* ⊂*F*−(*V*). This shows that *F* is lower *b-I*-continuous. The proof of second case is similar.

Definition 3.13. [4] *Let A and X*0 *be subsets of an ideal topological space*(*X*; *τ*; *I*) *such that A* ⊂*X*0 ⊂*X. Then* (*X*0; _|*X*0; *I*|*X*0) *is an ideal topological space with an ideal I*|*X*0= {*I* ∈*I*|*I* ⊂*X*0} = {*I* ∩*X*0|*I* ∈*I*}.

Theorem 3.14. *Let F* : (*X*; *τ* ; *I*) → (*Y*; _) *be a multifunction and A be anopen subset of X. If F is upper b-I-continuous (lower b-I-continuous), thenF*|*A* : (*A*; _|*A*; *I*|*A*) → (*Y*; _) *is upper b-I-continuous (lower b-I-continuous).*

Proof. Follows from Theorem 3.15 of [1]. _

Theorem 3.15. *Let F* : (*X*; *τ*; *I*) → (*Y*; *τ*;*J*) *be a multifunction and* {*U*_ :_ ∈△} *be an open cover of X. If the restriction function F*|*U*_ *is upperb-I-continuous for each* _ ∈△, *then F is upper b-I-continuous.*

Definition 3.16. *A collection* {*G*_ : _ ∈∇} *is called a b-I-open cover ofa subset A of an ideal topological space* (*X*; *τ*; *I*) *if A* ⊂∪{*G*_ : *X**G*_ ∈*BIO*(*X*); _ ∈∇}.

Definition 3.17. [1] *An ideal space* (*X*; _; *I*) *is said be b-I-compact if forevery b-I-open cover* {*W*_ : _ ∈△}, *there exists a _nite subset* △0 *of* △*such that* (*X* −∪{*W*_ : _ ∈△0}) ∈*I*.

Lemma 3.18. [7] *For any surjective multifunction F* : (*X*; _; *I*) → (*Y*; _),*F*(*I*) *is an ideal on Y* .

Theorem 3.19. *The image of a b-I-compact space under upper b-I-continuoussurjection is b-F*(*I*)*-compact.Proof.* Follows from their respective definitions. _

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ENGAGING IN SOCIAL ACTION AT WORK: DEMOGRAPHIC DIFFERENCES IN PARTICIPATION

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ABSTRACT

Numerous associations are using corporate social obligation activities that require representative support. These activities, which include social activity at work (SAW), can be a wellspring of reputational gains, advantage the network, and increment worker hierarchical recognizable proof [1]. In spite of the fact that examination has been led on worker volunteer projects (EVP), one part of SAW, those investigations have not recognized the attributes of representatives who are well on the way to take an interest in EVP nor have they thought about the extensive variety of SAW programs. In the field of Sociology, extensive research has been led to recognize attributes of volunteers, yet these volunteer projects are outside the setting of CSR activities. This exploration tends to this hole by recognizing the attributes of employees who en- gage in SAW over an extensive variety of exercises. The aftereffects of the examination can help sharpen future research questions and help experts in creating and promoting SAW programs that reverberate with workers and amplify cooperation for the benefit of the representatives, association, and network all in all.

Keywords: Corporate Social Responsibility; Employee Engagement; Employee Volunteerism.

1. INTRODUCTION

As a major aspect of their corporate social obligation programs, numerous associations use worker endeavors to connect with their networks. They direct United Way campaigns, support blood drives, and arrange volunteer projects, to give some examples models. Past papers [1,2] have given a typology of these CSR activities that require representative association and have talked about their effect on worker frames of mind and practices, finding that support in these exercises can give benefits be-yond the monetary main concern, including expanded organizational recognizable proof with respect to representatives. These papers did not, be that as it may, examine which representatives might be well on the way to take part in social activity at work (SAW). While others have examined specific parts of SAW, for example, representative volunteer projects (e.g., [3-6]), knowing the attributes of the workers who participate in assorted SAW projects will enable administrators to focus on these projects successfully and also distinguish other SAW openings that might be additionally speaking to the gatherings that don't as of now take part in organizations'

CSR activities. The aftereffects of this exploration will likewise help researchers of future examinations sharpen future research questions.

2. THEORY AND HYPOTHESES DEVELOPMENT

2.1. Social Ties

A social tie is a relationship between two people, An and B, the quality of which relies upon the time, power, closeness, and response which describes the tie. Podolny and Baron made a typology of social ties in the work environment, recognizing position-to-position and individual to-individual ties, the previous dependent on employment reliance and the last dependent on relational fascination, or companionship. In any case, they push that the qualification is "a matter of degree not kind" and ought not be exaggerated [7, p. 677]. Thus, I will utilize the expression "social tie" to allude to both formal and casual ties between two associates. To delineate the effect of social ties on SAW, I will examine social ties with regards to social developments, activism, and volunteering. Research on social activism has discovered that having asocial attach is identified with support in social developments. Schussman and Soule, in an investigation investigating reasons why individuals take an interest in dissent exercises (characterized as a challenge, walk, or exhibit in response to a neighborhood or national issue) found that being as-ked to partake is the most grounded indicator of cooperation. Social capital, as estimated by embeddedness in interpersonal organizations (e.g., network initiative) and social standards (i.e., social and interracial trust), is likewise identified with volunteerism and magnanimous giving. Brady and Colalliances found that scouts for political activism were increasingly effective in increasing positive reactions when they knew their objective. Close binds inclined focuses to consent to demands; moreover, enrollment specialists could use individual data to engage the objectives' close to home interests, qualities, and objectives. A contextual investigation of a labor strike at a substantial college grounds uncovered that employees were bound to strike in the event that others in their unit were additionally taking an interest.

Volunteering is almost certain when social ties exist, and, on the other hand, volunteering fortifies social ties. Besides, volunteering is reinforced through social collaborations. Social binds additionally influence reactions to volunteerism: Kulik showed that volunteers who delighted in family bolster making the most of their charitable effort more and endured less burnout than those without family bolster. Wilson and Musick saw that volunteers with increasingly visit participation at gatherings of religious or altruistic gatherings were less inclined to drop out of volunteer exercises.

Non-benefit bunches have used this learning of social ties. In its writing for work environment blood drive coordinators, the United Blood Services focuses on the importance of shared selecting for a fruitful occasion. In light of the effect social ties have on an assortment of classifications of social activity, I offer the accompanying:

Hypothesis 1: Employees with social connections to others engaged in SAW will be bound to take an interest in SAW than workers without social ties.

2.2. Past Volunteering/Donating Experiences

Representatives who have volunteered or given in the past likely will be bound to rehash their conduct. In an investigation of blood contributors, Lee, Piliavin, and Call found that past conduct was prescient of the giving of time, cash, and blood, recommending that these exercises helped frame a job character, and consistency in real life built up and keep up the personality. Utilizing the Theory of Planned Behavior to dissect beneficent giving, Smith and McSweeney revealed a connection among past and current gifts. Given these outcomes in a general setting, I expect a similar drive for job personality consistency will be available in the work environment, prompting the accompanying:

Hypothesis 2: Past encounters volunteering or donating to foundations will be identified with current volunteering and giving at work.

2.3. Demographic Differences

Singular contrasts factors likewise likely assume a job in a worker's choice to take an interest in SAW. Sex, for instance, is identified with the probability to volunteer with ladies volunteering at a higher rate than men. Contrasts in expectation, appreciation, and unselfishness are likewise likely identified with SAW. To outline, an investigation of 308 desk workers by Andersson, Giacalone, and Jurkie-wicz showed a connection among seek and appreciation with worry after CSR. Perspective, especially the morals of consideration (a worry for others dependent on compassion and need) versus the morals of equity (a widespread viewpoint), is another individual variable anticipated that would impact SAW. Proof demonstrates that ladies are bound to have a morals of consideration which is identified with more elevated amounts of volunteerism. Together, these discoveries recommend:

Hypothesis 3: Women will be bound to participate in

SAW than men.

A few examinations demonstrate that when all is said in done, more seasoned grown-ups volunteer more regularly than more youthful grown-ups. These outcomes may be clarified by the measure of time more established grown- ups, frequently resigned, have accessible to dedicate to volunteer exercises. Inside the setting of SAW, nonetheless, age may not demonstrate a similar association with investment. Age is regularly seen as an intermediary for residency, and usually, the longer the residency at work, the more duties one has. In the meantime, workers with shorter residency may feel more strain to perform and in this manner give additional time and vitality to at work concerns. In spite of the mixed discoveries, on account of the solid association with age and social activity, I propose:

Hypothesis 4: Older workers will be bound to take an interest in SAW than more youthful representatives.

Racial contrasts additionally rise while considering cooperation in volunteerism. As indicated by Sun-deen, Garcia, and Raskoff, Caucasians volunteer at the most astounding rate. Wilson recommends this might be because of other racial gatherings' entrance to human capital or in light of the fact that they are not as inserted in interpersonal organizations and there-fore are not requested to volunteer. Therefore, we recommend:

Hypothesis 5: Ethnic and racial gatherings will indicate different dimensions of investment in SAW.

3. METHODS

3.1. Sample

This exploration was executed as a major aspect of a bigger report led at a Southwest area of a noteworthy semiconductor maker which we will call Chipmaker to master tect the character of the example site. Around the world, the company has right around 100,000 representatives as indicated by their 2006 yearly report with more than 10,000 of those representatives at the example site. Chipmaker produces microchips, motherboards, streak memory, items for system storage, and remote items. For our examination, 1000 utilizes from the chose site were haphazardly decided to receive solicitations to take an interest. Of these members, 314, or 31.4%, finished the review.

3.2 Procedure

A best positioning individual from Chipmakers' supervisory crew gave a letter of underwriting to the examination, send by means of email, that featured the advantages of the overview to the firm, requests that workers round out the review, which was accessible through the business online program Survey Monkey, and focused on that representative reactions will stay private. To mollify a few worries about socially alluring reacting, the directions additionally re-disapproved of members that reactions would remain confidential and showed that there were no set in stone answers; yet that we were keen on members' ho-settle suppositions. Seven days after the underlying welcome was sent, an agent from Chipmaker messaged a second notice to all members expressing gratitude toward them for taking an interest and requesting that they finish the overview on the off chance that they had not done as such. Members were likewise requested to finish a subsequent study two weeks after the underlying study. Of those respondents, 210 (21% of whole example, 66.8% of Part 1 respondents) finished Part 2. Thus, I had finish information for 210 respondents and utilized just this coordinated information in my examinations. A contact from Chipmaker contemplated that the low reaction rate could be because of various variables: low assurance because of staff decreases, a feeling that the study didn't identify with a center business and consequently respondents' everyday exercises, and absence of a motivator for finishing the study. Moreover, however the welcome to take an interest was sent from the Corporate Vice President for Corporate Affairs, that VP might not have been commonplace to beneficiaries since he isn't in their immediate line of direction. A couple of members responded to the welcome to finish section 2 with messages like, "also long" or "not intrigued". At long last, as talked about prior, the review was regulated amid December and January, a season when different commitments may overshadow a deliberate overview. Given the cons-traints with our exploration structure, and the unavoidable attrition in multi-part overviews, a 21% reaction rate with 210 usable cases appears to be reasonable.

In spite of the fact that gathering information from a solitary source, i.e. a self-detailed study, can be a wellspring of basic technique difference [34], in this investigation the develops all reflect singular discernments and comprehensions; in this way, no reason-capable elective wellsprings of data exist. To the ex-tent conceivable, I controlled for normal technique predisposition through control factors and study plan.

3.3. Sample Characteristics

Two hundred nine members gave their activity class: 165 (78.6%) singular donors, 28 (13.3%) directors, 12 (5.7%) heads, and 4 (1.9%) official administrators. A scope of residency classes was available in my example. Out of the185 members who respondents to this inquiry, 7 (3.3%) had worked in their present position at Chipmaker under1 year, 13 (6.2%) for 1 year, 12 (5.7%) for a long time, 19 (9%) for 3, 13 (6.2%) for 4, 22 (10.5%) for 5, 36 (17.1%) for 6, 11 (5.2%) for 7, 18 (8.6%) for 8, 12 (5.7%) for 9, 7 (3.3%) for 10, and 15 (7.1%) for a long time. Two (1%) of the example has a secondary school degree, 42 (20%) some school, 81 (38.6%) an advanced education, 59 (28.1%) a graduate degree, 10 (4.8%) a Ph.D. or on the other hand J.D., and 2 (1%) are present understudies. Of the 193 members giving data about sex, 59 (28.1%) were female and 134 (63.8%) were male. One hundred eighty-five members gave their age go: 2 (1.0%) 18 - 24 years of age, 51 (24.3%) 25 - 34, 72 (34.3%) 35 - 44, 43 (20.5%) 45 - 54, 10 (4.8%) 55 - 64, 5 (2.4%) 65 - 74, and 2 (1%) more than 75. One hundred eighty-two respondents offered their race or ethnic foundation: 12 (5.7%) Black or African-American, 14 (6.7%) Asian, 129 (61.4%) Caucasian, and 18 (8.6%) Hispanic. Six (2.9%) respondents gave numerous classes while 3 (1.4%) determined "other". One hundred ninety-one members gave their conjugal status: 27 (12.9%) single, 142 (67.6%) wedded, 2 (1%) household organization, and 20 (9.5%) separated.

3.4. Measures

Social Action at Work. Social activity at work (SAW) is specifically identified with circumstances given by the participating site, so things were composed to mirror the sorts of magnanimous and charitable open doors the organization gives. A starter list dependent on investigation of Chipmaker's distributed material was given to our center gathering who looked into the things for dialect and pertinence. Members were asked nine things, "How regularly do you take an interest in the accompanying exercises?" Activities included things, for example, "I reuse at work" and "I give to a philanthropy of decision through my work". Res-ponses were evaluated utilizing a five-thing likert scale with the grapples inconsistently and much of the time.

Magnanimous Giving Outside the Workplace. In view of the Social Capital Community Benchmark Survey (as referred to by Brady et al., [16]), things to evaluate members' altruistic giving and volunteerism were incorporated: 1) "I have given cash, property or different resources for charitable purposes in the previous a year" and 2) "I have performed unpaid work to help individuals other than my family, companions, or colleagues in the previous a year." These were estimated on a 5-point Likert scale extending from firmly consent to unequivocally oppose this idea. Furthermore, respondents were asked how essential giving and volunteering are to them.

Social Ties. Social ties were estimated through three things grew explicitly for this investigation: 1) I am bound to take an interest in CSR exercises when my collaborate ers visit, 2) I typically volunteer for CSR exercises without anyone else, and 3) I ordinarily volunteer for CSR exercises with my associates. I had considered approaching these questions for each SAW depicted on the study, yet because of the likelihood of study weariness and on the recommendation of my delegates at Chipmaker, I picked to utilize less things. While this forfeits some fine-grained information, the likelihood of losing a member at Time 2 was a more prominent expense.

3.5. Data Analysis

SPSS adaptation 22.0.0 was utilized for the information examination. Regression examination was utilized to test speculations including social ties and past volunteering and altruistic gifts. ANOVA investigations were used to test the speculations identifying with statistic contrasts and direct post hoc tests to test critical contrasts among gatherings. To test the speculations, I utilized a normal of the diverse SAW exercises and additionally broke down contrasts among gatherings in individual SAW exercises.

4. RESULTS

Distinct measurements for the factors are displayed in Table 1 and a synopsis of the consequences of the information investigation are introduced in Tables 1 through 4. Speculation 1 was upheld. In the wake of controlling for sex, age, and race, representatives with increasingly social binds were bound to partake in SAW, clarifying 42.6% of the change ($R2 = 0.426$, $F(4,241) = 46.501$, $p < 0.001$). Theory 2 was likewise upheld. Workers showing an example of cooperation in volunteering or giving were probably going to keep on doing as such. In the wake of controlling for sex, race, and age, this clarified 21.5% of change, $R2 = 0.215$, $F(5,167) = 10.397$, $p < 0.001$; be that as it may, just past volunteering was critical ($\beta = 0.308$, $p < 0.001$). Hypo- proposition 3, that ladies will have more elevated amounts of SAW than men, was bolstered (see Table 2 for means). Overall, ladies were almost certain than men to take an interest in SAW ($F(1,269)=18.487$, $p < 0.001$). Ladies were bound to give to a philanthropy of decision, $F(1,269) = 6.315$, $p = 013$ volunteer at a neighborhood school, $F(1,269) = 10.291$, $p = 0.001$, take an interest in a maintainability gathering, $F(1,269) = 5.819$, $p= 0.017$, join the ChipMaker supported EVP, $F(1,269) = 28.452$, $p< 0.001$, volunteer with their worker gathering, $F(1,269)= 11.401$, $p < 0.001$, or give their aptitudes or skill to network associations, $F(1,269) = 14.989$, $p < 0.001$.

There were no critical contrasts among people concerning giving to the United Way, $F(1,269) = 1.365$, p = n.s., reusing at work $F(1,269) =0.219$, p = n.s., or giving blood at work $F(1,269) =0.756$, p = n.s. Theory 4, that more established representatives would be bound to take an interest in SAW than workers in more youthful age bunches was not upheld ($F(7,255) = 1.630$, p = n.s), nor was Hypothesis 5, that racial and ethnic gatherings will demonstrate diverse examples of cooperation ($F(5,250) = 0.268$, p = n.s); see Table 4. I tested for contrasts for every one of a kind SAW activity. Since the most elevated age bunches had little numbers, I crumbled them into a solitary classification. ANOVA tests indicated contrasts in investment among age bunches in United Way donations, $F(5,257) = 5.353$, $p < 0.001$, gifts to the burnity of decision, $F(5,257) = 3.090$, $p = 0.01$, and giving blood at work $F(5,257) = 2.659$, $p = 0.023$. Post- hoc tests, including Tukey, Bonferroni, Scheffe, and LSD uncover that the most youthful age gatherings (18 to 24) are the to the least extent liable to en gage in SAW, while representatives 45 and more seasoned are the well on the way to partake. Indeed, even in the wake of examining each SAW separately, no distinctions among race or ethnic gatherings in interest levels were watched.

Table 1. Descriptive Statistics.

	N	Mean	Std. Deviation
Donate to United Way	314	3.37	1.770
Donate to Charity of Choice	314	2.99	1.734
Recycle at Work	314	4.57	0.892
Local School Volunteer	314	2.83	1.584
Employee Sustainability Network	314	1.68	1.051
Company Sponsored Volunteer	314	2.89	1.495
Employee Group Volunteer	314	2.59	1.446
Blood Drive Participant	314	1.86	1.301
Donate Expertise	314	2.44	1.413
SOCTIE1	314	4.74	1.664
SOCTIE2	314	3.45	1.081
SOCTIE3	314	2.88	1.170
Past Donations	210	3.90	1.217
Past Volunteering	210	2.97	1.546

Table 2. SAW by Gender.

	Mean—Males N = 190	Mean—Females N = 81
Donate to United Way	3.39	3.67
Donate to Charity of Choice	2.83	3.41
Recycle at Work	4.63	4.58
Local School Volunteer	2.64	3.31
Employee Sustainability Network	2.45	3.09
Company Sponsored Volunteer	2.66	3.65
Employee Group Volunteer	2.45	3.09
Blood Drive Participant	1.78	1.93
Donate Expertise/Skill	2.23	2.93
SAWAverage	**2.69**	**3.16**

5. DISCUSSION

Knowing the qualities of representatives prone to take an interest in SAW can enable specialists to progress pertinent and fitting examinations and administrators create and advertise CSR activities that reverberate with workers. Of course, workers who had social ties with others taking part in SAW programs were bound to partake themselves, while the individuals who had take part in the past were likewise bound to join SAW activities. As far as statistic contrasts, ladies were almost certain than men to take an interest in SAW. In past research, ladies have shown higher inclinations to give to magnanimous associations and to volunteer and also to show more elevated amounts of morals of consideration, which is identified with these exercises. Nonetheless, take note of that this induces a pivotal inquiry: are ladies troubled by the desire to think about others to the detriment of exercises that may give them greater perceivability at work or to help advance their profession inside. Truly, ladies have been appointed to "occupied work" which can be impeding to long haul profession movement. Also, this sort of work may include progressively passionate work, and enthusiastic work prerequisites of ladies who have been in positions are increasingly serious.

Contrasts in SAW cooperation among ethic/racial gatherings or in age bunches were not seen in this investigation. It is conceivable because of the little numbers in the number of inhabitants in a portion of the gatherings; for instance, the two age bunches speaking to the most seasoned workers contained just four members. More difference in the example could deliver progressively powerful outcomes. Nonetheless, it is conceivable that the cultural standards at the example site overwhelm the impacts that have prompted contrasts among these gatherings in volunteering and giving outside of the work setting.

5.1. LIMITATIONS

The present examination was led in the United States and may not be generalizable to different nations. Lee and Chang for instance, found distinctive examples in Tawainese residents' giving and volunteering conduct from those saw in Western nations.

It is reasonable to figure that with regards to SAW, national and social contrasts would likewise rise.

Table 3. SAW by Age.

	18 - 24 N = 3	25 - 34 N =74	35 - 44 N=99	45 - 54 N = 63	55 - 64 N = 14	65 - 74 N = 7	75 - 84 N = 2	85+ N = 1
Donate to United Way	1.00	2.95	3.37	4.17	4.07	2.71	4.0	5.0
Donate to Charity	1.00	2.54	3.18	3.38	3.43	2.14	4.5	1.0
Recycle at Work	4.67	4.50	4.67	4.68	4.57	4.57	5.0	5.0
Local School Volunteer	3.00	2.55	3.01	2.87	2.79	3.71	2.50	1.00
Sustainability Group	2.00	1.65	1.64	1.71	2.07	1.57	1.00	1.00
Company EVP	2.67	2.74	3.04	3.08	2.93	3.00	2.50	3.00
Group EVP	2.67	2.47	2.64	2.97	2.43	2.29	2.00	3.00
Blood Drive Participant	1.33	1.81	1.75	2.11	2.71	1.00	1.00	2.00
Donate Skills/Expertise	2.67	2.34	2.54	2.57	2.21	2.43	3.00	1.00
SAW Average	**2.4444**	**2.6171**	**2.8698**	**3.0617**	**3.0238**	**2.6032**	**2.8333**	**2.4444**

Table 4. SAW by Race.

	Black/African-American N = 14	Asian N = 24	Caucasian N = 179	Hispanic N = 26	Multiple Races N = 7	Other N = 6
Donate to United Way	3.79	3.75	3.37	3.50	2.29	3.33
Donate to Charity	3.00	3.63	2.90	2.69	2.86	2.00
Recycle at Work	4.36	4.25	4.69	4.35	5.00	4.67
Local School Volunteer	3.29	2.54	2.75	3.08	2.86	3.50
Sustainability Group	2.21	1.75	1.61	1.69	2.00	1.50
Company EVP	3.00	2.71	2.94	3.04	3.43	2.67
Group EVP	3.0	2.92	2.60	2.46	3.00	2.67
Blood Drive Participant	1.21	1.88	1.93	1.85	1.57	2.17
Donate Skills/Expertise	3.14	2.75	2.28	2.19	3.29	2.67
SAW Average	**3.00**	**2.90**	**2.79**	**2.76**	**2.91**	**2.81**

Because of study limitations at the example site, it was unrealistic to gather information with respect to a portion of the measurements fundamental the speculations created here. For instance, it is valuable to have possessed the capacity to gauge morals of consideration straight forward as opposed to utilizing sexual orientation as an intermediary.

5.2. Future Research

Various different attributes can be recognized to explore as precursors of SAW. As referenced, the immediate instruments, for example, morals of consideration, could Furthermore, some statistic bunches had little populaces. Having all the more equally circulated gathering membership would give more confirmation in the example of results. At long last, while I endeavored to limit the imagreement of normal strategy predisposition, it remains a worry.

be studied to more readily comprehend the qualities of representatives who take part in SAW. Future research might have the capacity to fuse coordinate, instead of self- revealed, proportions of investment in SAW. Moreover, hierarchical culture likely impacts workers in this procedure. Since the present examination was directed at a solitary site, it was impractical to research this road, however I urge others to look at a solitary association at different locales and additionally numerous associations to perceive how the way of life influences SAW contribution. In open-finished inquiries, a bunch of members referenced they took part in social activity outside of work. It is intriguing to see the connections among the inspirations and involvement in social activity at work and outside of work. Given the examination plan, I couldn't explore the motivations to participate in SAW, which is a basic advance in the exploration stream around there.

5.3. Managerial Implications

Administrators can make their SAW programs more successful on the off chance that they can get progressively male workers involved. No matter how you look at it, interest in SAW is low (see Table 1). The most famous SAW programs are reusing at work and giving to the United Way. These projects likely observe the most abnormal amounts of cooperation since they are entrenched, all around advanced, and simple to utilize. These projects can fill in as a model to im-demonstrate the support in other SAW activities.

Directors ought to likewise examine reasons why ladies are bound to take an interest in these projects and develop programs that would increase parallel investment among sexual orientations. Realizing that social ties help SAW, managers can use kinship and interpersonal organizations to star bit and carryout CSR activities including representatives. They can use the data from this paper to help a greater amount of their representatives take an interest in advancing SAW exercises that give advantages to the worker him or herself, the organization, and the network all in all.

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AN ANALYTIC HP-TRANSFORMATION IN ALMOST KAEHLERIAN SPACES

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ABSTRACT:

In the present paper, we have defined and studied an analytic holomorphically projective transformation in Almost Kaehlerian spaces and several theorems have been established.

KEY WORDS: Kaehlerian space, H-Projective, Recurrent, Symmetric and Transformation.

2010MSC: 32C15, 53B35, 53C55.

1. INTRODUCTION

An almost Kaehlerian space is first of all an almost complex space, that is, a 2n-dimensional space with an almost complex structure $F^h{}_i$ with following properties:

$$F^i{}_j\, F^h{}_i = \delta^h{}_j, \tag{1.1}$$

$$F^a{}_j\, F^b{}_i\, g_{ab} = g_{ji}, \tag{1.2}$$

$$F_{ji} = -\, F_{ij}, \tag{1.3}$$

$$F_{ji} \overset{\text{def}}{=} F^a{}_j\, g_{ai} \tag{1.4}$$

And finally has the property that the differential form

$$F_{ji}\, d\xi^j \wedge d\xi^j \text{ is closed, that is, } \quad F_{jih} \overset{\text{def}}{=} \nabla_j F_{ih} + \nabla_i F_{hj} + \nabla_h F_{ji} = 0$$

And the property of the skew-symmetric F_{ih} is a killing tensor

$$\nabla_j F_{ih} + \nabla_i F_{hj} = 0 \tag{1.5}$$

From which

$$\nabla_j F^j{}_i + \nabla_j F^h{}_j = 0 \tag{1.6}$$

and $$F i = \nabla_j F^j{}_i = 0 \tag{1.7}$$

Here ∇ denotes the operation of covariant differentiations with respect to the Riemannian connection $\{^{\,h}_{j\ i}\}$.

The Nijenhuis tensor N^h_{ji} is written in the form:

$$N^h_{ji} = -4\,(\nabla_j F^t_i)\,F^h_t + 2\,G^t_{ji}\,F^h_i + F^t_j\,G^h_{ti} - F^t_i\,G^h_{tj}. \qquad (1.8)$$

A Contravariant almost analytic vector field is defined as a vector field v^i, satisfying Tachibana (1959):

$$£_v F^h i \equiv v^j \partial_j F^h_i - F^j_i \partial_j v^h + F^h_j \partial_i v^j = 0,$$

Where $£_v$ stands for the Lie-derivative with respect to v^i.

Let R^h_{kji} be the Riemannian curvature tensor and put

$$R_{ji} = R^r_{rji}, \quad R_{kjih} = R^r_{kji}\, g_{rh},\; R = R_{ji}\, g^{ji} \text{ and } S_{ji} = F^r_j R_{ri},$$

Then the following identities are satisfied (Yano 1957)

$$R^r_{kji} F^h_r = R^h_{kjr} F^r_i, \quad R_{kjir} F^r_h = R_{kjhr} F^r \qquad (1.9)$$

$$R_{kjih} = R_{kjtr} F^t_i F^r_h, \quad R_{ji} = R_{tr} F^t_j F^r_i \qquad (1.10)$$

$$S_{ji} + S_{ij} = 0, \quad S_{ji} = S_{tr} F^t_j F^r_i, \qquad S_{ji} = -\frac{1}{2} F^{tr} R_{trji}. \qquad (1.11)$$

The holomorphically projective curvature tensor P^h_{kji}, which will be briefly called HP-curvature tensor, is given by

$$P^h_{kji} = R^h_{kji} + \frac{1}{n+2}\left(R_{ki}\,\delta^h_j - R_{ji}\,\delta^h_k + S_{ki} F^h_j - S_{ji} F^h_k + 2S_{kj} F^h_i\right) \qquad (1.12)$$

We can obtain the following identities

$$P^h_{(kj)i} = 0, \quad P^h_{(kji)} = 0, \qquad (1.13)$$

$$P^r_{rji} = 0, \qquad (1.14)$$

$$P^r_{kji} F^h_r = P^h_{kjr} F^r_i, \qquad P^h_{rji} F^r_k = P^h_{rki} F^r_j \qquad (1.15)$$

From which, we have

$$P^r_{kjr} = 0, \qquad (1.16)$$

$$P^t_{rji} F^r_t = 0,\; P^t_{Kjr} F^r_t = 0 \qquad (1.17)$$

A necessary and sufficient condition for $P^h_{kji} = 0$, is that the space is a space of constant holomorphically curvature (Tashiro 1957), i.e., a space whose curvature tensor R^h_{kji} takes the form

$$R^h_{kji} = -\frac{R}{n(n+2)}\left(g_{ki}\,\delta^h_j - g_{ji}\,\delta^h_k + F_{ki} F^h_j - F_{ji} F^h_k + 2F_{kj} F^h_i\right) \qquad (1.18)$$

For a vector field V^i and a tensor field α^h_i, the following identities are known (Yano 1957)

$$£_v \nabla_j \alpha^h_i - \nabla_j £_v \alpha^h_i = \alpha^r_i\, £_v \{ {}_j{}^h{}_r \} - \alpha^h_r\, £v\, \{ {}_j{}^r{}_i \} \qquad (1.19)$$

$$\nabla_k £_v \{ {}_j{}^h{}_i \} - \nabla_j £_v \{ {}_k{}^h{}_i \} = £_v R^h_{kji} \qquad (1.20)$$

Where $£_v$ denotes the operator of Lie-differentiation with respect to V^i.

A killing vector or an infinitesimal iso-metry V^i is defined by

$$£_v g_{ji} = \nabla_j V_i + \nabla_i V_j = 0$$

Here we shall identity a Contra-variant vectors V^i with a covariant vector $V_i = g_{ir} V^r$. Hence we shall say V_i is a killing vector, or that ρ^i is gradient, for example.

An infinitesimal affine transformation V^i is defined by

$$£_v\{{}_j{}^h{}_i\} - \nabla_j \nabla_i V^h + R^h{}_{rji} V^r = 0$$

We shall say a vector field V^i an infinitesimal holomorphically projective transformation or, for simplicity, and HP-transformation, if it satisfies

$$£_v\{{}_j{}^h{}_i\} = \rho_j \delta^h{}_i + \rho_i \delta^h{}_j - \bar{\rho}_j F^h{}_i - \bar{\rho}_i F^h{}_j,$$

Where ρ_i is a certain vector and $\bar{\rho}_i = F^r{}_i \rho_r$. In this case, we shall call ρ_i the associated vector of the transformation, If ρ_i vanishes, then the HP-transformation reduces to an affine one.

Contracting the last equation with respect to h and i, we get

$$\nabla_j \nabla_r V^r = (n+2) \rho_j$$

Which shows that the associated vector is gradient.

A vector field V^i is called Contravariant analytic or, for simplicity, analytic, if it satisfies

$$£_v F^h{}_i = -F^r{}_i \nabla_r V^h + F^h{}_r \nabla_i V^r = 0.$$

2. AN ANALYTIC HP-TRANSFORMATION IN ALMOST KEHKERIAN SPACES.

In a differentiable space M, we consider a tensor valued function V depending not only on a point P of M but also on k vectors $u_1, u_2 \ldots\ldots u_k$ at the point and denote it by $V(P, u_1, u_2 \ldots, u_k)$. We assume that the value of this function V lies in the tensor space associated to the tangent space of M at P and that it depends differentiably on its arguments.

Assuming the space M to be affine connected, we take an arbitrary curve C: $x^i = x^i(t)$ and denote its successive derivatives by

$$\frac{dx^i}{dt}, \frac{d^2x^i}{dt^2}, \frac{d^3x^i}{dt^3}, \ldots\ldots\ldots \qquad (2.1)$$

Then if we substitute (2.1) into the function V instead of $u_1, u_2 \ldots\ldots u_k$. We have a family of tensors

$$V(C) = V(\dot{x}, \frac{dx}{dt}, \ldots\ldots, \frac{d^kx}{dt^k})$$

along the curve C.

Let V^i be an infinitesimal transformation i.e., a vector field, and $x^i = x^i + \varepsilon v_i$ be the infinitesimal point transformation determined by v_i, ε being an infinitesimal constant. Given a curve C: $x^i = x^i(t)$, the image C of F is expressed by

$$x^i = x^i(t) + \varepsilon\, V^i(x(t)).$$

We shall call the limiting value

$$£_v\, V(C) \equiv \lim_{\varepsilon\to 0} \frac{V('c) - {}'V(c)}{\varepsilon}$$

The Lie-derivative of V(C) with respect to V^i, where we have denoted by 'V(C) the family of tensors induced from V(C) by the transformation

$$x^k = x^i + \varepsilon\, V^i$$

In a Almost Kaehlerian space, a curve $x^i = x^i(t)$ defined by

$$\frac{d^2x^h}{dt^2} + \left\{\frac{h}{j\ \ i}\right\} \frac{dx^j}{dt}\frac{dx^i}{dt} = \alpha\ \frac{dx^h}{dt} + \beta\ F_j^h\ \frac{dx^j}{dt} \tag{2.2}$$

is, by definition, a holomorphically planar curve, or an H-plane curve, where α and β are certain functions of t.

Let V^i be an infinitesimal transformation and assume that any ε the infinitesimal point transformation $x^i = x^i\ \varepsilon\ V^i$ maps any H-plane curves.

Now we ask for the condition that V^i preserve that H-plane curves. For such a vector V^i taking account of (2.2), we have

$$£_v \left[\frac{d^2x^h}{dt^2} + \left\{\frac{h}{j\ \ i}\right\} \frac{dx^j}{dt}\frac{dx^i}{dt} - \alpha\ \frac{dx^h}{dt} + \beta\ F_j^h\ \frac{dx^j}{dt}\right] = \gamma\ \frac{dx^h}{dt} + \delta\ F_j^h\ \frac{dx^j}{dt} \tag{2.3}$$

along any H-plane curve, where γ and δ are certain functions of t.

Denoting the Lie-derivative of the Christoffel's symbols and the complex structure $F^h{}_i$, respectively, by

$$t^h{}_{ji} = £_v \left\{\frac{h}{j\ i}\right\},\ \alpha^h{}_i = £_v\, F^h{}_i,$$

We have from (2.3)

$$t^h{}_{ji}\, \dot{x}^j \dot{x}^i + \alpha\, \dot{x}^h + b\, F^h{}_j\, \dot{x}^j - \beta\, \alpha^h{}_j\, \dot{x}^j = 0 \tag{2.4}$$

Where we have put

$$\alpha = -(\gamma + £_v\, \alpha),\ b = -(\delta + £_v\, \beta),\quad \dot{x} = \frac{dx^i}{dt}$$

Since the relation (2.4) holds for any H-plane curve C, it must hold identically for any values of x^i and $\dot{x}^i$.

By means of the definition of the H-plane curve, we see further that the identity (2.4) holds for any value of the coefficient β.

Taking account of these arguments, we can easily see that relation

$$a^h_j \dot{x}^j = f \dot{x}^h + g F^h_j \dot{x}^j, \qquad (2.5)$$

$$t^h_{ji} \dot{x}^j \dot{x}^i = p \dot{x}^h + q F^h_j \dot{x}^j, \qquad (2.6)$$

hold for any values x^i and $\dot{x}^i$, where f, g, p and q are certain functions of x^i and $\dot{x}^i$.

Let α^i_j be a tensor on V such that

$$F^r_j a^i_r + a^r_j F^i_r = 0$$

We obtain by means of (2.5)

$$\alpha^h_i \equiv £_v F^h_i = 0 \qquad (2.7)$$

On the other hand, if we substitute (2.7) and $\nabla_j F^i_j = 0$ into the identify

$$\nabla_j £_v F^h_i - £_v \nabla_j F^h_i = F^h_r \; £_v \left\{ {r \atop j\,i} \right\} - F^r_i \; £_v \left\{ {h \atop j\,r} \right\},$$

Then we get

$$t^r_{ji} F^h_r = t^h_{jr} F^r_i \; . \qquad (2.8)$$

From (2.6) and (2.8), taking account of the fact that

$$t^h_{ji} = \alpha_j \delta^h_i + \alpha_i \delta^h_j - \overline{\alpha}_j F^h_i - \overline{\alpha}_i F^h_j ,$$

Where α_i is certain vector and $\overline{\alpha}_i = F^r_i \alpha_r$ we get

$$t^h_{ji} = £_v\{ {}_j{}^h{}_i \} = \rho_j \delta^h_i + \rho_i \delta^h_j - \overline{\rho}_j F^h_i - \overline{\rho}_i F^h_j, \qquad (2.9)$$

Where ρ_i is certain vector field. Therefore, the infinitesimal transformation V^i is an analytic HP-transformation.

Conversely, it is obvious that an analytic HP-transformation preserves the H-plane curves.

Thus we have the following:

THEOREM (2.1): In an almost Kaehlerian space, an infinitesimal transformation preserves the H-plane curves, if and only if it is an analytic HP-transformation.

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STOCHASTIC ANALYSIS OF A SYSTEM WITH REPLACEMENT TIME AND INSPECTION OF THE FAILED UNIT BY POSSIBLE MAINTENANCE OF STANDBY UNIT

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ABSTRACT

A stochastic system of two identical units has been probed by conducting inspection of the standby unit to examine its possible goodness for operation at the failure of operating unit. There is a single server who visits the system immediately to rectify the faults which occur during system operation. The inspection of the failed unit is done to see the feasibility of its repair. If repair of the unit is not feasible to the system, it is replaced by new one after a time 't'. All random variables are statistically independent. The expressions for the time to failure of the unit follows negative exponential distribution while the distributions for maintenance, inspection repair and replacement times are taken as arbitrary with different probability density functions. The maintenance and repair of the unit are perfect. The semi-Markov process and regenerative point technique are used to derive the expressions for some reliability measures of vital significance. The graphical behavior of MTSF, availability and profit function have been observed for particular values of different parameters and costs.

Keywords: Stochastic System, Maintenance, Immediate Replacement, Inspection, Repair and Reliability Measures.

INTRODUCTION

Due to industry needs and huge operating cost for maintenance and repair of stochastic system, competitive market force to cut down either reliability of the system or innovate new functioning techniques. As a result several studies have been conducted on identification of improvement in traditional policies or implementation of advancement. Therefore, stochastic models of systems have been considered broadly by the scholars counting Malik and Barak(2014),Goyal et al. (2015) and Chauhan(2016) on reliability modeling of redundant systems, they reflected a communal postulation that failed unit is replaced or repaired immediately. But, sometimes repair of a failed unit may not be possible and in such case inspection can play a dynamic role to reveal its possibility. Also, the require unit can be avail as some time is disbursed at inspection . Recently, Malik and Sudesh (2014) had developed reliability model of a cold standby system with priority to preventive maintenance over repair, Malik (2013) and Upma (2016) evaluated performance measures of cold standby systems with maintenance and repair.

The present study is devoted to considering idea of inspection on failure ,maintenance and chances of replacement after inspection after a specific period of time. There is a single server who inspects the unit at its failure to see the feasibility of repair or replacement of failed unit. However ,the failed unit is replaced by new one after some time, if the server declares the impossibility of repair. The random variables are statistically independent. The time to failure of the unit follows negative exponential distribution while the distributions for maintenance, inspection and repair times are taken as arbitrary with different probability density functions. The maintenance and repair of the unit are perfect. The semi-Markov process and regeneration point technique are used to derive the expressions for some important reliability measures such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server for inspection, maintenance and repair, expected number of visits of the server and profit function. The graphical behavior of MTSF, availability and profit function have been observed for particular values of different parameters and costs.

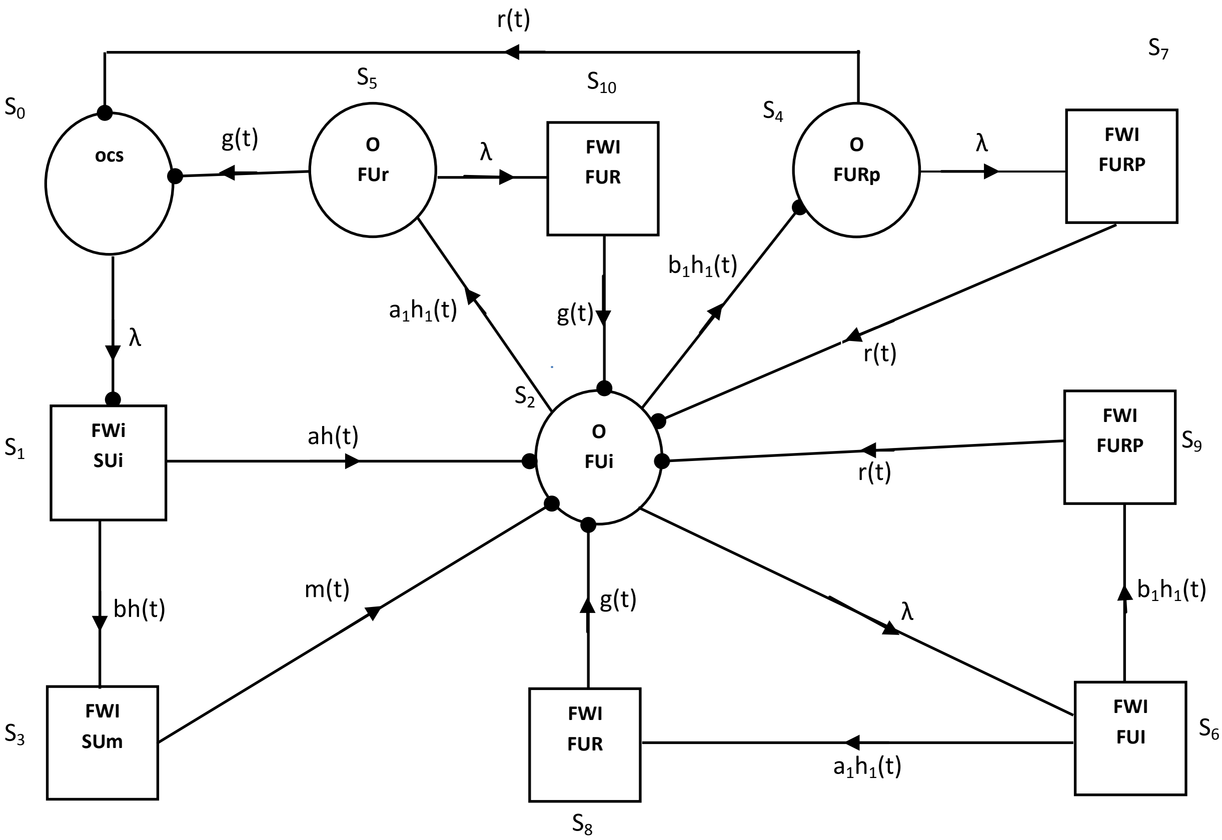
2. NOTATIONS:

E	:	Set of regenerative states.
o/cs	:	The unit is in operative/cold standby mode.
λ	:	Constant failure rate of the unit.
a/b	:	Probability that standby unit is operable/non-operable.
a_1/b_1	:	Probability that the failed unit is repairable/replaceable.
$h_1(t)/H_1(t)$	:	pdf/cdf of inspection time of the unit.
g(t)/ G(t)	:	pdf/cdf of repair time of the unit.
r(t)/ R(t)	:	pdf/cdf of replacement time of the unit.
SU_i/SU_m	:	Standby unit under inspection/under maintenance.
FUr /FUR	:	The unit is failed and under repair / under repair continuously from previous state.
FURp /FURP	:	The unit is failed and under replacement / under replacement Continuously from previous state.
FUi /FUI	:	The unit is failed and under inspection / under inspection continuously from previous state.
FWr/FWR	:	The unit is failed and waiting for repair/waiting for repair continuously from previous state.
FWi/FWI	:	The unit is failed and waiting for inspection/waiting for repair continuously from previous state
$q_{ij}(t)/Q_{ij}(t)$	:	pdf /cdf of direct transition time from a regenerative state S_i to regenerative state Sj or to a failed state without visiting to any other regenerative state in (0,t].
$q_{ij.k}(t)/Q_{ij.k}(t)$	:	pdf/cdf of first passage time for a regenerative state S_i to regenerative state S_j or to failed state $S_j$visiting state S_k once in (0,t].
m(t)/M(t)	:	pdf/cdf of maintenance time of standby unit.
$W_i(t)$	:	Probability that the server is busy in state S_i up to time twithout making transition to any other regenerative state or returning to the same via one or more regenerative states.
$M_i(t)$	:	Probability that the system is up initially in state $S_i \in E$ is up at the time "t" without visiting to any other regenerative state.
m_{ij}	:	Contribution to mean sojourn time in state S_i when system transits directly to state $S_j(S_i, S_j \in E)$so that $\mu_i = \sum m_{ij}$ where $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*'}(0)$
μ_i	:	The mean sojourn time spent in state $S_i \in E$ before transition to any other state.
pdf/cdf	:	probability density function/cumulative density function.

$\boxed{S}$/©	:	Laplace Stieltjes convolution/ Laplace convolution.
~/*	:	Symbol for Laplace Stieltjes transform (LST)/ Laplace transform (LT)
′	:	Symbol for derivative of the function.

The possible transition states of the system model is shown in figure 1

State Transition Diagram



3.TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\, dt \quad \text{as}$$

$p_{01} = 1,\ \ p_{12} = \mathrm{ah}^*(0),\ p_{13} = \mathrm{bh}^*(0),\ p_{24} = \mathrm{b_1h_1}^*(\lambda),\ p_{25} = \mathrm{a_1h_1}^*(\lambda),$

$p_{26} = \left(1 - \mathrm{h_1}^*(\lambda)\right),\ p_{32} = \mathrm{m}^*(0)p_{40} = \mathrm{r}^*(\lambda),\ \ p_{47} = \left(1 - \mathrm{r}^*(\lambda)\right)p_{50} = \mathrm{g}^*(\lambda),$

$p_{5,10} = 1 - \mathrm{g}^*(\lambda), p_{67} = \mathrm{g}^*(\lambda), p_{68} = \mathrm{a_1h_1}^*(0), p_{69} = \mathrm{b_1h_1}^*(0),\ p_{72} = \mathrm{r}^*(0), p_{82}{=}\mathrm{g}^*(0), p_{92}$
$= \mathrm{r}^*(0), p_{10,2} = \mathrm{g}^*(0) \dots (1)$

It can easily be verified that

$$p_{01} = p_{12} + p_{13} = p_{12} + p_{12.3} = p_{24} + p_{25} + p_{26} = p_{24} + p_{25} + p_{22.68} + p_{22.69} = p_{32} = p_{40} + p_{47}$$
$$= p_{40} + p_{42.7} = p_{50} + p_{57} = p_{68} + p_{69} = p_{72} = p_{82} = p_{92} = p_{10,2} = 1 \quad \dots (2)$$

The mean sojourn times μ_i in state S_i is given by

$$\mu_0 = \int_0^{\infty} P(T > t)dt = \mathrm{m}_{01} = \frac{1}{\lambda}$$

$\mu_1 = m_{12} + m_{13} = -h^{*'}(0), \mu_2 = m_{24} + m_{25} + m_{26} = \frac{1}{\lambda}(1 - h_1{}^*(\lambda)),$
$\mu_3 = m_{32} = -m^{*'}(0), \mu_4 = m_{40} + m_{47} = \frac{1}{\lambda}(1 - r^*(\lambda)),$
$\mu_5 = m_{50} + m_{510} = \frac{1}{\lambda}(1 - g^*(\lambda), \mu_6 = m_{68} + m_{69} = -h_1{}^{*'}(0), \mu_7 = m_{72} = -r^{*'}(0),$
$, \mu_8 = m_{82} = -g^{*'}(0), \mu_9 = m_{92} = -g^{*'}(0), \mu_{10} = m_{10,2} = -g^{*'}(0)$

$\mu_1{}' = m_{12} + m_{12.3} = [-ah^{*'}(0) - bm^{*'}(0)h^*(0) - bm^*(0)h^{*'}(0)], \mu_2' = m_{24} + m_{25} + m_{22.68} +$
$m_{22.69} = \frac{1}{\lambda}(1 - h^*(\lambda))\left(1 - a_1\lambda g^{*'}(0) - b_1\lambda r^{*'}(0) - \lambda h_1{}^{*\prime}(0)\right)$

$$\mu_4' = m_{40} + m_{42.7} = (1 - r^*(\lambda))r^{*'}(0) + r^{*'}(\lambda) + \frac{1}{\lambda}[1 - r^*(\lambda)],$$

$$\mu_5' = m_{50} + m_{52.10} = (1 - g^*(\lambda))\frac{1}{\lambda} \quad \ldots (3)$$

4. MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\emptyset_i(t)$ be the cdf of the first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\emptyset_i(t)$:

$$\emptyset_0(t) = Q_{01}(t) \quad \ldots (4)$$

Taking LST of the relations (4.4) and solving for $\widetilde{\phi_0}(s)$ we get

$$MTSF(T_0) = \lim_{s \to 0} \frac{1 - \widetilde{\phi_0}(s)}{s} = \frac{N_1}{D_1} \quad \cdots (5)$$

Where

$N_1 = \mu_0$ and $D_1 = 1$

5. AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is in upstate at instant $'t'$ given that the system entered regenerative state S_i at $t = 0$.The recursive relations for $A_i(t)$ are given as

$A_0(t) = M_0(t) + q_{01}(t)©A_1(t)$
$A_1(t) = (q_{12}(t) + q_{12.3}(t))©A_2(t)$
$A_2(t) = M_2(t) + q_{24}(t)©A_4(t) + q_{25}(t)©A_5(t) + (q_{22.68}(t) + q_{22.69}(t))©A_2(t)$
$A_4(t) = M_4(t) + q_{40}(t)©A_0(t) + q_{42.7}(t)©A_2(t)$
$A_5(t) = M_5(t) + q_{50}(t)©A_0(t) + q_{52.10}(t)©A_1(t) \quad \ldots (6)$

Where

$M_0(t) = e^{-\lambda t}, M_1(t) = e^{-\lambda t}\overline{H_1(t)}, \ M_4(t) = e^{-\lambda t}\overline{R(t)}$ and $M_5(t) = e^{-\lambda t}\overline{G(t)}$

Taking LT of relations (6) and solving for $A_0^*(s)$, we get steady-state availability as

$$A_0 = \lim_{s\to 0} s\, A_0^*(s) = \frac{N_2}{D_2} \qquad \cdots (7)$$

$N_2 = \mu_0(p_{25}p_{50} + p_{24}p_{40}) + \mu_2 + p_{24}\mu_4 + p_{25}\mu_5$ and

$D_2 = (p_{25}p_{50} + p_{24}p_{40})(\mu_0 + \mu_1^{'}) + \mu_2^{'} + p_{24}\mu_4^{'} + p_{25}\mu_5^{'}$.

6. BUSY PERIOD ANALYSIS OF THE SERVER

6.1. Due to Inspection and Maintenance:

Let $B_i^{im}(t)$ be the probability that the server is busy at instant 't' due to inspection and maintenance given that the system entered regenerative state S_i at t=0.The recursive relations for $B_i^{im}(t)$ are:

$B_0^{im}(t) = q_{01}(t)©B_1^{im}(t))$

$B_1^{im}(t) = (q_{12}(t) + q_{12.3}(t))©B_2^{im}(t)$

$B_2^{im}(t) = W_2^{im}(t) + q_{24}(t)©B_4^{im}(t) + q_{25}(t)©B_5^{im}(t) + (q_{22.68}(t) + q_{22.69}(t))©B_2^{im}(t)$

$B_4^{im}(t) = q_{40}(t)©B_0^{im}(t) + q_{42.7}(t))©B_2^{im}(t)$

$B_5^{im}(t) = q_{50}(t)©B_0^{im}(t) + q_{52.10}(t))©B_2^{im}(t) \qquad \ldots (8)$

Where

$W_1^{im}(t) = \overline{H(t)} + (bh(t)©1)\overline{M(t)}$

$W_2^{im}(t) = e^{-\lambda t}\overline{H_1(t)} + (\lambda e^{-\lambda t}©1)\overline{H_1(t)}$

Taking LT of relations (8)and solving for $B_0^{im^*}(s)$. In the long run, the time for which the system is under inspection and maintenance is given by

$$B_0^{im}(\infty) = \lim_{s\to 0} s\, B_0^{im^*}(s) = \frac{N_3}{D_2} \qquad \ldots (9)$$

Where

$N_3 = (p_{25}p_{50} + p_{24}p_{40})W_1^{im*}(0) + W_2^{im*}(0)$ and D_2 is already defined.

6.2. Due to repair:

Let $B_i^r(t)$ be the probability that the server is busy at instant 't' due to repair given that the system entered regenerative state S_i at t=0. The recursive relations for $B_i^r(t)$ are as follows:

$B_0^r(t) = q_{01}(t)©B_1^r(t)$

$B_1^r(t) = (q_{12}(t) + q_{12.3}(t))©B_2^r(t)$

$B_2^r(t) = q_{24}(t)©B_4^r(t) + q_{25}(t)©B_5^r(t) + (q_{22.68}(t) + q_{22.69}(t))©B_2^r(t) \qquad \cdots (10)$

$B_4^r(t) = W_4^{rp}(t) + q_{40}(t)©B_0^r(t) + q_{42.7}(t)©B_2^r(t)$

$B_5^r(t) = W_5^r(t) + q_{50}(t)©B_0^r(t) + q_{52.10}(t)©B_2^r(t)$

Where

$$W_5^r(t) = e^{-\lambda t}\overline{G(t)} + (\lambda e^{-\lambda t}©1)\overline{G(t)}$$

$$W_4^{rp}(t) = e^{-\lambda t}\overline{R(t)} + \left(\lambda e^{-\lambda t} ©1\right)\overline{R(t)}$$

Taking LT of relations (10)and solving for $B_0^{r^*}(s)$. In the long run, the time for which the system is under repair is given by

$$B_0^r(\infty) = \lim_{s\to 0} s\, B_0^{r^*}(s) = \frac{N_4}{D_2} \;\; (i = 1,2) \qquad \ldots(11)$$

Where,

$$N_4 = p_{24}W_4^{rp*}(0) + p_{25}W_5^{r*}(0)$$

and D_2 is already defined.

7. EXPECTED NUMBER OF VISITS BY THE SERVER

Let $N_i(t)$ be the expected number of visits by the ordinary server in $(0, t]$ given that the system entered the regenerative state S_i at $t = 0$. The recursive relations for $N_i(t)$ are given by

$N_0(t) = Q_{01}(t)\boxed{S}[N_1(t)]$

$N_1(t) = [Q_{12}(t) + Q_{12.3}(t)]\boxed{S}N_2(t)$

$N_2(t) = Q_{24}(t)\boxed{S}N_4(t) + Q_{25}(t)\boxed{S}[1 + N_5(t)] + \left(Q_{22.68}(t) + Q_{22.69}(t)\right)\boxed{S}N_2(t)$

$N_4(t) = Q_{40}(t)\boxed{S}N_0(t) + Q_{42.7}(t)\boxed{S}N_2(t)$

$N_5(t) = Q_{50}(t)\boxed{S}N_0(t) + Q_{52.10}(t)\boxed{S}N_1(t)$ $\cdots(12)$

Taking LST of relations (12) and solving for $\widetilde{N_0}(s)$, we get the expected number of visits by server per unit time as

$$N_0 = \lim_{s\to 0} s\, \widetilde{N_0}(s) = \frac{N_5}{D_2} \qquad \ldots(13)$$

Where

$N_5 = (p_{24}p_{10} + p_{25}p_{10})andD_2$is already defined

8. COST-BENEFIT ANALYSIS

Profit incurred to the system model in steady state is given by:

$P = K_1A_0 - K_2B_0^{im} - K_3B_0^{r} - K_4N_0$

where,

K_1	=	Revenue per unit uptime of the system.
K_2	=	Cost per unit time for which server is busy due to inspection and maintenance.
K_3	=	Cost per unit time for which server is busy due to repair and replacement.
K_4	=	Cost per unit time visits by the server.

9. PARTICULAR CASE

Let us consider $g\,(t) = \alpha e^{-\alpha t},\ h(t) = \beta e^{-\beta t},\ h_1(t) = \beta_1 e^{-\beta_1 t}, \mathrm{m}(t) = \gamma e^{-\gamma t}, r(t) = \theta e^{-\theta t}$

By using the non-zero element p_{ij}, we obtain the following results:

$N_1 = 1, D_1 = \lambda$

$$N_2 = \left[\frac{\alpha}{\alpha+\lambda}\cdot\frac{a_1\beta_1}{\beta_1+\lambda} + \frac{b_1\beta_1}{\beta_1+\lambda}\cdot\frac{\theta}{\theta+\lambda}\right]\cdot\frac{1}{\lambda} + \frac{1}{\beta_1+\lambda} + \frac{a_1\beta_1}{\beta_1+\lambda}\cdot\frac{1}{\alpha+\lambda} + \frac{b_1\beta_1}{\beta_1+\lambda}\cdot\frac{1}{\theta+\lambda}$$

$$N_3 = \left[\frac{\alpha}{\alpha+\lambda}\cdot\frac{a_1\beta_1}{\beta_1+\lambda} + \frac{b_1\beta_1}{\beta_1+\lambda}\cdot\frac{\theta}{\theta+\lambda}\right]\cdot\left[\frac{1}{\beta} + \frac{b\beta}{\gamma(\gamma+\beta)}\right] + \frac{1}{\beta_1}$$

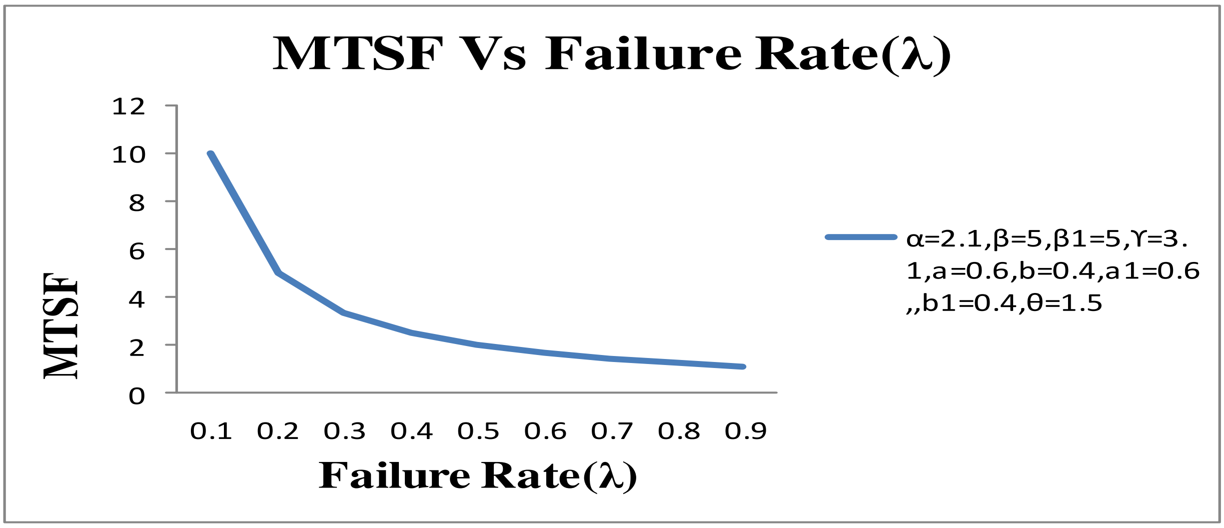
$$N_4 = \left[\frac{a_1}{\alpha} + \frac{b_1}{\theta}\right]\cdot\frac{\beta_1}{\beta_1+\lambda}$$

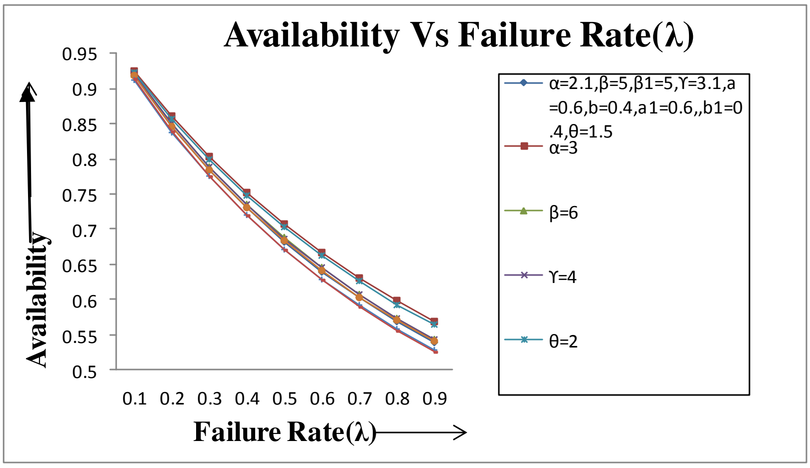
$$N_5 = \left[\frac{\alpha}{\alpha+\lambda}\cdot\frac{a_1\beta_1}{\beta_1+\lambda} + \frac{b_1\beta_1}{\beta_1+\lambda}\cdot\frac{\theta}{\theta+\lambda}\right]$$

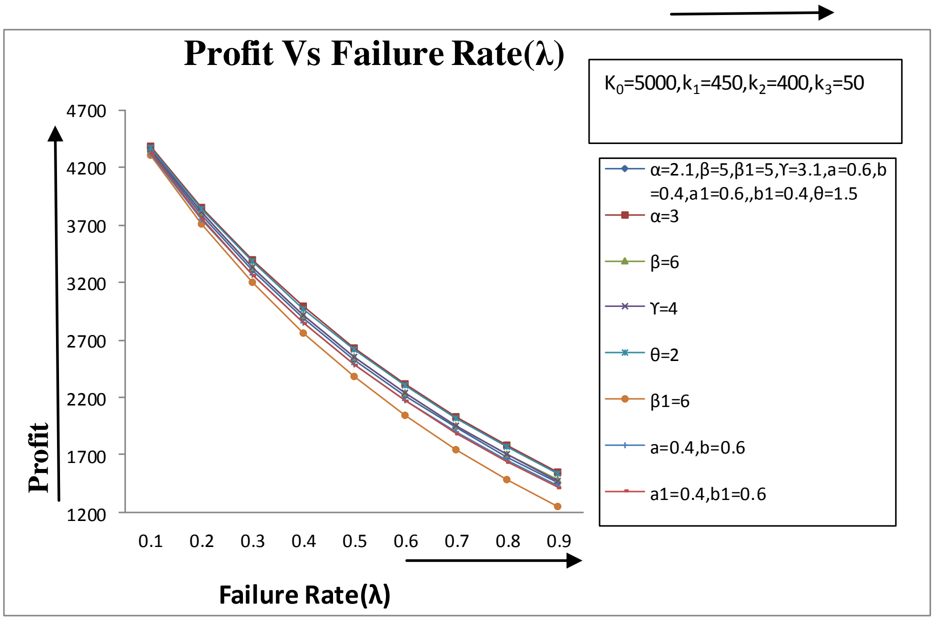
$$D_2 = \left[\frac{\alpha}{\alpha+\lambda}\cdot\frac{a_1\beta_1}{\beta_1+\lambda} + \frac{b_1\beta_1}{\beta_1+\lambda}\cdot\frac{\theta}{\theta+\lambda}\right]\cdot\left[\frac{1}{\lambda} + \frac{1}{\beta} + \frac{b}{\gamma}\right] - \frac{\beta_1}{(\beta_1+\lambda)^2} + \left[\frac{a_1}{\alpha} + \frac{1}{\beta_1} + \frac{b_1}{\theta}\right] + \frac{b_1.\beta_1}{(\beta_1+\lambda)}\cdot\frac{1}{\theta} + \frac{1}{\beta+\lambda} + \frac{1}{\alpha}\cdot\frac{a_1\beta_1}{\beta_1+\lambda}$$

10.CONCLUSION:

The stochastic model has been observed for different values of parameters like ($\alpha,\beta,\beta_1,\gamma$,a,b,$a_1$,$b_1$and θ). Also, graphs of some performance measures mean time to system failure (MTSF), availability and profit function have been drawn with respect to failure rate(λ) as shown in figures 2to 4. The trend of graphs of these three go on decreasing with the increase of failure rate (λ) while they increase with the increase of repair rate (α), inspection rate (β_1), maintenance rate (γ) and replacement rate(θ). But performance measures follows decreasing pattern if standby unit has probably more chances of its maintenance (a>b) after inspection.. Again, system becomes less profitable by making replacement of the failed unit rather than its repair ($a_1<b_1$). Hence, on the basis of the results obtained for a particular cases reveal that a system in which cold standby unit has more chances of its maintenance before operation can be made more profitable by making repair of the failed unit by new one instead of its repair







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INVENTORY MODEL WITH WEIBULL DISTRIBUTED DETERIORATING ITEMS, VARIABLE DEMAND RATE AND TIME VARYING HOLDING COST

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1. ABSTRACT

An Inventory model with variable demand, Weibull distributed deterioration rate, time dependent holding cost has been explored under without shortage constraints. The demand is considered a quadratic function of time started from zero and ending also on zero. The objective of the paper is to find the optimal value of production cycle time which minimizes the total average cost over the time horizon [0,T].

2. INTRODUCTION

Most of the inventory models developed in the past under the assumption that the holding cost is constant for the entire inventory cycle but the assumption is not significant in the real life situation. In the storage of deteriorating items such as food products, the longer these food products are kept in storage, the more sophisticated storage facilities and services are needed and therefore higher holding cost. So in these cases, holding cost should be assumed as a function of time. Madhu Jain (2010) developed EOQ model for deteriorating items which has been exponential declining rate of demand under inflation and delay in payments. Goswami and Chaudhuri (1981) discussed EOQ model for deteriorating item with shortages and linear trend in demand. Covert and Philip (1973) developed an EOQ model for items with variable rate of deterioration. Sharma Anil etal (2012) and Harikishan etal (2012) in their separately study developed the price dependent inventory model under different parameters and distributed function form. This paper investigates simple inventory model with variable demand rate, Weibull distributed deterioration rate, without shortages and time dependent holding cost .To consider deterioration rate in form of Weibull distributed is justified in context of inventory problem since items or materials in bulk contain a lot of defective materials which deteriorate early over times as such items are weeded out of population. A value of $\beta < 1$ indicates that the deterioration rate decreases over time which is Lindy effect. In case some external events cause deterioration value of $\beta=1$ indicating the deterioration rate to be constant. Also in inventory problem aging process or parts of items which move likely to deteriorate as time horizon goes on has also been observed. This is the case of $\beta >1$ which implies that deterioration rate increases with respect to time horizon. The holding cost has been taken as a linear function of time while the demand rate is supposed to be a quadratic function of time starting from zero and ending also on zero.

This paper differs with the study of Harikishan etal (2013) that the inventory holding cost is a linear function of time while the demand rate is quadratic function of time and Weibull distributed deterioration rate. The cost minimization technique has been approached to obtain the optimum value of stock level and time horizon.

2. ASSUMPTIONS AND NOTATIONS:

The model is developed under the following assumptions;

1) Replenishment size is constant and the replenishment rate is infinite.
2) Lead time is zero.
3) T is the fixed length of each production cycle;
4) C_1 = b+ct is the inventory holding cost per unit per unit time;
5) C_3 is the cost of each deteriorated unit;
6) The deterioration rate function θ(t) represents the on-hand inventory deteriorates per unit time and Moreover in the present study the function assumed in the form $\theta(t) = \alpha\beta t^{\beta-1}$; $0 < \alpha < 1$, $\beta > 0$, $t > 0$.
   When β = 1, θ(t) becomes a constant which is a case of exponential decay. When β < 1, the rate of deterioration is decreasing with t and when β > 1, the rate of deterioration is increasing with t.
7) The demand rate starts from zero and ends at zero during the inventory period. It is assumed of the form D(t) = at(T−t) where T is the cycle period.

3. MATHEMATICAL MODEL AND ANALYSIS

Let us assume we get an amount S (S>0) as an initial inventory. Inventory level gradually diminishes due to reasons of market demand and deterioration of the items and ultimately falls to zero at time T . Let I(t) be the on hand inventory at any time t. The differential equations which the on hand inventory I(t) must satisfy the following :

$$\frac{dI(t)}{dt} + \theta\, I(t) = -D(t), \qquad 0 \le t \le T \qquad \text{.........(1)}$$

By using D(t) = at(T−t) the differential equation (1) can be rewritten as

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1}\, I(t) = -at(T-t), \qquad 0 \le t \le T \qquad \text{.........(2)}$$

Solution of differential equation (2) is

$$I(t)\, e^{\alpha t^{\beta}} = -\int at(T-t)\, e^{\alpha t^{\beta}}\, dt + C$$

$$I(t)\, e^{\alpha t^{\beta}} = -\int at(T-t)\,(1 + \alpha t^{\beta})\, dt + C$$

$$I(t)\, e^{\alpha t^{\beta}} = -a\left(\frac{Tt^2}{2} + \frac{\alpha T t^{\beta+2}}{\beta+2} - \frac{t^3}{3} - \frac{\alpha t^{\beta+3}}{\beta+3}\right) + C \qquad \text{.........(3)}$$

Since I(0) =S, we have C = S then Equation (3) gives

$$I(t)\, e^{\alpha t^{\beta}} = -a\left(\frac{Tt^2}{2} + \frac{\alpha T t^{\beta+2}}{\beta+2} - \frac{t^3}{3} - \frac{\alpha t^{\beta+3}}{\beta+3}\right) + S \qquad \text{.........(4)}$$

Using the boundary condition I(T)=0 we get

$$S = a\frac{T^3}{6} + a\,\frac{\alpha}{(\beta+2)(\beta+3)}\, T^{\beta+3} \qquad \text{.........(5)}$$

Putting the value of S in (4), we get

$$I(t)\, e^{\alpha t^{\beta}} = -\,a\left[\frac{Tt^2}{2}+\frac{\alpha T t^{\beta+2}}{\beta+2}-\frac{t^3}{3}-\frac{\alpha t^{\beta+3}}{\beta+3}-\frac{T^3}{6}-\frac{\alpha}{(\beta+2)(\beta+3)}T^{\beta+3}\right]$$

$$I(t) = -\,a\left[\frac{Tt^2}{2}+\frac{\alpha T t^{\beta+2}}{\beta+2}-\frac{t^3}{3}-\frac{\alpha t^{\beta+3}}{\beta+3}-\frac{T^3}{6}-\frac{\alpha}{(\beta+2)(\beta+3)}T^{\beta+3}\right] e^{-\alpha t^{\beta}}$$

$$I(t) = -\,a\left[\frac{Tt^2}{2}+\frac{\alpha T t^{\beta+2}}{\beta+2}-\frac{t^3}{3}-\frac{\alpha t^{\beta+3}}{\beta+3}-\frac{T^3}{6}-\frac{\alpha T^{\beta+3}}{(\beta+2)(\beta+3)}\right](1-\alpha t^{\beta})$$

$$I(t) = -\,a\left[\frac{Tt^2}{2}+\frac{\alpha T t^{\beta+2}}{\beta+2}-\frac{t^3}{3}-\frac{\alpha t^{\beta+3}}{\beta+3}-\frac{T^3}{6}-\frac{\alpha T^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{\alpha T t^{\beta+2}}{2}-\frac{\alpha^2 T t^{2\beta+2}}{\beta+2}+\frac{\alpha t^{\beta+3}}{2}+\frac{\alpha^2 t^{2\beta+3}}{\beta+3}+\frac{\alpha t^{\beta}T^3}{6}+\frac{\alpha^2 T^{\beta+3}t^{\beta}}{(\beta+2)(\beta+3)}\right] \quad ....(6)$$

Hence total amount of deteriorated units (D)=I(0)−stock loss due to demand

$$= S - \int_0^T at(T-t)\; dt$$

$$= \frac{a\alpha}{(\beta+2)(\beta+3)}T^{\beta+3} \quad ....(7)$$

Total Inventory held $(I_1) = \int_0^T (b+ct)I(t)\; dt$

$$= \int_0^T \Big\{-ab\left[\frac{Tt^2}{2}+\frac{\alpha T t^{\beta+2}}{\beta+2}-\frac{t^3}{3}-\frac{\alpha t^{\beta+3}}{\beta+3}-\frac{T^3}{6}-\frac{\alpha T^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{\alpha T t^{\beta+2}}{2}-\frac{\alpha^2 T t^{2\beta+2}}{\beta+2}+\frac{\alpha t^{\beta+3}}{2}+\frac{\alpha^2 t^{2\beta+3}}{\beta+3}+\frac{\alpha t^{\beta}T^3}{6}+\frac{\alpha^2 T^{\beta+3}t^{\beta}}{(\beta+2)(\beta+3)}\right] - ac\left[\frac{Tt^3}{2}+\frac{\alpha T t^{\beta+3}}{\beta+2}-\frac{t^4}{3}-\frac{\alpha t^{\beta+4}}{\beta+3}-\frac{T^3 t}{6}-\frac{\alpha T^{\beta+3}t}{(\beta+2)(\beta+3)}-\frac{\alpha T t^{\beta+3}}{2}-\frac{\alpha^2 T t^{2\beta+3}}{\beta+2}+\frac{\alpha t^{\beta+4}}{2}+\frac{\alpha^2 t^{2\beta+4}}{\beta+3}+\frac{\alpha t^{\beta+1}T^3}{6}+\frac{\alpha^2 T^{\beta+3}t^{\beta+1}}{(\beta+2)(\beta+3)}\right]\Big\}\, dt$$

$$I_1 = a\, c\alpha^2 \frac{3\beta-\beta^2-6}{(\beta+2)^2(\beta+3)}T^{2\beta+5} + a\,\alpha^2 \frac{18+17\beta-3\beta^2-\beta^3}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(2\beta+3)}T^{2\beta+4} + a\, c\alpha \frac{2\beta^2+10\beta+12}{2(\beta+2)(\beta+3)(\beta+4)(\beta+5)}T^{\beta+5} + a\alpha\frac{5\beta^2+20\beta+9}{6(\beta+1)(\beta+3)(\beta+4)}T^{\beta+4} + ac\frac{1}{40}T^5 + a\frac{1}{8}T^4 \quad ....(8)$$

Cost of deteriorated items = C_3 × total amount of deteriorated units

$$= C_3 \frac{a\alpha}{(\beta+2)(\beta+3)}T^{\beta+3} \quad ....(9)$$

Average total cost per unit time C(T)= $\frac{1}{T}$[Total cost per unit time] = $\frac{1}{T}$[Total Inventory held +Cost of deterioration items]

$$= ac\alpha^2 \frac{3\beta-\beta^2-6}{(\beta+2)^2(\beta+3)}T^{2\beta+4} + a\alpha^2 \frac{18+17\beta-3\beta^2-\beta^3}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(2\beta+3)}T^{2\beta+3} + ac\alpha\frac{2\beta^2+10\beta+12}{2(\beta+2)(\beta+3)(\beta+4)(\beta+5)}T^{\beta+4} + a\alpha\frac{5\beta^2+20\beta+9}{6(\beta+1)(\beta+3)(\beta+4)}T^{\beta+3} + ac\frac{1}{40}T^4 + a\frac{1}{8}T^3 + C_3\frac{a\alpha}{(\beta+2)(\beta+3)}T^{\beta+2} \quad ....(10)$$

$$\frac{dC(T)}{dT} = ac\alpha^2 \frac{(3\beta-\beta^2-6)(2\beta+4)}{(\beta+2)^2(\beta+3)}T^{2\beta+3} + a\alpha^2 \frac{18+17\beta-3\beta^2-\beta^3}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)}T^{2\beta+2} + ac\alpha \frac{2\beta^2+10\beta+12}{2(\beta+2)(\beta+3)(\beta+5)}T^{\beta+3} + a\alpha\frac{5\beta^2+20\beta+9}{6(\beta+1)(\beta+4)}T^{\beta+2} + ac\frac{1}{10}T^3 + a\frac{3}{8}T^2 + C_3\frac{a\alpha}{(\beta+3)}T^{\beta+1}$$

…….(11)

$$\frac{d^2C}{dT} = a\,c\alpha^2 \frac{(3\beta-\beta^2-6)(2\beta+3)(2\beta+4)}{(\beta+2)^2(\beta+3)}T^{2\beta+2} + a\,\alpha^2 \frac{2(18+17\beta-3\beta^2-\beta^3)}{(\beta+2)(\beta+3)(\beta+4)}T^{2\beta+1} + ac\alpha \frac{2\beta^2+10\beta+12}{2(\beta+3)(\beta+5)}T^{\beta+2} + a\alpha\frac{(5\beta^2+20\beta+9)(\beta+2)}{6(\beta+1)(\beta+4)}T^{\beta+1} + ac\frac{1}{5}T^2 + a\frac{3}{4}T^1 + C_3\frac{a\alpha(\beta+1)}{(\beta+3)}T^{\beta}$$

……..(12)

For minimum $C(T)$,the necessary condition is $\frac{dC(T)}{dT} = 0$

After solving we get an equation of odd degree whose last term is negative, then there exists a unique solution $T^* \in (0,T)$ can be solved from equation (11) also clearly $\frac{d^2C}{dT^2} > 0$ at $T = T^*$ $\therefore$ C(T) is minimum at $T = T^*$ So optimum value of T is T^*.

CONCLUSION:

An Inventory model of deteriorating products has been explored in which demand rate is quadratic function of time while the deterioration rate follows Weibull Deterioration. The case of without shortage of items has been discussed.

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APPLICATIONS OF MULTI-FUNCTIONS

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ABSTRACT

In this paper, we introduce and study a new topological game for multi-functions.

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Key words and phrases. Topological game, !-open set, multi-functions.

1. INTRODUCTION

A topological game is a game in which positions are the points of a topological space, the set of possible moves from any such points varies continuously with the point. The play can start from any point of the space and at each point it is specified which player has the initiate, and ends when the set of positions from which the player with the initiate can choose is empty. The payo to each player depends on the set of positions met in the play. The concept of !-open sets was studied by Sundaram and Sheik John [4]. This notion was further studied by Sheik John and Sundaram in [6] and Noiri and Popa in [3]. In this paper, we have introduced and study a new topological game for multi-functions.

2. PRELIMINARIES

In this paper, consider each X_i is a topological space for each i = 1; 2; :::; n and X is a topological sum of fX_1; X_2; :::; X_ng.

Definition 2.1.

A game on X for the players (1), (2), ...(n) is de ned as follows

(1) A partition fN^+; N g of the set N of players.

(2) A partition fX_1; X_2; :::X_ng of X.

(3) An !-irresolute multifunction F of X onto itself such that F (X_i)\ X_i = ; for i = 1; 2; 3; :::; n.

(4) n-bounded real valued functions ff_1; f_2; :::; f_ng on X.

The points of X are the positions of the game, play can start from any position. If x 2 X_i is the move of player (i) at the position x. A play with x_0 as initial position proceeds as follows: If x 2 X_i, player (i) chooses a position x_1 in the set F (x_0). If x_1 2 X_j, player (j) chooses a position x_2 2 F (x_1), and so on. If in the course of the play a position x is reached where F (x) = ;, then the play terminates at x. Thus a play is a sequence < x_0; F (x_0); x_1; F (x_1)::: > such that x_0 2 F (x_0), x_1 2 F (x_1) and so on.

Definition 2.2.

For a finite sequence < x_0; F (x_0); x_1; F (x_1):::x_k; F (x_k) > with k + 1 elements of a play, the length of the play is said to be k, note that the last element x_k must satisfy F (x_k) = ;. If the length of each play of the game is finite, the game is called locally finite. If S is the set of positions in a play, the pay off to player (i) for the play is supff$_i$(x) : x 2 Sg or infff$_i$(x) : x 2 Sg according as (i) 2 N^+ or (i) 2 N . The aim of each player is to obtain as large pay off as possible.

Definition 2.3.

Player (i) is said to guarantee (where is a real number) from the initial position x if he can ensure that, whatever the other players do, his pay off for all plays starting at x is greater than or equal to . If he can ensure a pay off is greater than for a play begins at x, he is called strictly guarantee from x.

By a multifunction F : (X;) ! (Y;), we shall denote the upper and lower inverse of a set B of Y by F $^+$(B) and F (B), respectively, that is,

F $^+$(B) = x X : F (x) B and F (B) = x X : F (x) B = .
f 2 g f 2 \ 6 ;g
f 2 2 g 2
In particular, F (y) = x X : y F (x) for each point y Y and
if F (X) = Y . S
for each A X, F (A) = $_{x2A}$ F (x). Then F is said to be surjection

Definition 2.4.

[2] A subset S of a topological space (X;) is called a semi-open set if S Cl(Int(S)).

Definition 2.5.

[4] A subset A of a topological space (X;) is called an -closed set if Cl(A) U whenever A U and U is semi- open in X. The complement of an !-closed set is called an !-open set. The family of all !-open (!-closed) sets of (X;) is denoted by !O(X) (!C(X)).

Definition 2.6.

[2] A subset M(x) of a topological space (X;) is called an !-neighbourhood of a point x 2 X if there exists an !-open set S such that x 2 S M(x).

Definition 2.7.

[1] A multifunction F : (X;) ! (Y;) is said to be:

(1) upper !-irresolute if for each point x 2 X and each !-open set V containing F (x), there exists U 2 !O(X; x) such that F (U) V ;

(2) lower !-irresolute if for each point x 2 X and each !-open set V such that F (x) \ V 6= ;, there exists U 2 !O(X; x) such that U F (V).

Definition 2.8.

[5] A subset A of a space (X;) is said to be !-compact if every cover of A by !-open sets of (X;) has a nite subcover.

3. APPLICATIONS OF MULTIFUNCTIONS

Definition 3.1.

For a topological space X, a real valued function f : X ! R is upper (lower) !-continuous if for every x 2 X and every real number r satisfying f(x) < r (f(x) > r), there exists an !-open neighbourhood U X of x such that $f(x^0) < r$ $(f(x^0) > r)$ for every x^0 2 U.

Definition 3.2.

The game is said to be

(1) lower topological for player (i) if in addition the real valued function is lower !-continuous.

(2) upper topological for player (i) if f_i is upper !-continuous.

Theorem 3.3.

If a game is lower topological for (1) 2 N^+, then the set of positions from which (1) can strictly guarantee a gain is !-open in X. Proof. Let A be the set of initial positions from which (1) can strictly guarantee . Then (X_1 \ F (A) [([(X_j \ F^+(A))) A_F . Let j=1 us note that F^+(A) = fx 2 X : F (x) A g and F (A) = fx 2 : F (x) \ A 6= ;g. We construct by trans finite induction an !-open set A( ) such that A( ) A for each ordinal as follows: Let A(0) = fx 2 X : f_1(x) > g. Since the game is lower topological, then f_1 is lower !-continuous. Thus A(0) is !-open and A(0) A. Suppose that we have defined an !-open set A( ) A for all ordinal < . If is a limit ordinal, let A( ) = [A( ), then A( ) is !-open and A( )< A . If is not a limit ordinal, this means $=^0$+ 1 say, let A( ) = n A(^{0})(X_1 \ F (A(^{0}))) [[(X_j \ F^+(A($_0$))) by hypothesis, A(^{0}) is n=2 -open and hence X_1 \F (A(^{0})), X_j \F $^+$(A(^{0})) for each j = 2; 3; :::n are !-open. Since X is the topological sum of fX_1; X_2; :::; X_ng and F is an !-irresolute multifunction, thus A( ) is !-open and A( ) A .n For A(^{0}) A and (X_1 \ F (A(^{0}))) [[(X_j \ F^+(A(^{0}))) (X_1 \ j=2 n F (A) [[(X_j \F $^+$(A)) A. Therefore for each ordinal we have n=2 an !-open set A( ) and A( ) A. The transfinite sequence fA( )g is increasing and so must became ultimately constant. This means that, we have A($_0$) = A($_0$ + 1) = ::: for some $_0$. Let A^0 = XnA($_0$). If x 2 A^0 \ X_1, then F (x) \ A^0 whilst if x 2 A^0 \ X_j, where j 6= 1, then F (x) \ A^0 6= ;. herefore if a play begins from a point in A^0, whatever (1) does, players (2), (3),...,(n) can ensure that a position in A($_0$) is never reached. But A($_0$) A(0) = fx : f_1(x) > g. Thus if x 2 A^0 implies x 6= A($_0$), then x 6= A and so A A($_0$). But A($_0$) A by construction and so we have A = A($_0$). Hence A 2 !O(X).

Remark 3.4. Since the complement of each !-open set is !-closed, and is irresolute, F $^+$(X) is !-open, then X_0 = XnF $^+$(X) is !-closed.

Theorem 3.5. Suppose that a game is locally finite and upper topological for (1) 2 N^+ and that X_0 = fx : F (X) = ;g is a !-open set. Then the set of positions from which (1) can guarantee a gain is !-closed. Proof. We de ne an !-open set X( ) for each ordinal . Let X(0) = X_0 = fx : F (x) = ;g. Then X(0) is !-open. Now suppose we have constructed !-open sets X( ) for all ordinals < . If is a limit ordinal, let X( ) = [X( ) which is !-open. If has an immediate < predecessor ^{0}, that is = ^{0} + 1, let X( ) = X(^{0}) [F $^+$(X(^{0})), since F is an !-irresolute multifunction, then X( ) is !-open. Thus for each ordinal we have by transfinite induction !-open set X( ). We note that if < , X( ) < X( ). Now, we de ne H to be the set of positions from which (1) can guarantee . Then (X_1 \ F (H)) [n[(X_j \ F $^+$(H)) H . We de ne a set H( ) for each such that (i)j=2 H( ) H ; (ii) f < , H( ) H( ); (iii) if < , H( ) \X( ) = H( ) \ X( ) and (iv) H( ) \ X( ) is !-closed in X( ). Claim (1): Let H(0) = fx : f_1(x) g, since f_1 is upper !-continuous, then H(0) is !-closed in X; also H(0) H ; and so H(0) \ X(0) is-closed in X(0). Suppose that the set H( ) satisfying (i) (iv) have been constructed for all ordinals < . Claim (2): If is a limit ordinal, let H( ) = [H( ). Since each < H( ) H, then H( ) H . Also if < , then H( ) H( ) and if ^{0} < , H( ) \ X(^{0}) = ([H( )) \ X(^{0}) = [(H( )) \ X(^{0}). < < If < ^{0}, then H( ) \ X(^{0}) H(^{0}) \ X(^{0}), and if ^{0} < , H( ) \ X(^{0}) = H(^{0}) \ X(^{0}). Hence H( ) \ X(^{0}) = H(^{0}) \ X(^{0}) and (iii) is satisfied. If x 2 X( ) and x 2= H( ), then x 2 X( ) for some < and x 2= H( ). Now H( ) \ X( ) is !-closed in X( ) and so there is an !-open neighbourhood A of x in X( ) such that A \ H( ) = ;. Now, since X( ) and A are !-open sets in X and X( ) X( ), then A is !-open in X( ). By (iii), X( )\H( ) = X( ) \ H( ) and so A is an !-open neighbourhood of x in X( ) such that A \ H( ) = ;. Thus (iv) is satisfied. Claim (3) If has an immediate predecessor ^{0}, that is, = ^{0} + 1, n let H( ) = H(^{0}) [(X_1 \ F (H(^{0}))) [[(X_j \ F $^+$(H(^{0}))). Since j=2 n H(^{0}) H and X_1 \ F (H(^{0})) [[(X_j \ F $^+$(H(^{0}))) (X_1 \ j=2 n F (H)) [[(X_j \ F $^+$(H(^{0}))) H , then (i) satisfied, (ii) is clearly j=2 satisfied. Suppose ^{0} < . If x 2 X( ) and F (x) 6= ;, then F (x) X( ) for some < ^{0}. Thus if x 2 X(^{0}) \ (X_1 \ F (H(^{0}))), and so (iii) is satisfied. Finally, we prove (iv), suppose that x 2 X( ) and2= H( ). If x 2 X(^{0}), then x 2= H(^{0}) and since H(^{0}) \ X(^{0}) is ! -closed in X(^{0}), there is an !-open neighbourhood A of x in X(^{0}) such that A \ H(^{0}) = ;. Since X(^{0}) is !-open in X and A X(^{0}) X( ), then A is an !-open neighbourhood of x in X( ), and since X(^{0}) \ H( ) = X(^{0})H(^{0}), by (iii), then A \ H( ) = ;. If x 2 (X( )nX(^{0})) \ X_1, then F (x) (X(^{0})nH(^{0})). X(^{0})nH(^{0}) is -open in X(^{0}) and so is !-open in X. X_1 \F $^+$(X(^{0})nH(^{0})) is an !-open neighbourhood of x such that (X_1 \ F $^+$(X(^{0})nH(^{0}))) \ H( ) = ;. If x 2 (X( )nX(^{0})) \ X_j, then F

(x) \ (X( )nH(^{0})) 6= ; and X_j \ F (X(^{0})nH(^{0})) is an !-open neighbourhood of x such that (X_j \ F (X(^{0})nH(^{0}))) \ H( ) = ;. In either cases, if x 2 X( ) and 2= H( ), there is an !-open neighbourhood of x in X( ) not meeting H( ). Therefore H( ) \X( ) is !-closed in X( ) and (iv) is satisfied. Thus, by transfinite induction we can construct H( ) for each ordinal such that (i) (iv) are satisfied. Since the game is locally finite, then X = X($_0$) for some ordinal $_0$. Thus H($_0$) is !-closed set and if > $_0$, H( ) = H( ) \ H($_0$) = H($_0$). Let H^0 = XnH($_0$). If 2 H^0 \ X_1, then F (x) H^0, and if x 2 H^0 \ X_j where j 6= 1, then F (x)\H^0 6= ;. Thus if a play begins from a position in H^0, whatever (1) does, players (2), (3),...,(n) can ensure that a position in H($_0$) is never reached. But H($_0$) H(0) = fx : f_1(x) g and so H H($_0$). But H($_0$) H by construction and so we have H($_0$) = H . Thus H is !-closed. This complete the proof.

Example 3.6. Consider two players ONE and TWO played on the space consisting of the topological sum of X_1 and X_2 of the segment (1; m] of the real line. Let (x; i) be the point x 2 X_i and suppose that

F (x; i) = Suppose that (1) 2 N^+ and

f_1(x) = (x 1; j) if i 6= j, x > 0 if x 0.

1 if x 2 X_2, x 0

0 otherwise.

Then f_1 is upper !-continuous, and so the game is upper topological for (1) 2 N^+. The set of initial positions from which (1) can guarantee unit gain is f(x; 1) : 0 < x 1; 2 < x 3; ::g [f(x; 2) : 1 < x 2; 3 < x 4; ::g which is not !-closed.

Corollary 3.7.

If a game is upper topological for (1) 2 N , the set of positions from which (1) can guarantee is !-closed. Proof. Let A denotes the set of initial positions from which (1) can not guarantee . Similar to the proof of Theorem 3.3, we can construct an !-open set A such that fx : f_1(x) < g A A . Then XnA is !-closed in X. If x 2 XnA \ X1, then F (x) \ XnA 6= ;, which if x 2 XnA \ X_j such that j 6= 1, then F (x) XnA. Therefore if a play begins at a position in XnA, (1) can ensure that a position in H is never reached. Thus if H is the set of initial positions from which (1) can guarantee , H XnA. But A A and H \ A = ; and so H = XnA. Thus H is !-closed.

Corollary 3.8.

Suppose that a game is locally finite and lower topological for (1) 2 N and that X_0 = fx : f(x) = ;g is !-open set. Then the set of positions from which (1) can strictly guarantee a gain is -open set. Proof. Let K denotes the set of initial positions from which (1) can not strictly guarantee . By the modification of the argument used in proving Theorem 3.5, we can show that K is !-closed. But if A is the set of initial positions from which (1) can strictly guarantee , A = XnK and so A is !-open.

Definition 3.9.

Let X_0 = fx : F (x) = ;g, a tactic for player (i) is function : X_inX_0 ! X such that (x) 2 F (x) for all x 2 X_inX_0. A tactic for each player and an initial position determine a play of the game.

Definition 3.10.

A tactic for player (1) is said to guarantee him from an initial position x if whenever play begins at x and (1) uses a tactic he obtains a pay off greater than or equal to whatever tactics the other players are employed.

Definition 3.11.

If x 2 X, let (x) = supff : x 2 H g, where H is the set of positions from which (1) can guarantee . A tactic for player (1) is called optimal if it guarantees that (x) from the initial position for all x 2 X. Now, let denotes the set of tactics for player (1). Since each tactic for is a function from X_1 X_0 to X. If X^X1 $^X0$ is the set of functions from X_1 X_0 to X, then X^X1 $^X0$. We suppose that have the relativized product topology.

Theorem 3.12.

Suppose we have either

(a). a locally finite game upper topological for (1) 2 N^+ such that X_0 is an !-open set in X, or

(b). an upper topological game for (1) 2 N . If F (x) is !-compact for all x 2 X_1 X_0, then the set of optimal tactics for (1) is nonempty and !-closed in . Proof. Since each F (x) is !-compact for all x 2 X_1 X_0, then = F (x) is !-compact. Let S denotes the set of tactics with which x2X_1 X_0 can guarantee from any initial position in H, where H is the set of positions from which (1) can guarantee . Clearly, S 6= ; if H 6= ;. Now, we prove that S is !-closed in . Suppose 2 , 2= S . Then for some x 2 X_1 \ H , (x) 2= H . By condition (a) and Theorem 3.5 or condition (b) and Corollary 3.8, we get H is !-closed, so that there is an !-open neighbourhood N of (x) in X such that N \ H = ;. If M( ) = f : 2 g and (x) 2 N, M( ) is an !-open neighbourhood of and M( )\S = ;. Thus S is !-closed in . Let x_0 2 X, consider

the family fS : < (x_0)g. Suppose $_1$ < $_2$ < ::: < $_n$ < (x_0).

Then x_0 2 H $_i$ for each i and H $_1$ H $_2$ ::: H $_n$. Suppose H $_k$ \ (X_1 X_0) 6= ; and that k n the largest integer for which this is true. Then for k < j n, (H $_j$) \ (X_1 X_0) = ; and S $_j$ = . Let $_i$ 2 , where for x 2 X_1 X_0, 8 $_k$(x) : x 2 H $_j$ (x) = $_i$(x) : x 2 H $_i$ H $_{i+1}$, i = 1; 2; ::k 1 $_1$(x) : x 2 H $_2$. Then 2 S $_1$ \S $_2$ \:::\S $_n$. Thus for each x_0 2 X, fS : < (x_0)g is a family of nonempty !-closed sets with the finite intersection property. Let S(x) = \ S . So S(x) is nonempty !-closed. Now, consider < (x_0) the family fS(x) : x 2 Xg. Suppose x$_;$x$_2$; ::x$_n$ 2 X. If (x_m) = max$_{1\ i\ n}$ (x_i), S(x_m) S(x_i). Thus fS(x) : x 2 X g is the family of nonempty !-closed sets with the finite intersection property. Let S(x) = \ S(x). Thus S is nonempty and !-closed in . But S is x2X precisely the set of optimal tactics for (1). For if 2 S(x) for all < (x) and so guarantees for (1) that (x) if the play begins from x. Thus if 2 \ S(x), is an optimal tactic. Conversely, if is x2X an optimal tactic for (1), guarantees (1) if the play begins from x and so 2 <\(x)^s This is true for all x2X and so 2 x\X so 2 <\(x)^s

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ANALYSIS OF TWO NON-IDENTICAL DIE CASTING UNITS DIFFERING IN FAILURE MODE

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ABSTRACT

The present paper is an attempt to find out reliability of two non identical die casting units in which one is in working state and other is cold standby. The working unit has three modes; normal, partial failure ,total failure while standby unit has two modes normal and total failure .The primary unit may fail partially or completely ,when the unit goes for repair, the cold standby unit starts. The partial failed unit after repair is O.K. its starts in operative mode and other operative unit returns to standby mode .If the repair of partial failed unit is continued and other working unit is also failed after that repair facility stop repairing and put it under operation and start immediately repair other failed unit. Further, the other unit is O.K to operative mode repair facility restarts the repair of partial failed unit. There is a single repair facility which is always available with the system and the makes an attempt to repair the failed unit with full satisfaction. The repaired unit works as good as new. There are some states which are called up states and some down states. The up states are those states in which at least one machine is in operative mode either primary unit or secondary unit. The down states are states in which both machines are not in operative mode. The standby unit is used to replace the operative failed unit instantaneously. The Markov process and Regenerative point technique are used to explore about the reliability and profitability of the system.

***Keyword**: MTSF, Markov process, regenerative state, reliability, standby system.*

INTRODUCTION

A die casting two unit standby system consisting primary unit and secondary has been considered in this paper. We analyzed two non identical units, operative and cold standby .The primary operative unit has three modes; normal, partial failure total failure and second unit is cold standby have two modes normal, total failure. Single repair facility has been provided priority for primary unit. There are up and down states in system.

A large number of researchers in the field of reliability modeling including Nakagawa and Osaki (1975), Goel and Agnihotri (1992), Mokaddis and Labib (1997), Tuteja (2001), Sharma and Taneja (2011), Kumar and Bhatia (2011), Kumar and Rani (2013), V. Kumar and P. Bhatia with S. Ahmed (2014), etc. analyzed the one/ two unit redundant systems. Kumar and Vashistha (2001) explained the two unit redundant system with degradation and replacement of the faulty. Kumar and Bhatia (2011) discussed the behavior of the single unit centrifuge system considering the concepts of inspections, halt of system, degradation, minor/major faults, neglected faults, online/offline maintenances, repairs of the faults. Kumar and Rani (2013) explained the cost benefit analysis for a redundant system. V. Kumar and P. Bhatia with S. Ahmed (2014) explained in very detail the profit analysis for a two unit standby centrifuge system having a single repairman. Jain (2014) explained the different failures in a repairable redundant system. Recently Narender Singh etal (2018) studied the cost benefit analysis of two cold standby units under the influence of the snow storm.

A Pressure Die casting Machine is a mechanical device used for assembling of parts or some specified work in industry .The die casting machine are two types;1.cold chamber 2.hot chamber but we use horizontal high pressure cold chamber machines. It works automatically without any rest and more efficiently in comparison of any other means of working. These machines are used in most of the industry applications like car part assembling, bicycle part assembling etc. Nowadays, die casting machine works for replacement of a large group of people, the die casting machine works very fast in comparison of people's work. Also the die casting machine is less costly in comparison of work done cost by people. The work will not be affected just like human problems. The high pressure die casting machines are used more frequently in industries so there is a need of analysis of robotic machine to improve the reliability of the system. As the system may be in failed state due to some problem in the machine, one solution is to use the standby unit. In this paper we explain this concept in detail.

None of the researchers have analyzed so far taking real data for such die casting two-unit cold standby system with occurrence of various faults. To fill up this gap, the paper discussed an analysis of a stochastic model for two-unit die casting system with failure and measures the affects in terms of MTSF and Profitability.

MODEL DESCRIPTION:

We have taken two non identical die casting units in which one is in working state and other is cold standby. The working unit has three modes; normal, partial failure ,total failure while standby unit has two modes normal and total failure .The primary unit may fail partially or completely ,when the unit goes for repair, the cold standby unit starts. The partial failed unit after repair is O.K. it starts in operative mode and other operative unit returns to standby mode .If the repair of partial failed unit is continued and mean while other working unit is also failed, the repair facility stop repairing and put it under operation and starts immediately repair for failed unit. Further, the other unit is O.K to operative mode repair facility restarts the repair of partial failed unit. There is a single repair facility which is always available with the system and the makes an attempt to repair the failed unit with full satisfaction. The repaired unit works as good as new. There are some states which are called up states and some down states. The up states are those states in which at least one machine is in operative mode either primary unit or secondary unit. The down states are states in which both machines are not in operative mode. The standby unit is used to replace the operative failed unit instantaneously. The Semi-Markov process and Regenerative point technique are used to explore about the reliability and profitability of the system.

NOTATIONS

X_0 : Priority unit is operative.

Y_0 : Non priority unit is operative.

Y_S : Non priority unit is in standby mode.

X_{P0} : Priority unit is partially failed under operative mode .

X_{PFr} : Priority unit is partially failed sent for repair.

X_{PFR} : The repair of the partially failed is go for previous state.

X_{TFr} : The totally failed unit is sent for repair.

X_{TFR} : The repair of the totally failed is go for previous state.

Y_{FW} : Non priority is failed and waiting for repair

Y_R : Non priority is under repair.

$G(.)$: c d f of repair time when priority unit is totally failed.

$F(.)$: c d f repair time when priority unit is partially failed.

$H(.)$: c d f of repair time of non priority unit.

μ_i : Mean sojourn time in state S_i.

α_1 : Parameter of failure time distribution for main unit operative to partial failure.

α_2 : Parameter of failure time distribution for main unit partial to total failure.

α_3 : Parameter of failure time distribution for main unit operative to total failure.

p_1 : Probability of repaired unit in working unit.

p_2 : Probability of repaired unit require post repair.

* : Symbol for Laplace transformation $F^*(s) = \int_0^\infty e^{-st} f(t)\, dt$

~ : Symbol for Laplace Stieltjes transformation $F^{\sim}(s) = \int_0^\infty e^{-st}\, dF(t)$

Ψi : Mean sojourn time in state S_i.

$Mi(t)$: Probability that the sojourns in state S_i up to time t.

$\Phi i(t)$: Where starting from upstate S_0 ,c d f of time to the system.

© : Symbol for Laplace convolution

THE STATES OF THE SYSTEM:

The different states of the system having all the possibilities either main unit is in operative state, failed state, repairing state, inspection state and similarly for second standby unit, the possibilities are in operative state, repairing state, inspection state, failed state, waiting state are taken into account.

$S_0= [X_0,Y_s]$; $S_1= [X_{PFr},Y_0]$; $S_2= [X_{TFr},Y_0]$;

$S_3= [X_{PQ} ,Y_r]$; $S_4= [X_{PFR},Y_{FW}]$; $S_5= [X_0,Y_{Fr}]$;

$S_6= [X_{TFR},Y_{FW}]$;

The figure showing all the possible states of the system, some states are up states and some are down states. The states S_0, S_1, S_2, S_3, S_5 are up states and the states S_4, S_6 are down states. The all possibilities are shown in this figure;

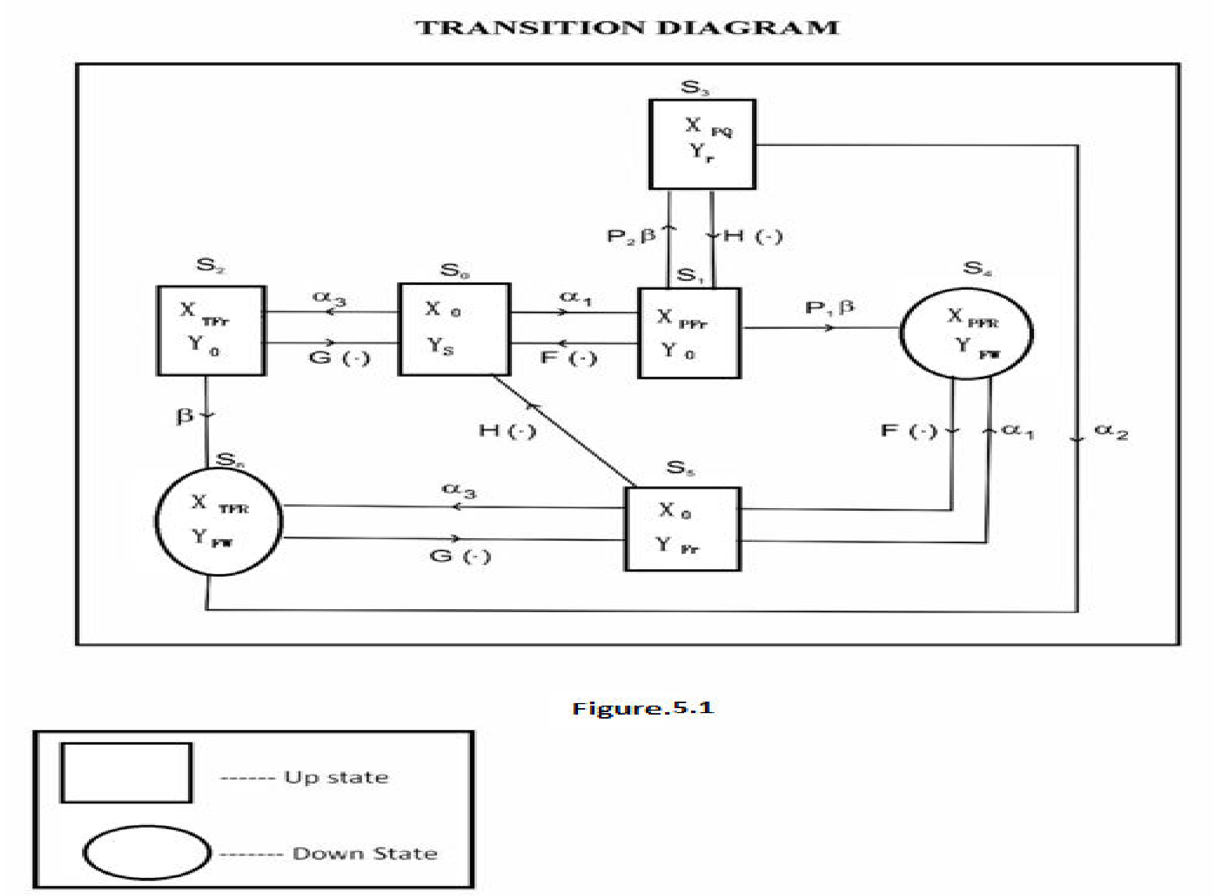

Figure.5.1

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Here $Q_{ij}(t)$ denotes the $c\ d\ f$ (cumulative distribution function) of transition time from state Si to Sj in 0 to t .To determine the transition probabilities of states. Let T_0, T_1, T_2 ,....denotes the regenerative epochs (a particular time period of the state). Then $\{\ Xn, Tn\}$ constitute a space E, set of regenerative states and $Q\ ij\ (t) = P[Xn + 1 =\ j, Tn + 1 - Tn\ \leq\ t\ / Xn = i]$ is the semi Markov over E .The various transition probabilities are:

$Q_{01}(t) = \alpha_1 \int_0^t e^{-(\alpha_1+\alpha_3)u} du$ $\qquad$ $Q_{02}(t) = \alpha_3 \int_0^t e^{-(\alpha_1+\alpha_3)u} du$

$Q_{10}(t) = \int_0^t\ e^{-\beta u}\ d\mathrm{F}(u)$ $\qquad$ $Q_{15}^{(4)}(t) = p_1 \int_0^t [1 - e^{-\beta v}]\ d\mathrm{F}(v)$

$Q_{13}\ (t) = p_2 \beta \int_0^t\ e^{-\beta t} \overline{F}\ (u) du$ $\qquad$ $Q_{25}^{(6)}(t) =\ \beta \int_0^t [1 - e^{-\beta v}]\ d\mathrm{G}(u)$

$Q_{20}(t) = \int_0^t\ e^{-\beta t}\ d\mathrm{G}(t)$

$Q_{31}(t) = \int_0^t e^{-\alpha_2 u} d\mathrm{H}\ (u)$ $\qquad$ $Q_{36}(t)\ = \alpha_2 \int_0^t e^{-\alpha_2 u} \overline{H}\ (u) du$

$Q_{45}(t) = \int_0^t d\mathrm{F}(u)$ $\qquad$ $Q_{50}(t) = \int_0^t e^{-(\alpha_1+\alpha_3)u}\ d\mathrm{H}\ (u)$

$Q_{54}(t) = \alpha_1 \int_0^t e^{-(\alpha_1+\alpha_3)u}\ \overline{H}\ (u) du$

$Q_{56}(t) = \alpha_3 \int_0^t e^{-(\alpha_1+\alpha_3)u}\ \overline{H}\ (u) du$ $\qquad$ $Q_{65}(t) = \int_0^t d\mathrm{G}(u)$ $\qquad$ (1-14)

STEADY STATE TRANSITION PROBABILITIES:

We generally take limit from 0 to t while calculating the cumulative distribution function (c d f) for the system. Here we are taking in steady state the limit of t tends to infinity. And then the transition probabilities are calculated (1-14).

$P_{01} = \frac{\alpha_1}{\alpha_1+\alpha_3}$ $\qquad$ $P_{02} = \frac{\alpha_3}{\alpha_1+\alpha_3}$ $\qquad$ $P_{10} =\ \tilde{F}\ (\beta)$

$P_{13} =\ p_2[1 - \tilde{F}\ (\beta)]$ $\qquad$ $P_{15}^{(4)}\ = p_1[1 - \tilde{F}\ (\beta)$

$P_{20}\ =\ \tilde{G}\ (\beta)$ $\qquad$ $P_{25}^{(6)} = 1 - \tilde{G}\ (\beta)$

$P_{31} =\ \tilde{H}\ (\alpha_2)$ $\qquad$ $P_{36}\ =\ 1 - \tilde{H}\ (\alpha_2)$ $\qquad$ $P_{45} = P_{65} = 1$

$P_{50} = \tilde{H}(\alpha_1 + \alpha_3)$ $\qquad$ $P_{54} = \frac{\alpha_1}{\alpha_1+\alpha_3}\ [1 - \tilde{H}(\alpha_1 + \alpha_3)]$

$P_{56} = \frac{\alpha_3}{\alpha_1+\alpha_3}\ [1 - \tilde{H}(\alpha_1 + \alpha_3)]$ $\qquad$ (15-27)

By these transition probabilities, it can be verified that

$P_{01} + P_{02} = 1$; $\qquad$ $P_{10} + P_{13} + P_{15}^{(4)} = 1$; $\qquad$ $P_{20} + P_{25}^{(6)} = 1$;

$P_{31} + P_{36} = 1$; $\qquad$ $P_{45} = P_{65} = 1$; $\qquad$ $P_{50} + P_{56} = 1$; $\qquad$ (28)

MEAN SOJOURN TIME

The sojourns time Si , denoted by Ψi which is the time spent in a particular state before going to another state. The sojourn times are $\Psi_0, \Psi_1, \Psi_2, \Psi_3, \Psi_4, \Psi_5, \Psi_6,$ and they are calculated as:

$$\Psi i = E[Ti] = \int P(Ti > t)\, dt$$

$\Psi_0 = \int_0^\infty e^{-(\alpha_1+\alpha_3)} dt = 1/ (\alpha_1 + \alpha_3)$ $\quad$ $\Psi_1 = \int_0^\infty e^{-\beta t}\overline{F}\,(t)dt = \frac{1}{\beta}\,[1 - \tilde{F}\,(\beta)]$

$\Psi_2 = \int_0^\infty e^{-\beta t}\overline{G}\,(t)dt = \frac{1}{\beta}\,[1 - \tilde{G}\,(\beta)]$ $\quad$ $\Psi_3 = \int_0^\infty e^{-\alpha_2 u}\overline{H}\,(t)dt = \frac{1}{\alpha_2}[1 - \widetilde{H}\,(\alpha_2)]$

$\Psi_4 = \int_0^\infty \overline{F}\,(t)dt$ $\quad$ $\Psi_5 = \int_0^\infty e^{-(\alpha_1+\alpha_3)t}\,\overline{H}\,(t)dt = \frac{\alpha_3}{\alpha_1+\alpha_3}\,[1 - \widetilde{H}(\alpha_1 + \alpha_3)]$

$\Psi_6 = \int_0^\infty \overline{G}\,(t)dt$ (29-35)

PROFIT ANALYSIS

The profit analysis of the system can be carried out by considering the all the factors in time period (0, t). Therefore, the expected profit of system is:

P(t) = expected total revenue in (0,t)-expected total expenditure(cost) in (0,t)

In steady state, expected no of profit per unit time

$P = \lim_{t\to\infty} [P(t)/t] = \lim_{s\to 0} s^2 P^*(s)$

$P = L_0 A_0 - L_1 B_0$

L_0=Revenue per unit for up state of system

L_1= is the cost per unit time for which repair man is busy in repair of the failed unit.

PARTICULAR CASE

The repair time's distributions of different failures were assumed to be arbitrary while analyzing the proposed model. If we assume repair time distribution of total and partially failure of unit X, and failure unit Y are exponential but different values $\lambda_1, \lambda_2, \lambda_3$.

$M_i(t) = \lambda_i e^{-\lambda_i(t)}$ for i=1,2,…

STEADY STATE TRANSITION PROBABILITIES

$P_{10} = \frac{\lambda_1}{(\alpha_2+\lambda_1)}$ $\quad$ $P_{13} = \frac{p_2\beta}{(\beta+\lambda_2)}$

$P_{14} = \frac{p_1\beta}{(\beta+\lambda_2)}$ $\quad$ $P_{20} = \frac{\lambda_1}{(\beta+\lambda_1)}$ $\quad$ $P_{26} = \frac{\beta}{(\beta+\lambda_1)}$ $\quad$ $P_{31} = \frac{\lambda_3}{(\alpha_2+\lambda_3)}$

$P_{36} = \frac{\alpha_2}{(\alpha_2+\lambda_1)}$ $\quad$ $P_{50} = \frac{\lambda_3}{(\alpha_1+\alpha_3+\lambda_3)}$ $\quad$ $P_{54} = \frac{\alpha_1}{(\alpha_1+\alpha_3+\lambda_3)}$

$P_{56} = \frac{\alpha_3}{(\alpha_1+\alpha_3+\lambda_3)}$ $\quad$ $P_{45} = P_{65} = 1$

MEAN SOJOURN TIMES

$\mu_0 = \frac{1}{(\alpha_1+\alpha_3)}$ $\mu_1 = 1/(\beta + \lambda_2)$

$\mu_2 = 1/(\beta + \lambda_1)$ $\mu_3 = \frac{1}{(\alpha_2+\lambda_3)}$

$\mu_4 = \frac{1}{\lambda_2}$ $\mu_5 = \frac{1}{(\alpha_1+\alpha_3+\lambda_3)}$ $\mu_6 = \frac{1}{\lambda_1}$

$$\text{MTSF} = \frac{(\beta+\lambda_1+\alpha_3)[(\beta+\lambda_2)(\alpha_2+\lambda_3)- p_2\beta\lambda_3]+\alpha_1(\alpha_1+\alpha_3)+\alpha_1 p_2\beta](\beta+\lambda_1)}{[((\beta+\lambda_2)(\alpha_2+\lambda_3)- p_2\beta\lambda_3][(\alpha_1+\alpha_3)(\beta+\lambda_2)-\alpha_3\lambda_1]-\alpha_1\lambda_2(\beta+\lambda_1)(\alpha_2+\lambda_3)}$$

$$A_0 = \frac{\begin{gathered}\lambda_1\lambda_2[\lambda_3(\beta+\lambda_1+\alpha_3)[(\beta+\lambda_2)(\alpha_2+\lambda_3)- p_2\beta\lambda_3]+\alpha_1\lambda_3(\beta+\lambda_1)(\alpha_2+\lambda_3)+p_2\beta] \\ +(\beta+\lambda_1)[\alpha_1\alpha_2 p_2\beta+\alpha_1 p_1\beta(\alpha_2+\lambda_3)]+ \\ \alpha_3\beta[(\beta+\lambda_2)(\alpha_2+\lambda_3)- p_2\beta\lambda_3]]\end{gathered}}{\begin{gathered}\lambda_1\lambda_2\lambda_3[(\beta+\lambda_2)+\lambda_3][(\beta+\lambda_2)(\alpha_2+\lambda_3)- p_2\beta\lambda_3]+\lambda_1\lambda_2\lambda_3\alpha_1(\beta+\lambda_1)[(\alpha_2+\lambda_3)+p_2\beta] \\ +\alpha_1\alpha_2\alpha_3 p_2\beta\lambda_2(\beta+\lambda_1)[(\beta+\lambda_2)(\alpha_2+\lambda_3)- p_2\beta\lambda_3] \\ \{[(\alpha_1+\alpha_3)(\beta+\lambda_2)-\alpha_3\lambda_1]-\alpha_1\lambda_2(\beta+\lambda_1)(\alpha_2+\lambda_3)\}[\alpha_3\lambda_2+\alpha_1\lambda_1+\lambda_1\lambda_2(\alpha_1+\alpha_2+\lambda_3)\end{gathered}}$$

$$B_0 = \frac{\begin{gathered}\lambda_1\lambda_2[\lambda_1\lambda_2\lambda_3\alpha_1(\beta + \lambda_2)(\alpha_2 + \lambda_3) + \lambda_1\lambda_2\lambda_3\alpha_3 \\ [(\beta + \lambda_2)(\alpha_2 + \lambda_3) - p_2\beta\lambda_3] + \lambda_1\lambda_2\lambda_3\alpha_1 p_2\beta(\beta + \lambda_1)(\alpha_2 + \lambda_1) \\ +\{\alpha_1\alpha_2 p_2\beta(\beta + \lambda_1) + \alpha_1 p_1\beta(\beta + \lambda_1)(\alpha_2 + \lambda_3) + \alpha_3\beta[(\beta + \lambda_2)(\alpha_2 + \lambda_3) \\ - p_2\beta\lambda_3][\alpha_3\lambda_2 + \alpha_1\lambda_1 + \lambda_1\lambda_2(\alpha_1 + \alpha_2 + \lambda_3)]\end{gathered}}{\begin{gathered}\lambda_1\lambda_2\lambda_3[(\beta + \lambda_2) + \lambda_3][(\beta + \lambda_2)(\alpha_2 + \lambda_3) - p_2\beta\lambda_3] + \lambda_1\lambda_2\lambda_3\alpha_1(\beta + \lambda_1) \\ [(\alpha_2 + \lambda_3) + p_2\beta] + \alpha_1\alpha_2\alpha_3 p_2\beta\lambda_2(\beta + \lambda_1) \\ [(\beta + \lambda_2)(\alpha_2 + \lambda_3) - p_2\beta\lambda_3] \\ \{[(\alpha_1 + \alpha_3)(\beta + \lambda_2) - \alpha_3\lambda_1] - \alpha_1\lambda_2(\beta + \lambda_1)(\alpha_2 + \lambda_3)\}[\alpha_3\lambda_2 + \alpha_1\lambda_1 + \lambda_1\lambda_2(\alpha_1 + \alpha_2 + \lambda_3)]\end{gathered}}$$

$$V_0 = \frac{\lambda_1\lambda_2\lambda_3[(\beta+\lambda_2)(\alpha_2+\lambda_3)- p_2\beta\lambda_3](\alpha_1+\alpha_3)(\beta+\lambda_1)}{\begin{gathered}\lambda_1\lambda_2\lambda_3[(\beta+\lambda_2)+\lambda_3][(\beta+\lambda_2)(\alpha_2+\lambda_3)- p_2\beta\lambda_3]+\lambda_1\lambda_2\lambda_3\alpha_1(\beta+\lambda_1)[(\alpha_2+\lambda_3)+p_2\beta] \\ +\alpha_1\alpha_2\alpha_3 p_2\beta\lambda_2(\beta+\lambda_1)[(\beta+\lambda_2)(\alpha_2+\lambda_3)- p_2\beta\lambda_3] \\ \{[(\alpha_1+\alpha_3)(\beta+\lambda_2)-\alpha_3\lambda_1]-\alpha_1\lambda_2(\beta+\lambda_1)(\alpha_2+\lambda_3)\}[\alpha_3\lambda_2+\alpha_1\lambda_1+\lambda_1\lambda_2(\alpha_1+\alpha_2+\lambda_3)\end{gathered}}$$

GRAPHICAL STUDY OF MODEL

In this model we study the system behavior, we plot the graph and table of MTSF, Availability, Profit function with respect to failure rate ($\boldsymbol{\beta}$) for different values of repair rate($\boldsymbol{\lambda_3}$).

Figure 5.2 Mean Time To System Failure of the System for given value of (0.01 to 0.95) and λ_3 *as values* 0.02,0.04 ,0.06 , λ_1=0.35,$\lambda_2 = 0.55$,
$\lambda_3 = 0.15$, $\alpha_1 = 0.30$, $\alpha_2 = 0.50$,$\alpha_3 0.50$, $p_1 = 0.01$, $p_2 = 0.02$.It observed and plot graph MTSF decrease and failure rate increase and increase with the repair rate.

Figure 5.3 Availability of the System for given value of (0.01 to 0.95) and λ_3 *as values* 0.02,0.04 ,0.06 , λ_1=0.35,$\lambda_2 = 0.55$,
$\lambda_3 = 0.15$, $\alpha_1 = 0.30$, $\alpha_2 = 0.50$,$\alpha_3 0.50$, $p_1 = 0.01$, $p_2 = 0.02$.It observed and plot graph Availability decrease and failure rate increase and increase with the repair rate.

Figure 5.4 Profit function of the System for given value of (0.01 to 0.95) and λ_3 *as values* 0.02,0.04 ,0.06 , λ_1=0.35,$\lambda_2 = 0.55$,
$\lambda_3 = 0.15$, $\alpha_1 = 0.30$, $\alpha_2 = 0.50$,$\alpha_3 0.50$, $p_1 = 0.01$, $p_2 = 0.02$.It observed and plot graph Profit function decrease and failure rate increase and increase with the repair rate.

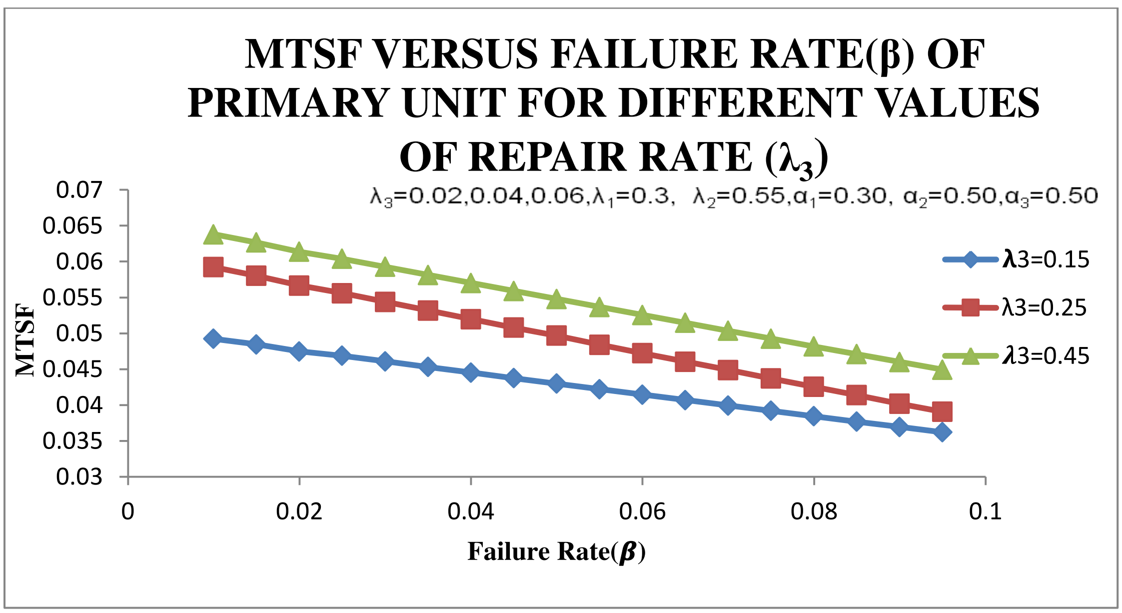

Fig. 5.2

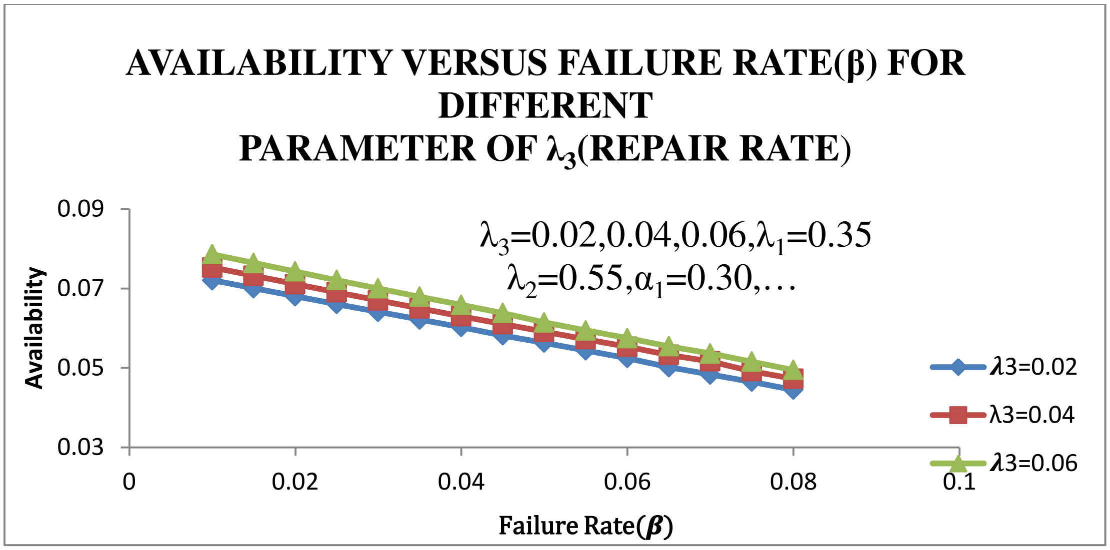

Figure 5.3

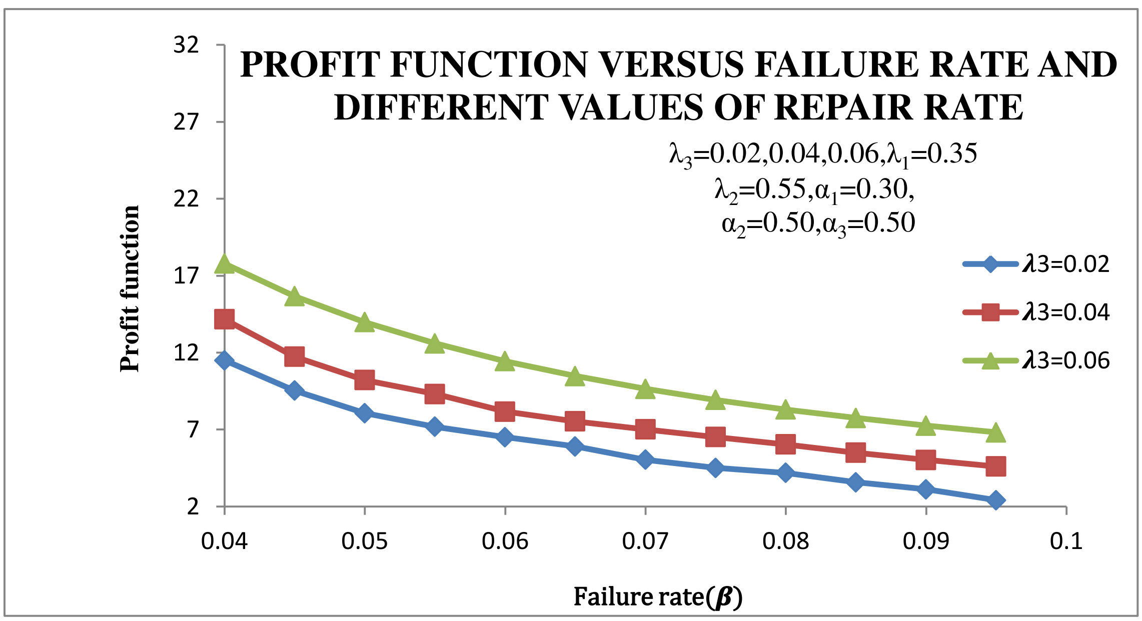

Figure 5.4

CONCLUSION:

This paper develops a two unit reliability model on high pressure die casting machine system and analyzes the expression for transition probabilities and various system performance measures under all possible failure modes and repair rate. The Profit function graph reveals that profit decreases with the increase in failure rate as well as repair rate.

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MODELING AND COST ANALYSIS OF A QUEUE DYNAMICAL SYSTEM WITH IMPATIENT BEHAVIOUR OF CUSTOMERS.

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ABSTRACT

The Present communication explores the cost effectiveness of M/M/1/N Queue system with impatient nature of customers. The most dominant difference between queue with impatient customers & ordinary queue is the impatient acts (balking & reneging) in queue. Some arriving customers looking large queue in the system do not like to stand in queue, balk with some probability from the system and others stand in the queue for a while but leave or renege without getting service. Assuming Reneging rate exponentially distributed, the steady state probability equations have been developed with the help of Markov process technique and Matrix form solution has been derived. A central challenge in designing & managing any service operation in a queue system is to achieve a cost balance between operational efficiency & service quality .The objective of this paper is to develop a mathematical model in order to find the optimal service rate and the equilibrium point between the degree of waiting time and the total expected cost.

Keywords: Balking, Reneging, Cost analysis, Exponential distribution steady state etc.

1. INTRODUCTION

The earlier researchers did not count the rate of impatient customer in the study of queue system but as the time goes on, civilization progress, the population density not only of human beings but also vehicles on an increase, the impatient behavior of customers rate became significant. It could not be ignored in situations arising hospital emergency room, toll booth, inventory control system storing valuable goods, call centre system etc. Practically it has been observed that the cost factor play vital role to survey & study a queue system, for which equilibrium level queue decision model is required. It is because the two conflicting cost offering the service i.e. the cost of providing service with waiting time cost against the cost in preventing the queue, with computation of total expected operational cost per unit time for the system is required.

Haight (1957,1959) made the first attempt to discuss the effect of balking and reneging phenomenon in queuing problem and assumed that customer had a threshold value N of queue length before he arrives at service facility; in case he observes < N he joins queue, otherwise goes away. The work was further studied by Ancker & Gafarin (1963) and Abou & Harire (1992) Yechiali (1971) shown that the threshold type balancing rule is more realistic in some specific situations. Singh TP (1985) incorporated the reneging concept (reluctant a customer remains in line after joining the queue & waiting) in serial queue network and obtained transient solution. Ashok & Taneja (1983) studied the cost analysis of multichannel queue system where in both the arrival & service intensities are subject to alterations. Man Singh & Umed Singh (1994) studied the steady state behavior for impatient customers.

Neetu Gupta etal. (2009),Singh T.P.&Arti (2014) in their separate study made an effort of the balking and reneging effect on the performance of a queue system under certain parametric constraints and discussed the cost analysis of a queue system with impatient customers but these authors failed to search out the equilibrium point between the various cost structure. This study is further an extended work of above said authors and explores the cost analysis of queue dynamical system in a wider sense. The degree of waiting time equilibrium phenomenon has been derived. The concept has been made clear through numerically illustration along with graphically presentation, making analysis more meaningful and relevant.

2. MODEL FORMULATION

The Mathematical modeling for the stated system under study can be depicted as below:

2.1 ASSUMPTIONS

1. Customers arrive at the system one by one in a Poisson fashion with expected arrival rate λ. On arrival a customer either decides to join the queue with probability b_n or balk from the system on observing the long queue with probability $1-b_n$ when n customers are ahead (n=1,2,3,4,-------N-1) where N is maximum number of customers in the system i.e.

   $0 \leq b_{n-1} \leq b_n < 1 \qquad 1 \leq n \leq N-1$

   $bn=0 \qquad n \geq N$

2. On joining queue each customer has to wait a certain time period T in order to start the service. In case customers has not received service at once he feels irritated, after a while gets impatient and reneges from queue without being served. Here, the time T is a random variable following exponential distribution and its probability distribution function is given by $f(t) = \alpha e^{-\alpha t} \quad t \geq 0, \alpha \geq 0$

   Where α is the degree of waiting rate. Since the arrival and departure of impatient customers without service are independent, the function of customer's average reneging rate r_n is directly proportional to degree of waiting rate α mathematically given by

$r(n) = (n - i)\alpha \qquad i \leq n \leq N$

$\quad = 0 \qquad n > N \qquad i = 0,1,2,3\ldots\ldots\ldots$

3. Queue discipline is FIFO, once the service starts, it precedes till its completion.
4. The service time has been assumed to be exponentially distributed whose probability distribution function is given by

   $G(t) = \mu e^{-\mu t}, \qquad t \geq 0,\ \mu > 0$ where μ is mean service rate.

3. FORMATION OF DIFFERENTIAL DIFFERENCE EQUATION

Define, P_n = Probability that there are n customers in the system,

b_n = Probability that on arrival a customer decides to join the queue or balk with the probability $1\text{-}b_n$.

The steady state probability differential difference equation governing the model can expressed as

$\lambda\, b_{n-1} P(n-1) + (\mu + n\alpha)\, P(n+1) = [\lambda\, b_n + \mu + (n-1)\alpha]\, P(n).$ For $n = 1,2,3 \ldots\ldots..N-1$

$\lambda\, b_{N-1} P(N-1) + [\mu + (N+1)\,\alpha]\, P(n),$ For $n = N$

$\mu\, p(1) = \lambda\, p(0)$ For $n = 0$

Let us discuss the cost model for N = 5

The following equation are obtained

$\mu\, p(1) = \lambda\, p(0)$ For $n = 0$ (1)

$\lambda\, P(0) + (\mu + \alpha)\, P(2) = [\lambda\, b_1 + \mu]\, P(1)$ For $n = 1$ (2)

$\lambda\, b_1 P(1) + (\mu + 2\alpha)\, P(3) = [\lambda\, b_2 + \mu + \alpha]\, P(2)$ For $n = 2$ (3)

$\lambda\, b_2 P(2) + (\mu + 3\alpha)\, P(4) = [\lambda\, b_3 + \mu + 2\alpha]\, P(3)$ For $n = 3$ (4)

$\lambda\, b_3 P(3) + (\mu + 4\alpha)\, P(5) = [\lambda\, b_4 + \mu + 3\alpha]\, P(4)$ For $n = 4$ (5)

$\lambda\, b_4 P(4) = [\mu + 4\alpha]\, P(5)$ For $n = 5$ (6)

4. SOLUTION METHODOLOGY

Writing the above equations in matrix form

$$\begin{vmatrix} -\lambda & \mu & 0 & 0 & 0 & 0 \\ \lambda & -(\lambda b_1+\mu) & (\mu+\alpha) & 0 & 0 & 0 \\ 0 & \lambda b_1 & -(\lambda b_2+\mu+\alpha) & (\mu+2\alpha) & 0 & 0 \\ 0 & 0 & \lambda b_2 & -(\lambda b_3+\mu+2\alpha) & (\mu+3\alpha) & 0 \\ 0 & 0 & 0 & \lambda b_3 & -(\lambda b_4+\mu+3\alpha) & (\mu+4\alpha) \\ 0 & 0 & 0 & 0 & \lambda b_4 & -(\mu+4\alpha) \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

On solving by usual method, we get

$P(0) = K$

$P(1) = \dfrac{\lambda}{\mu} K$

$P(2) = \lambda^2 \dfrac{b_1 K}{\mu(\mu+\alpha)}$

$P(3) = \lambda^3 \dfrac{b_1 b_2 K}{\mu(\mu+\alpha)(\mu+2\alpha)}$

$P(4) = \lambda^4 \dfrac{b_1 b_2 b_3 K}{\mu(\mu+\alpha)(\mu+2\alpha)(\mu+3\alpha)}$

$P(5) = \lambda^5 \dfrac{b_1 b_2 b_3 b_4 K}{\mu(\mu+\alpha)(\mu+2\alpha)(\mu+3\alpha)(\mu+4\alpha)}$

Applying initial conditions

$$\sum_{i=1}^{5} P_i = 1$$

i.e. $P(0) + P(1) + P(2) + P(3) + P(4) + P(5) = 1$

Putting the values we get

$$K = \left[1 + \frac{\lambda}{\mu} + \lambda^2 \frac{b_1}{\mu(\mu+\alpha)} + \lambda^3 \frac{b_1 b_2}{\mu(\mu+\alpha)(\mu+2\alpha)} + \lambda^4 \frac{b_1 b_2 b_3}{\mu(\mu+\alpha)(\mu+2\alpha)(\mu+3\alpha)} + \lambda^5 \frac{b_1 b_2 b_3 b_4}{\mu(\mu+\alpha)(\mu+2\alpha)(\mu+3\alpha)(\mu+4\alpha)}\right]^{-1}$$

Cost analysis and performance measure

Expected number of cost in Queue i.e. waiting customer in Queue and system

E (Nq) = $\sum_{n=1}^{N}(n-1)P(n)$

E (N) = $\sum_{n=1}^{N}nP(n)$

Balking Rate (B.R.) = $\sum_{n=1}^{N}\lambda(1-b_n)P(n)$

Reneging Rate (R.R.) = $\sum_{n=1}^{N}(n-1)\alpha P(n)$

L.R. = B.R. + R.R.

Where L.R is cost incurred due to customers loss.

μ^* = Control variable

Our objective is to control the service rate to minimize system's total average cost per unit.

C_1 = Cost per unit time when server is busy.

C_2 = Cost per unit time when customer join in the queue and waits for service.

C_3 = Cost per unit time when a customer balks or reneges.

F(μ^*) = Expected functional cost of the system per unit time.

Total expected functional cost of the system per unit time

T F (μ^*) = C_1P(B) + C_2 E(NL) + C_3 L.R

P(B) = busy probability of server

5. NUMERICAL ILLUSTRATION

Consider maximum number of customer in system N = 5

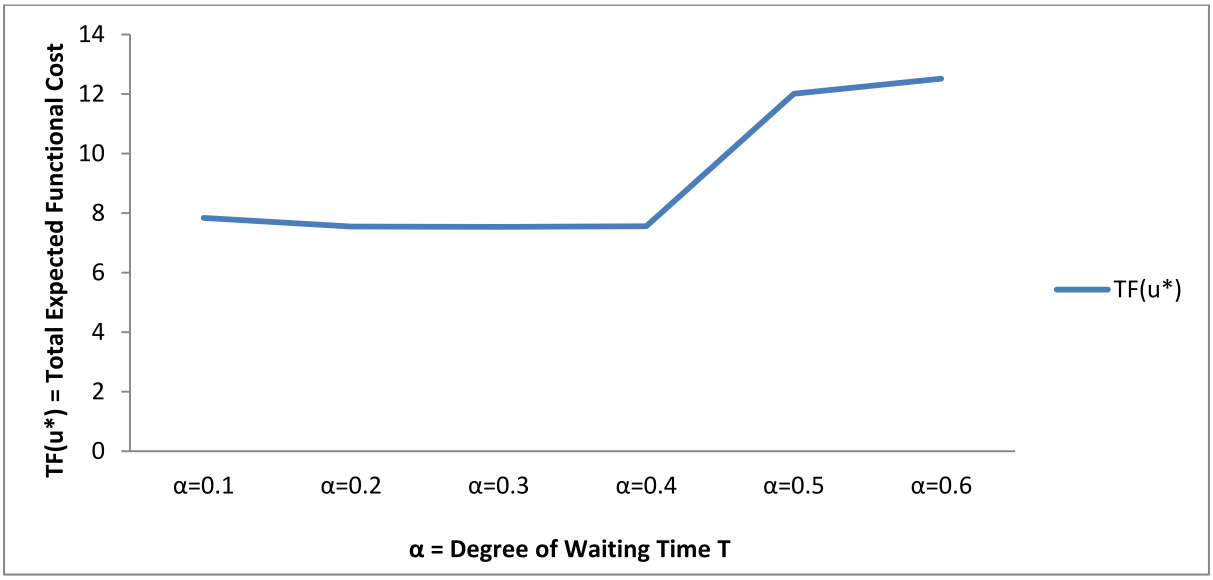
The probability **bn**= $\frac{1}{n+1}$ & cost element $\mathbf{C_1}$ = 10, $\mathbf{C_2}$ = 8, $\mathbf{C_3}$ = 16

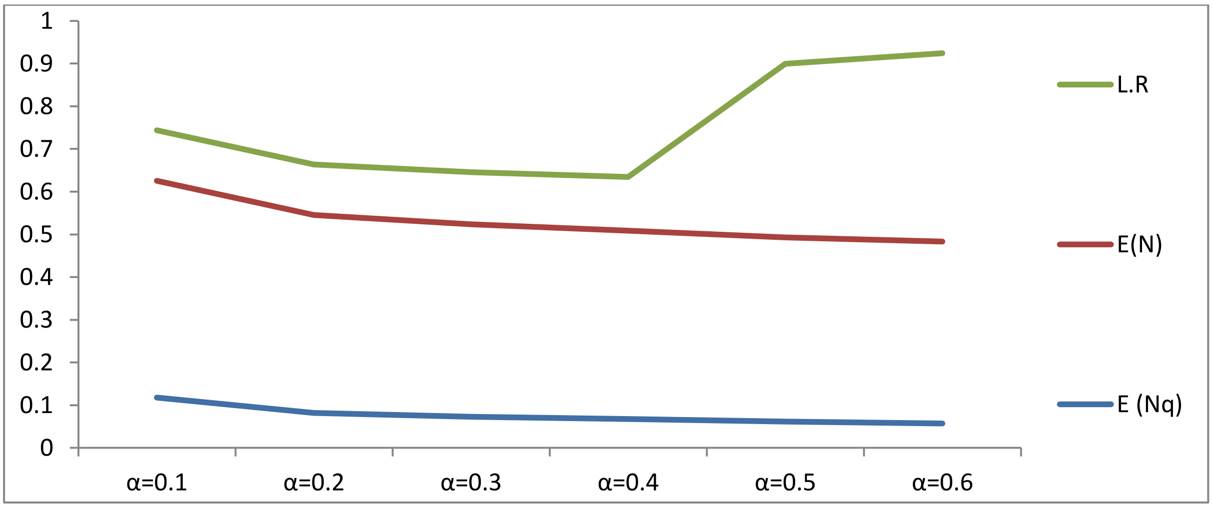
Table – 1 for α = 0.1

λ ⟶	**.4**	**.5**	**.6**	**.7**	**.8**
E(Nq)	0.061055	0.117363	0.1286619	0.16658565	0.210273
E(N)	0.386512	0.508244	0.5739055	0.659896	0.74826574
L.R.	0.0751841	0.1180925	0.1572375	0.20920	0.261520409
T F(μ*)	5.690945	7.82838	9.54515	11.6434	13.8665

Table – 2 for λ = 0.5

α ⟶	0.1	0.2	0.3	0.4	0.5	0.6
E(Nq)	0.117363	0.081815	0.72591	0.067199	0.061837	0.057326
E(N)	0.508244	0.463636	0.451379	0.44144	0.431094	0.426142
L.R.	0.1180925	0.1182845	0.1220313	0.12579	0.406961	0.440762
T F(μ*)	7.82834	7.547056	7.53322	7.55023	12.0060	12.51086





6. ANALYSIS OF GRAPH AND TABLE

Keeping the fix degree of waiting time rate α = 0.1 & changing value of arrival rate of customers λ, the results have been summarized in Table – 1.

Table 1 shows that as the value of λ increases, expected number of customers in queue and expected number of customer in system increases corresponding the total expected functional cost increases, i.e. balking and reneging phenomenon is on increase there will be customer loss in the system.

While the graph of Table – 2 shown that on increasing value of α expected number of customer in system as well as expected number of customer in queue and mean rate all decreases. Since the Reneging rate is the function of α, as the value of degree of waiting time rate α increases we find total expected functional cost in the beginning slowly decrease. But when value of α increase beyond 0.4 there is sudden Jump in the total expected cost. We say at $\alpha = 0.4$ is point of equilibrium where the degree of waiting time rate and total expected cost balance each other.

7. CONCLUSION

We have discussed queue system with impatient customers and developed the steady state probability equations. The Matrix form of the solution has been derived. We formulate a cost model to determine the optimal service rate and total expected cost of the system per unit time. Although this function is too complicated to derive in explicit expression for optimal service rate, even than we have made an attempt to evaluate numerically the performance measures & the optimal service rate for the system.

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GRAPH COLORING AND ITS APPLICATIONS

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ABSTRACT:

Graph coloring is an important topic in graph theory. In graph coloring we have assign colors to certain elements of the graph along with certain constraints. Suppose we are given 'n' colors, then we have to color the vertices in such a way that no two adjacent vertices of the graph have the same color; this is known as vertex coloring. Similarly we have edge coloring and face coloring. The coloring problem has a huge number of applications in modern computer science such as making schedule of time table, sudoku, bipartite graphs, map coloring, data mining, networking, final exam timetabling, aircraft scheduling, guarding an art gallery. In this paper we are going to focus on face coloring.

Keywords: Graph, Edges, Vertices, Coloring Process, chromatic number etc.

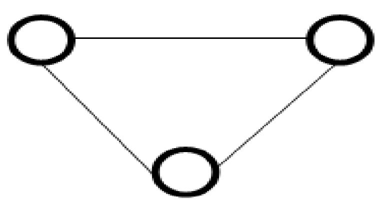
1. INTRODUCTION:

Graph theory is an important branch of mathematics, which has been applied to many problems related to mathematics, computer science, biochemistry, electrical engineering, operational research and other scientific and not-so-scientific areas. In this paper, we analyze the shape magic which has six faces, using graph coloring.

2. SOME DEFINITIONS:

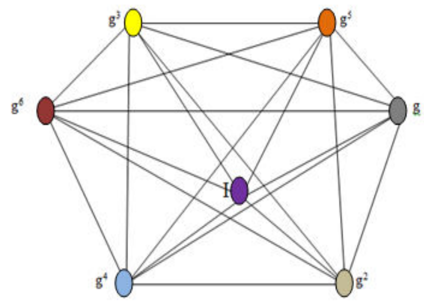
2.1 Graph: By a graph r = (V, E) we mean an undirected graph r with vertex set V, edge set E which has no loops or multiple edges.

2.2Example: The following figure is the example of undirected graph:-



2.3 Chromatic Number: The chromatic number of a graph G, written $\chi(G)$, is the minimum number of colors needed to label the vertices so that adjacent vertices receive different colors. A graph G is K-partite if V(G) can be expressed as the union of R independent sets. This generalizes the idea of bi-partite graphs which are 2-partite. If a graph has chromatic number two, then it is known as Bi-colorable. And if it has chromatic number three, then we call it as a three-colorable. And if a graph which has chromatic number 'k', is said to be k-colorable.

2.4 Example: This is an example of graph coloring whose chromatic number is 7.



2.5 Degree of a vertex: The number of edges incident on a vertex v with self loops counted twice is called the degree of v and is denoted by deg (v).
2.6 Isolated vertex: A vertex having no incident edge is called an isolated vertex. We can also say that isolated vertices are vertices with zero degree.
2.7 Pendent Vertex: A vertex, whose degree is 1, is known as pendent vertex.

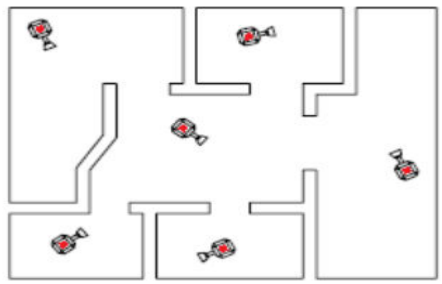
3 APPLICATION OF GRAPH COLORING:

3.1 Airline scheduling: Assume that we have k aircrafts, and we have to assign them to n flights, where the ith flight is during the time interval (a_i, b_i). Clearly, if two flights overlap, then we cannot assign the same aircraft to both flights. The vertices of the conflict graph correspond to the flights; two vertices are connected if the corresponding time intervals overlap. Therefore the conflict graph is an interval graph, which can be colored optimally in polynomial time.

3.2. Bi-processor tasks: Expect that we have a set of number of processors (machines) and a set of tasks, each one task must be executed on the two pre-assigned processors at the same time. A processor cannot chip away at 2 jobs at the same time. Case in point, such bi-processor tasks emerge when we need to calendar record exchanges between processors or on account of shared demonstrative testing of processors. Consider the graph whose vertices relate to the processors, and if there is a task that must be executed on processors i and j, then we include an edge between the two relating vertices. Presently the scheduling issue can be displayed as an edge coloring of this graph.

3.3 Frequency assignment: Accept that we have various radio stations, distinguished by x and y co-ordinates in the plane. We need to allot a frequency to each one station, however because of interferences; stations that are "close" to one another need to get diverse frequencies. Such issues emerge in frequency task of base stations in phone networks. At the outset, one may imagine that the conflict graph is planar in this issue, and the Four Color Hypothesis can be utilized, yet it is not genuine: if there are heaps of stations in little area, then they are all near one another, consequently they structure a huge coterie in the conflict graph. Rather, the conflict graph is a unit plate graph, where every vertex corresponds to a circle in the plane with unit measurement, and two vertices are connected if and only if the corresponding circles meet.

3.4 Guarding an Art Gallery: The application of Graph Coloring additionally utilized within guarding an art exhibition. Art exhibitions along these lines need to protect their collections precisely. The application of Graph Coloring also used in guarding an art gallery. Art galleries therefore have to guard their collections carefully. During the day the attendants can keep a look-out, but at night this has to be done by video cameras.



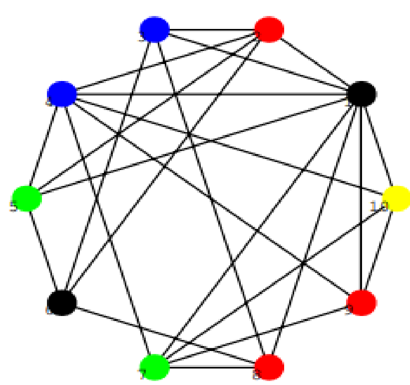
These cameras are usually hung from the ceiling and they rotate about a vertical axis. The images from the cameras are sent to TV screens in the office of the night watch. Because it is easier to keep an eye on few TV screens rather than on many, the number of cameras should be as small as possible. An additional advantage of a small number of cameras is that the cost of the security system will be lower.

3.5. Final Exam Timetabling Scheduling: Some scheduling problems induce a graph coloring, i.e., an assignment of positive integers (colors) to vertices of a graph. We discuss a simple example for coloring the vertices of a graph with a small number ‘k’ of colors and present computational results for calculating the chromatic number, i.e., the minimal possible value of such a ‘k’.

3.6 Example: Draw up an examination schedule involving the minimum number of days for the following problem:-

Set of students: S1, S2, S3, S4, S5, S6, S7, S8, S9 and Examination subjects for each group: {algebra, real analysis, and topology}, {algebra, operations research, and complex analysis}, {real analysis, functional analysis, and topology}, {algebra, graph theory, and combinatorics}, {combinatorics, topology, and functional analysis}, {operations research, graph theory, and coding theory}, {operations research, graph theory, and number theory}, {algebra, number theory, and coding theory}, {algebra, operations research, and real analysis}.

Solution : Let S be a set of students, P = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} be the set of examinations respectively algebra, real analysis, topology, operational research, complex analysis, functional analysis, graph theory, combinatorics, coding theory, and number theory. S(p) be the set of students who will take the examination p ∈ P. Form a graph G = G(P, E), where a, b ∈ P are adjacent if and only if S(a)∩S(b) ≠ Ø. Then each proper vertex coloring of G yields an examination schedule with the vertices in any color class representing the schedule on a particular day. Thus $\chi(G)$ gives the minimum number of days required for the examination schedule.

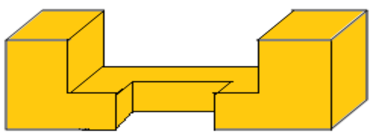


Now 5 days are required and you can see below the lessons in the same parenthesis which are on the same day {{1, 6}, {2, 8, 9}, {3, 4}, {5, 7}, {10}} It was very exciting to take 100-year old ideas, simple as they are, and implement them in Mathematics. But, there is more work to be done, both of a theoretical and practical nature.

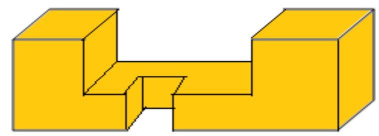
3.7 Theorem: Arrange the six Cuboids in such a way that they form a shape which is same in all sides.

Proof: Let we have six cuboids which have same length, breath and height. First we divide these cuboids into three pairs. Now we color each pair of cuboids with different colors in such a way that any two of them has same color for all faces. . Now we cut these cuboids in such a way that they formed a connected shape without any help of glue, fevicol, pin etc.

Now, first we have six cuboids and cut them according to the requirements.



(1)



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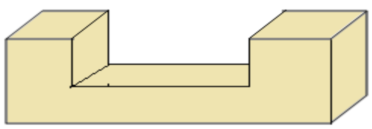

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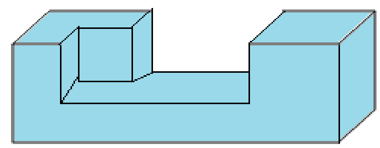

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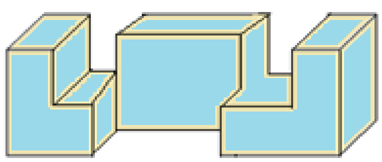
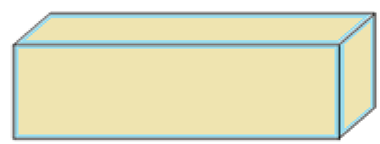
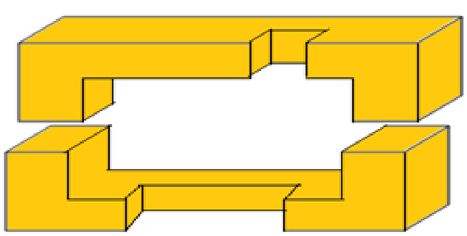


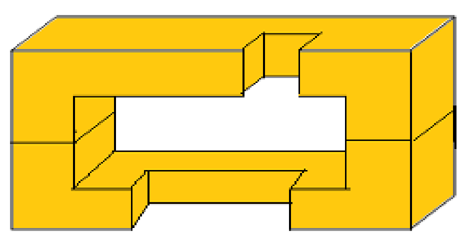
(5)



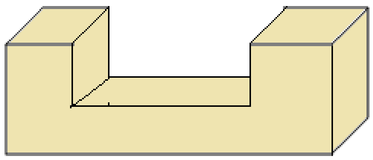
(6)

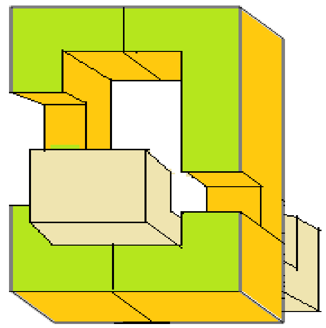
Now, first we take Cuboid (1) and Cuboid (2) and arrange them according to the following pattern:



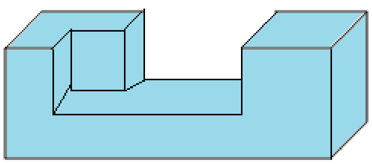


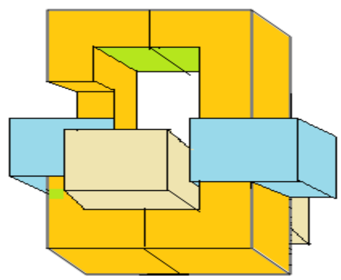
Now we take Cuboid (3) and arranged it according to the following pattern:



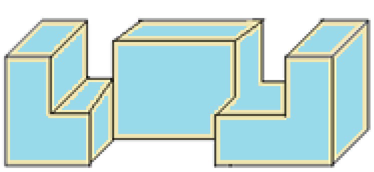


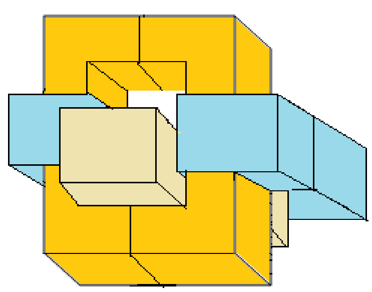
Now we take Cuboid (4) and arranged it according to the following pattern:



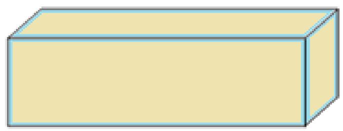


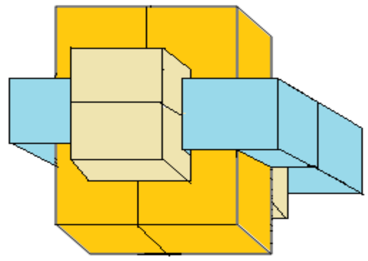
Now we take Cuboid (5) and arranged it according to the following pattern:





Now we take Cuboid (6) and arranged it according to the pattern:





CONCLUSION: An overview is presented especially to project the idea of graph coloring. Researches may get some information related to graph coloring and its applications in magic field and can get some ideas related to their field of research. This paper gives an overview of the applications of graph coloring in heterogeneous fields to some extent but mainly focuses on the magic applications that uses graph coloring concepts.

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APPLICATION OF CORRELATION AND INFORMATION THEORETIC MEASURES-KULLBACK-LEIBLER (KL) AND J-DIVERGENCE TO WORLD UNIVERSITIES RANKING PROBLEMS

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ABSTRACT

The main objective of this work is to analyze the existing patterns of parameters considered for world universities ranking problems using statistical tool, correlation among the parameters, Kullback- Leibler divergence measure (KL-Divergence) and J-Divergence for better pattern to raise the standard of rank among the universities or institutes. We have considered the current pattern and the past, to raise the standard of higher education of the concerned universities considering different main parameters. Correlation among them and using the Kullback-Leibler Divergence measure and J-Divergence, established, the better pattern to be utilized to establish the rank among the universities. Section 2 presents the basic concepts of divergence measures. Section 3 presents the data and patterns considered by QS and THE while section-4 presents The correlation coefficient combined. By as well as separately parameter wise for first 15 universities and section 5 represents the correlation among the parameters years wise as well as university wise for first five universities and suggests the correlation between citation and research is the best combination. Section 6 uses weighted Kullback-Leibler Divergence measure for parametric relation and section 7 presents the application of weighted J-Divergence, establishing which pattern is most suitable to raise the rank.

Key words: Kullback - Leibler Divergence Measure, J-Divergence, Correlation, Kerridge Inaccuracy..

2010 Mathematics Subject Classification. AMS 94A17, 26D15.

INTRODUCTION:

In the present communication, we have taken a case study of ranking problem for world universities. First, we have taken the data from TOI for 2009 onwards upto 2018 for first 15 universities in Rank. For ranking, the methodology, for five / six indicators which has been considered by the analyzer.

Shannon entropy [21] created vast awareness to measure the uncertainty in probabilistic phenomenon. Inaccuracy measure suggested by Kerridge [16] made the studies more applicable further and the concept of

3* Address for Correspondence

divergence measure was first considered by Kullback- Leibler [17]. Tsalli introduced the so-called Kullback- Leibler divergence measure, a pseudo- distance (or diseriminant function) between two probability distributions.

Recent application about polarimetric symthetic aperture (POLSAR) images, Frery et al [12] made use of complex Wishart distribution for modelling radar backscatt from forest and pasture areas. There they concluded that the Kullback- Leibler divergence measure is the best one with respect to Bhattacharya [6], chi-square, Hellinger or Renyi's distances and divergences. The study in information theory and image understanding [12] and [4] conclude that it is necessary to have appropriate statistics to compare multivariate distributions such as Wishart one. In addition [9] gives an extended theoretical analysis of the most important aspects of information theory for the multivariate normal and Student t- distribution among other distributions.

Divergence measures have also been used to examine data influencec and model perturbations [26] for a unified treatment and [1] for a review at some extensions of previous work on Bayesian influence measures based on the KL-Divergence. AIC (Akaike's Information Criteria), considers a good approximation of the KL-Divergence for selection model analysis. On the other hand asymptotic approximations of the KL-Divergence for the multivariate linear model are given in [8], where as asymptotic approximations of the Jeffreys's divergence (J-Diveregnce) for multivariate linear model, are given in [20]. In case of inter-tracking Vedic sequences [7], independent subspace analysis [18], image registration [11] at guessing moments [22].

Non- symmetric families of multivariate distributions as the multivariate skew-normal distributions [2, 3, 4, 5] have been developed. The real application for optimizing atmosphere monitoring network in the multivariate skew normal in [15]. Recently Javier et al [16] applying KL-Divergence and J- Divergence on a seismic catalogue analyzed, using non- parametric clustering (NPC) methods based on Kernel distribution, spatial weation of aftershock, compared the skew- distributed local magnitude among these clusters.

Recently Dagmar Markeehove and Boloslav Rieear [10], using KL-Divergence developed MV Algebra. Ruchi Nager ang Singh R.P [20] analysed, the theory of profit maximization in share market, comparing different divergences, proved that KL-Divergence [19] is the best among other divergences.

In this communication, we explore the possibility to raise the standard of higher education in deciding the suitability of parameters, correlation is used in ranking of world universities by the two organizations-QS and THE. In this context, we utilized the correlation among the parameters considered combined and separately university wise and year wise for decision and then used the KL-Divergence and J-divergence to support to decide the better pattern of correlated parameters in ranking, the universities globally. Section 2 presents the basic concepts for divergence measures and section-3 presents the case study of the data provided by QS and THE, while section 4 presents correlation of parameters combindly and separately to rank and section 5 presents the correlation among parameters year wise as well as Universities wise. Section 6 uses weighted IKL – Divergence for parametric relations while Section 7, presents the application of weighted J – Divergence.

SECTION 2: BASIC CONCEPTS

KULLBACK-LEIBLER (KL) AND J-DIVERGENCE

Let us consider the following Information Scheme:

$$IS = \begin{bmatrix} E_1 & E_2 & .... & E_n \\ p_1 & p_2 & .... & p_n \\ q_1 & q_2 & .... & q_n \end{bmatrix} = \begin{bmatrix} E \\ P \\ Q \end{bmatrix} \qquad (2.1)$$

where,

$E = [E_1, \quad E_2, \quad ...., \quad E_n]$, set of events,

$P = [p_1, \quad p_2, \quad ...., \quad p_n]$, set of corresponding probabilities, $p_i > 0, \forall i$

$\sum_{i=1}^{n} p_i = 1$,

$Q = [q_1, \quad q_2, \quad ...., \quad q_n]$, set of revised probabilities,

where

$$\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} q_i = 1, \ \forall i = 1,2,...,n$$

$\forall \ p_i \geq 0, \ q_i \geq 0.$

Shannon's Entropy [21] is defined as

$$H(P) = -k\sum_{i=1}^{n} p_i \log p_i. \qquad ... \qquad (2.2)$$

Later on Kerridge [16] defined the inaccuracy measure as

$$I(P;Q) = -\sum_{i=1}^{n} p_i \log q_i. \qquad ... \qquad (2.3)$$

Considering these, Kullback and Leibler [17] defined the non- symmetric divergence measure as:

$$K(P//Q) = \sum_{i=1}^{n} p_i \log \frac{p_i}{q_i} \qquad (2.4)$$

Later

Later on considering the importance or utility or weights of the concerned events, Guiasu, S. [ ] defined the weighted entropy corresponding to Shannon as follows:

$$H(P;W) = -\sum_{i=1}^{n} w_i p_i \log p_i, \forall i = 1,2,....n, \qquad \sum_{i=1}^{n} p_i = 1, \qquad ... \qquad (2.5)$$

$p_i \geq 0,$ $w_i > 0$ considering the weights w_i 's corresponding to each p_i ,

We have the revised information scheme as such:

$$I.S = \begin{bmatrix} E_1 & E_2 & .... & E_n \\ p_1 & p_2 & .... & p_n \\ q_1 & q_2 & .... & q_n \\ w_1 & w_2 & .... & w_n \end{bmatrix} = \begin{bmatrix} E \\ P \\ Q \\ W \end{bmatrix} \qquad ... \qquad (2.6)$$

Hooda and Tuteja (1985), [13A] studied non-additive relative information and inaccuracy measures.

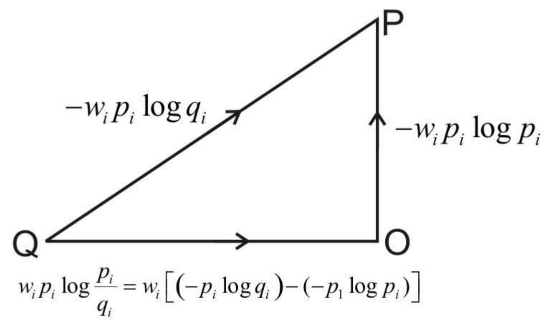
Taneja H.C. and Parkas, O. [23], defined Weighted Inaccuracy measure corresponding to Kerridge [16] considering (2.6) as follows:

$$I_W\left(P // Q;W\right) = -\sum_{i=1}^{n} w_i\, p_i \log q_i \qquad \ldots \qquad (2.7)$$

Singh and Bhardwaj [22] and HC Taneja [23] developed the weighted divergence as follows:

$$D_W\left(P // Q;W\right) = \sum_{i=1}^{n} w_i\, p_i \log \frac{p_i}{q_i} \qquad \ldots \qquad (2.8)$$

The victorial form is as follows:



Later on there is a vast sequence of divergence measures, weighted or unweigted.

For many other divergence measures refer Taneja, I.J. [24], Kullback-Leibler [17] also generalized (2.4) as a J-divergence, known as Jeffery's divergence as:

$$J(P // Q) = \sum_{i=1}^{n} (p_i - q_i) \log \frac{p_i}{q_i}$$

$$= \sum_{i=1}^{n} p_i \log \frac{p_i}{q_i} + \sum_{i=1}^{n} q_i \log \frac{q_i}{p_i}$$

$$= K(P // Q) + K(Q // P). \qquad \ldots \qquad (2.9)$$

(2.9) also was generalized as weighted J- divergence as:

$$J(P // Q;W) = \sum_{i=1}^{n} w_i (p_i - q_i) \log \frac{p_i}{q_i}.$$

$$= I_W(P // Q;W) + I_W(Q // P;W)$$

SECTION-3

EXISTING METHODOLOGIES - PATTERNS AND PARAMETERS

(a) QS- Quacquerelli Symonds

(b) THE- The Higher Education

3.1 (a) THE QS WORLD UNIVERSITY RANKING CRITERIA:

S.No	Factors	%	
1	Academic Peer Review	40%	Based on an international global Academic Survey
2	Faculty/ Staff Ratio	20%	A measurement of Teaching Community
3	Citation per Faculty	20%	A measurement of Research Impact

4	Employment Reputation	10%	Based on a survey on Graduate Employers
5	International Student Ratio	5%	A measurement of the Diversity of the student Community
6	International Staff Ratio	5%	A measurement of the Diversity of the Academic Staff

Ranking 2009 (THE SCORECARD) WIDE QS:

We have shown 6 indicators in this pie diagram which are given below:

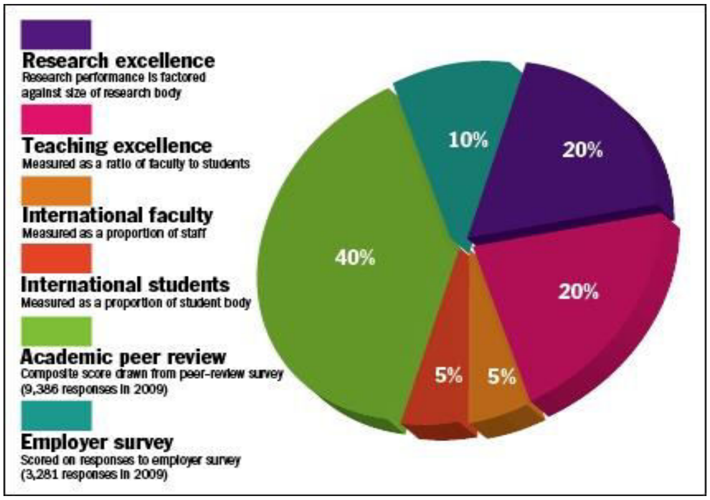


Fig 1.1

THE: TIMES HIGHER EDUCATION World University Rankings 2016-2017:

(b) <u>TIMES HIGHER EDUCATION CRITERIA:</u>

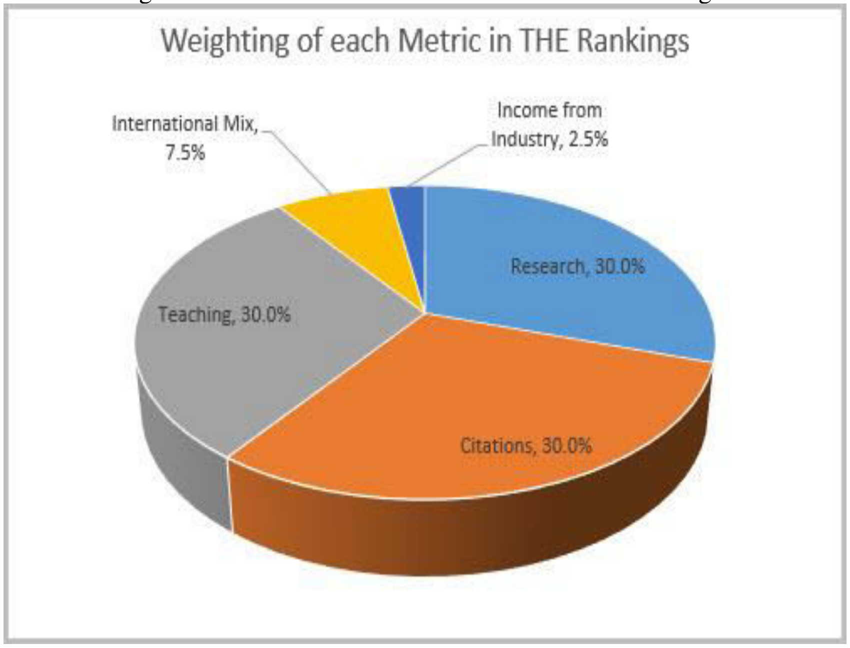
Ranking 2017 (THE SCORECARD)

The THE rankings are heavily weighted on the research metrics with a total of 60% weighting between the Citations and Research metrics.

Figure 1: Detailed breakdown of each metric in the THE Ranking

Overall	Category	Metric	Weighting
30%	Teaching	Doctorate to bachelor awarded	2.25%
		Doctorate awarded to academic staff	6%
		Teaching reputation	15%
		Institutional income to academic staff	2.25%
		Students to academic staff	4.50%
30%	Research	Papers to academic staff	6%
		Research income to academic staff	6%
		Research reputation	18%
30%	Citations	Citation impact	30%
2.50%	Industry Income	Industry income Industry income to academic staff	2.50%
7.50%	International Outlook	Percentage of international staff	2.50%
		International co-authorship	2.50%
		Percentage of international students 1	2.50%

Figure 2: Pie Chart of each metrics in the THE Ranking.



SECTION 4

4. APPLICATION OF STATISTICAL TOOL-CORRELATION AMONG THE PARAMETERS:

In this section, we have taken the data from Reuters for first 15 universities in the ranking of the world universities used by QS for 2009. Combining all 15 universities at a time and having correlation among the different parameters.

S.No	Institute	Country	Overall Score	Peer Review Score	Employer Review Score	Staff/Student Score	Citations/Staff Score	Int. Staff Score	Int. Student Score
1	Harvard University	US	100	100	100	98	100	85	78
2	University of Cambridge	UK	99.6	100	100	100	89	98	96
3	Yale University	US	99.1	100	99	100	94	85	77
4	University College London	UK	99	98	99	100	90	96	99
5	Imperial College London	UK	97.8	100	100	100	80	98	100
6	University of Oxford	UK	97.8	100	100	100	80	96	97
7	University of Chicago	US	96.8	100	99	97	88	77	83
8	Princeton University	US	96.6	100	96	82	100	89	81

9	Massachusetts Institute of Technology	US	96.1	100	100	89	100	31	95
10	California Institute of Technology	US	95.9	99	72	87	100	100	89
11	Columbia University	US	95.6	100	99	97	92	28	89
12	University of Pennsylvania	US	94.2	96	99	85	98	82	60
13	Johns Hopkins University	US	94.1	98	79	100	99	28	71
14	Duke University	US	92.9	95	97	100	93	29	62
15	Cornell University	US	92.5	100	99	85	94	28	73

Table- 4.1

4.1 APPLICATION OF STATISTICAL TECHNIQUES- CORRELATION

Correlation between different parameters for year 2009:

	Peer Rev Sc	**citations**			
S. No	**p**	**q**	**p2**	**q2**	**pq**
1	0.067294751	0.071581961	0.0045286	0.005124	0.00481709
2	0.067294751	0.063707946	0.0045286	0.0040587	0.00428721
3	0.067294751	0.067287044	0.0045286	0.0045275	0.004528065
4	0.065948856	0.064423765	0.0043493	0.0041504	0.004248674
5	0.067294751	0.057265569	0.0045286	0.0032793	0.003853672
6	0.067294751	0.057265569	0.0045286	0.0032793	0.003853672
7	0.067294751	0.062992126	0.0045286	0.003968	0.004239039
8	0.067294751	0.071581961	0.0045286	0.005124	0.00481709
9	0.067294751	0.071581961	0.0045286	0.005124	0.00481709
10	0.066621803	0.071581961	0.0044385	0.005124	0.004768919
11	0.067294751	0.065855404	0.0045286	0.0043369	0.004431723
12	0.064602961	0.070150322	0.0041735	0.0049211	0.004531919
13	0.065948856	0.070866142	0.0043493	0.005022	0.004673541
14	0.063930013	0.066571224	0.004087	0.0044317	0.004255899
15	0.067294751	0.067287044	0.0045286	0.0045275	0.004528065
			0.066683	**0.066999**	**0.066651669**
	r= 0.196815				

Analysis: Since r= 0.196815 is +ve i.e. the parameters Peer Rev Sc. and citations are positive. Hence this correlation is leading to raise the rank of the particular university/ institute.

Table- 4.2

Peer Rev Sc	**Emp. Rev. Sc**

S. No	p	Q	p2	q2	pq
1	0.067294751	0.069541029	0.0045286	0.004836	0.004679746
2	0.067294751	0.069541029	0.0045286	0.004836	0.004679746
3	0.067294751	0.068845619	0.0045286	0.0047397	0.004632949
4	0.065948856	0.068845619	0.0043493	0.0047397	0.00454029
5	0.067294751	0.069541029	0.0045286	0.004836	0.004679746
6	0.067294751	0.069541029	0.0045286	0.004836	0.004679746
7	0.067294751	0.068845619	0.0045286	0.0047397	0.004632949
8	0.067294751	0.066759388	0.0045286	0.0044568	0.004492556
9	0.067294751	0.069541029	0.0045286	0.004836	0.004679746
10	0.066621803	0.050069541	0.0044385	0.002507	0.003335723
11	0.067294751	0.068845619	0.0045286	0.0047397	0.004632949
12	0.064602961	0.068845619	0.0041735	0.0047397	0.004447631
13	0.065948856	0.054937413	0.0043493	0.0030181	0.00362306
14	0.063930013	0.067454798	0.004087	0.0045501	0.004312386
15	0.067294751	0.068845619	0.0045286	0.0047397	0.004632949

r=0.17270005

Table- 4.3

Analysis: Since the correlation between parameters Peer Rev Sc. and Emp. Rev. is +ve which shows this would raise the standard of the university/ Institute for increasing the Rank among world universities.

	Peer Rev Sc	**Int. Staff Sc.**		
S. No	**p**	**q**	**p2**	**q2**
1	0.067294751	0.080952381	0.0045286	0.0065533
2	0.067294751	0.093333333	0.0045286	0.0087111
3	0.067294751	0.080952381	0.0045286	0.0065533
4	0.065948856	0.091428571	0.0043493	0.0083592
5	0.067294751	0.093333333	0.0045286	0.0087111
6	0.067294751	0.091428571	0.0045286	0.0083592
7	0.067294751	0.073333333	0.0045286	0.0053778
8	0.067294751	0.084761905	0.0045286	0.0071846
9	0.067294751	0.02952381	0.0045286	0.0008717
10	0.066621803	0.095238095	0.0044385	0.0090703
11	0.067294751	0.026666667	0.0045286	0.0007111
12	0.064602961	0.078095238	0.0041735	0.0060989
13	0.065948856	0.026666667	0.0043493	0.0007111
14	0.063930013	0.027619048	0.004087	0.0007628
15	0.067294751	0.026666667	0.0045286	0.0007111
			0.066683	**0.078746**
			r=0 .2267101	

Analysis: The correlation r=0.2267101 between Peer Rev Sc. and Int. Staff Sc. is +ve which again shows that this correlation helps to raise the standard of university/ Institute for ranking among world universities.

Table- 4.4

Analysis:

(i) Correlation between Peer Review and citations r_1= 0.196815
(ii) Correlation between Peer Review and Emp. Review Score r_2= 0.1727005
(iii) Correlation between Peer Review and Int. Staff r_3= 0.226710
(iv) As $r_3 > r_1 > r_2$ i.e Peer Rev. Sc. and Int. Staff Sc. is more important.

Conclusion: All are positive but not leading to a perfect correlation as all correlation coefficients are even less than 0.5. Since all correlations are lying between 0 and 1, so we can consider them as probabilities for analysis.

SECTION 5

In this section, we have taken first 5 universities which have been analyzed year wise from 2014 - 2018, having the correlation between the parameters and conclude which is the best pair of parameters statistically.

5.1 APPLICATION OF STATISTICAL TECHNIQUES- CORRELATION

University wise Correlation between Teaching and Research:

1. UNIVERSITY OF OXFORD

Year	Rank	Teaching	Research	p	q	pq	p2	q2
2018	1	86.7	99.5	0.196866	0.201539	0.0396764	0.03875641	0.04061813
2017	1	89.6	99.1	0.203451	0.200729	0.0408386	0.04139248	0.04029221
2016	2	86.5	98.9	0.196412	0.200324	0.0393461	0.03857781	0.04012974
2015	3	88.6	97.7	0.201181	0.197893	0.0398124	0.04047369	0.03916182
2014	2	89	98.5	0.202089	0.199514	0.0403196	0.04083997	0.03980579
		440.4	**493.7**	**1**	**1**	**0.199993**	**0.2000404**	**0.2000077**

$\boxed{r_1= -0.39682}$

2. UNIVERSITY OF CAMBRIDGE

Year	Rank	Teaching	Research	p	q	pq	p2	q2
2018	2	87.8	97.8	0.196465	0.202652	0.039814	0.03859831	0.04106795
2017	4	90.6	97.2	0.20273	0.201409	0.0408316	0.04109942	0.0405656
2016	4	88.2	96.7	0.19736	0.200373	0.0395455	0.03895081	0.04014933
2015	5	89.7	95.6	0.200716	0.198094	0.0397606	0.04028693	0.0392411
2014	7	90.6	95.3	0.20273	0.197472	0.0400335	0.04109942	0.0389952
		446.9	**482.6**	**1**	**1**	**0.1999852**	**0.20003489**	**0.20001918**

$\boxed{r_2= -0.5721}$

3. CALIFORNIA INSTITUTE OF TECHNOLOGY

Year	Rank	Teaching	Research	P	q	pq	p2	q2
2018	3	90.3	97.5	0.192949	0.200164	0.0386214	0.03722921	0.04006572

2017	2	95.5	95.7	0.20406	0.196469	0.0400914	0.04164041	0.03860003
2016	1	95.6	97.6	0.204274	0.20037	0.0409302	0.04172766	0.04014795
2015	1	92.2	98.1	0.197009	0.201396	0.0396767	0.03881237	0.04056036
2014	1	94.4	98.2	0.201709	0.201601	0.0406649	0.04068668	0.04064309
		468	**487.1**	**1**	**1**	**0.1999846**	**0.20009634**	**0.20001715**

r_3= -0.37941

4. STANFORD UNIVERSITY

Year	Rank	Teaching	Research	p	q	pq	p2	q2
2018	3	89.1	96.7	0.193527	0.200498	0.0388018	0.03745284	0.04019929
2017	3	92.6	95.9	0.201129	0.198839	0.0399924	0.04045306	0.03953691
2016	3	92.5	96.2	0.200912	0.199461	0.0400741	0.04036573	0.03978466
2015	4	91.5	96.7	0.19874	0.200498	0.0398469	0.03949768	0.04019929
2014	4	94.7	96.8	0.205691	0.200705	0.0412831	0.04230867	0.04028248
		460.4	**482.3**	**1**	**1**	**0.1999984**	**0.20007797**	**0.20000263**

r_4= -0.11237

5. MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Year	Rank	Teaching	Research	p	q	pq	p2	q2
2018	5	87.3	91.9	0.194432	0.204222	0.0397073	0.03780383	0.04170672
2017	5	90.3	92.3	0.201114	0.205111	0.0412506	0.04044667	0.04207057
2016	5	89.4	88.6	0.199109	0.196889	0.0392024	0.03964445	0.03876523
2015	6	89.1	88.2	0.198441	0.196	0.0388944	0.03937882	0.038416
2014	5	92.9	89	0.206904	0.197778	0.0409211	0.04280936	0.03911605
		449	**450**	**1**	**1**	**0.1999758**	**0.20008313**	**0.20007457**

r_5= -0.3073

Conclusion: Although correlation coefficient is negative between Teaching and Research of Oxford University r = -0.39682 but even then it is on first position in ranking.

University wise Correlation between Research and Citation:

1. UNIVERSITY OF OXFORD

Year	Rank	Research	Citations	p	q	pq	p2	q2
2018	1	99.5	99.1	0.201539	0.203074	0.0409274	0.0406181	0.041239
2017	1	99.1	99.2	0.200729	0.203279	0.040804	0.0402922	0.0413222
2016	2	98.9	98.8	0.200324	0.202459	0.0405574	0.0401297	0.0409897
2015	3	97.7	95.5	0.197893	0.195697	0.0387271	0.0391618	0.0382972
2014	2	98.5	95.4	0.199514	0.195492	0.0390033	0.0398058	0.038217
		493.7	**488**			**0.200019**	**0.200008**	**0.200065**

r_1= 0.83318

2. UNIVERSITY OF CAMBRIDGE

Year	Rank	Research	Citations	p	q	pq	p2	q2

2018	2	97.8	97.5	0.202652	0.202198	0.0409759	0.041068	0.0408841
2017	4	97.2	96.8	0.201409	0.200747	0.0404322	0.0405656	0.0402992
2016	4	96.7	97	0.200373	0.201161	0.0403073	0.0401493	0.0404659
2015	5	95.6	95.2	0.198094	0.197428	0.0391093	0.0392411	0.038978
2014	7	95.3	95.7	0.197472	0.198465	0.0391914	0.0389952	0.0393885
		482.6	**482.2**			**0.2000161**	**0.2000192**	**0.2000157**

$\boxed{\mathbf{r_2= 0.92731}}$

3. CALIFORNIA INSTITUTE OF TECHNOLOGY

Year	Rank	Research	Citations	p	q	pq	p2	q2
2018	3	97.5	99.5	0.200164	0.199559	0.0399445	0.0400657	0.0398237
2017	2	95.7	99.8	0.196469	0.20016	0.0393253	0.0386	0.0400642
2016	1	97.6	99.8	0.20037	0.20016	0.0401061	0.040148	0.0400642
2015	1	98.1	99.7	0.201396	0.19996	0.0402711	0.0405604	0.039984
2014	1	98.2	99.8	0.201601	0.20016	0.0403526	0.0406431	0.0400642
		487.1	**498.6**			**0.1999996**	**0.2000171**	**0.2000003**

$\boxed{\mathbf{r_3= 0.17667}}$

4. STANFORD UNIVERSITY

Year	Rank	Research	Citations	p	q	pq	p2	q2
2018	3	96.7	99.9	0.200498	0.200643	0.0402284	0.0401993	0.0402575
2017	3	95.9	99.9	0.198839	0.200643	0.0398956	0.0395369	0.0402575
2016	3	96.2	99.9	0.199461	0.200643	0.0400204	0.0397847	0.0402575
2015	4	96.7	99.1	0.200498	0.199036	0.0399062	0.0401993	0.0396153
2014	4	96.8	99.1	0.200705	0.199036	0.0399475	0.0402825	0.0396153
		482.3	**497.9**			**0.1999981**	**0.2000026**	**0.2000031**

$\boxed{\mathbf{r_4= 0.6690}}$

5. MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Year	Rank	Research	Citations	p	q	pq	p2	q2
2018	5	91.9	100	0.204222	0.20016	0.0408771	0.0417067	0.0400641
2017	5	92.3	99.9	0.205111	0.19996	0.041014	0.0420706	0.039984
2016	5	88.6	99.7	0.196889	0.19956	0.0392911	0.0387652	0.0398241
2015	6	88.2	100	0.196	0.20016	0.0392314	0.038416	0.0400641
2014	5	89	100	0.197778	0.20016	0.0395872	0.039116	0.0400641
		450	**499.6**			**0.2000008**	**0.2000746**	**0.2000003**

$\boxed{\mathbf{r_5= 0.1691}}$

CONCLUSION: The correlation between Research and Citations is r= 0.83318 shows is nearly perfect due to citations depends upon high class research. Therefore for Ranking Trend High Class Research is more important than any other parameter viz. Teaching as it is very clear from the above trend. Hence OXFORD University has stood up on first place although University of CAMBRIDGE has correlation coefficient r=0.92731 which more than that of OXFORD but University of CAMBRIDGE has to jump from forth place to second.

The ordered sequence of correlation coefficient is

r_2= 0.92731> r_1=0.83318> r_3=0.6690> r_4>0.17667> r_5=0.1691

University of OXFORD has maintained its position i.e. FIRST in 2017 and 2018.

University wise Correlation between Research and Industry Income:

1. **UNIVERSITY OF OXFORD**

Year	Rank	Research	Industry Income	p	q	pq	p2	q2
2018	1	99.5	63.7	0.201539	0.175724	0.0354153	0.0406181	0.030879
2017	1	99.1	62.5	0.200729	0.172414	0.0346085	0.0402922	0.029727
2016	2	98.9	73.1	0.200324	0.201655	0.0403964	0.0401297	0.040665
2015	3	97.7	72.9	0.197893	0.201103	0.0397971	0.0391618	0.040443
2014	2	98.5	90.3	0.199514	0.249103	0.0496996	0.0398058	0.062053
		493.7	**362.5**	**1**	**1**	**0.1999169**	**0.2000077**	**0.203765**

r_1= -0.4880

2. **UNIVERSITY OF CAMBRIDGE**

Year	Rank	Research	Industry Income	p	q	pq	p2	q2
2018	2	97.8	51.5	0.202652	0.197469	0.0400176	0.041068	0.038994
2017	4	97.2	50.4	0.201409	0.193252	0.0389226	0.0405656	0.037346
2016	4	96.7	55	0.200373	0.21089	0.0422566	0.0401493	0.044474
2015	5	95.6	51.1	0.198094	0.195936	0.0388136	0.0392411	0.038391
2014	7	95.3	52.8	0.197472	0.202454	0.039979	0.0389952	0.040988
		482.6	**260.8**	**1**	**1**	**0.1999894**	**0.2000192**	**0.200193**

r_2= -0.174

3. **CALIFORNIA INSTITUTE OF TECHNOLOGY**

Year	Rank	Research	Industry Income	p	q	pq	p2	q2
2018	3	97.5	92.6	0.200164	0.20065	0.040163	0.0400657	0.04026
2017	2	95.7	90.8	0.196469	0.19675	0.0386552	0.0386	0.03871
2016	1	97.6	97.8	0.20037	0.211918	0.0424618	0.040148	0.044909
2015	1	98.1	89.1	0.201396	0.193066	0.0388827	0.0405604	0.037275
2014	1	98.2	91.2	0.201601	0.197616	0.0398397	0.0406431	0.039052
		487.1	**461.5**	**1**	**1**	**0.2000025**	**0.2000171**	**0.200207**

r_3= 0.0420

4. STANFORD UNIVERSITY

Year	Rank	Research	Industry Income	p	q	pq	p2	q2
2018	3	96.7	60.5	0.200498	0.19573	0.0392433	0.0401993	0.03831
2017	3	95.9	60.9	0.198839	0.197024	0.039176	0.0395369	0.038818
2016	3	96.2	63.3	0.199461	0.204788	0.0408472	0.0397847	0.041938
2015	4	96.7	63.1	0.200498	0.204141	0.0409298	0.0401993	0.041674
2014	4	96.8	61.3	0.200705	0.198318	0.0398033	0.0402825	0.03933
		482.3	**309.1**	**1**	**1**	**0.1999996**	**0.2000026**	**0.20007**

$\boxed{r_4 = -0.0296}$

5. MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Year	Rank	Research	Industry Income	p	q	pq	p2	q2
2018	5	91.9	88.4	0.204222	0.191259	0.0390594	0.0417067	0.03658
2017	5	92.3	88.4	0.205111	0.191259	0.0392294	0.0420706	0.03658
2016	5	88.6	95.4	0.196889	0.206404	0.0406387	0.0387652	0.042603
2015	6	88.2	95.7	0.196	0.207053	0.0405824	0.038416	0.042871
2014	5	89	94.3	0.197778	0.204024	0.0403515	0.039116	0.041626
		450	**462.2**	**1**	**1**	**0.1998613**	**0.2000746**	**0.20026**

$\boxed{r5 = 0.99}$

CONCLUSION: Although the correlation coefficient of UNIVERSITY OF OXFORD is Negative i.e. r= 0.488 between Research and Industry Income even then it is on First Place while Univ. of Cambridge which is on IInd place in 2018 which has jumped from IVth place to IInd place while Univ. of Oxford has maintained its first place from 2017 to 2018.

SECTION 6

APPLICATION OF INFORMATION THEORETIC MEASURE- KL DIVERGENCE:

Here we have considered the data from Reuters for the year 2016-17 for first 15 universities for ranking. The following table is for different parameters combinedly. The data has been transformed into probabilities and Kullback-Leibler Divergence (KL-Divergence) has been applied for maintaining the rank in world universities ranking. The present rank has been considered the weight as a parameter.

WORLD UNIVERSITY RANKING (THE) FOR YEAR 2016-17:

Rank	Name	Overall 100	Teaching 30%	Research 30%	Citations 30%	Industry Income 2.5%	International Outlook 7.5%
1	University of Oxford	95	89.6	99.1	99.2	62.5	94.5
2	California Institute of Technology	94.3	95.5	95.7	99.8	90.8	63.4

3	Stanford University	93.8	92.6	95.9	99.9	60.9	76.5
4	University of Cambridge	93.6	90.6	97.2	96.8	50.4	92.4
5	Massachusetts Institute of Technology	93.4	90.3	92.3	99.9	88.4	85.6
6	Harward University	92.7	87.5	98.3	99.7	47.3	77.9
7	Princeton University	90.2	89.5	88.4	99.2	49.9	77.2
8	Imperial College London	90	86.4	86.6	97.3	67.5	96.5
9	ETH Zurich – Swiss Federal Institute of Technology Zurich	89.3	81.5	93.7	92.5	63.7	98.1
10	University of California, Berkeley	88.9	82.4	96.1	99.8	37.6	59.6
10	University of Chicago	88.9	88.1	89.1	99.1	37.7	67.8
12	Yale University	88.2	88.5	87.8	97.8	44.5	64.3
13	University of Pennsylvania	87.1	85.9	88.9	98.6	49.9	50.1
14	University of California, Los Angeles	86.6	82.9	89	98.4	47.1	58
15	University College London	86.5	77.4	90	94	41.9	94.3

Table- 6.1

KL-WEIGHTED DIVERGENCE AMONG THE PARAMETERS FOR TWO METHODOLOGIES FOR YEAR 2016-17:

Here for KL-Weighted divergence, we use the result (2.8) i.e.

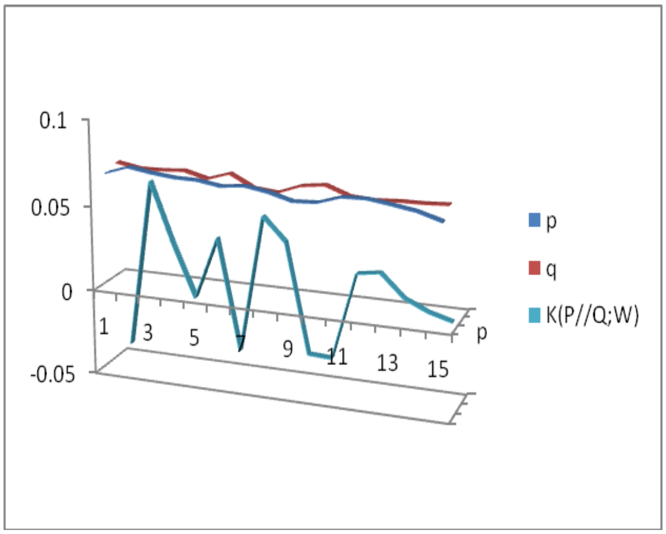
$$K(P \| Q;) = \sum_{i=1}^{n} w_i \mid x_i \log \frac{p_i}{q_i}$$

	Teaching	Research			
S. No	**p**	**q**	**w**	**Ln (p/q)**	**K(P//Q;W)**
1	-0.018185	-0.0186755	15	-0.041872494	**-0.043001935**
2	0.0246721	0.0252056	14	0.056809577	**0.058038056**
3	0.010373	0.00954157	13	0.023884787	**0.021970281**
4	-0.0049574	-0.00411837	12	-0.011414871	**-0.009482882**
5	0.0160667	0.01219459	11	0.036994946	**0.028079071**
6	-0.0249648	-0.01669154	10	-0.057483606	**-0.038433679**
7	0.0309514	0.01905048	9	0.071268283	**0.043865364**
8	0.0245765	0.01298027	8	0.056589488	**0.029888174**

9	-0.0350013	-0.01525809	7	-0.080593541	**-0.035133045**
10	-0.0412155	-0.01557038	6	-0.094902252	**-0.035852123**
11	0.0206789	0.00696037	5	0.047614826	**0.016026844**
12	0.0290294	0.00785238	4	0.066842679	**0.018080774**
13	0.0106721	0.00210147	3	0.024573314	**0.004838804**
14	-0.0052548	-0.00066574	2	-0.01209968	**-0.001532916**
15	-0.0399209	-0.00236103	1	-0.091921262	**-0.005436468**
		0.0225461			

Table- 6.2

GRAPHICAL REPRESENTATION OF K(P//Q;W):



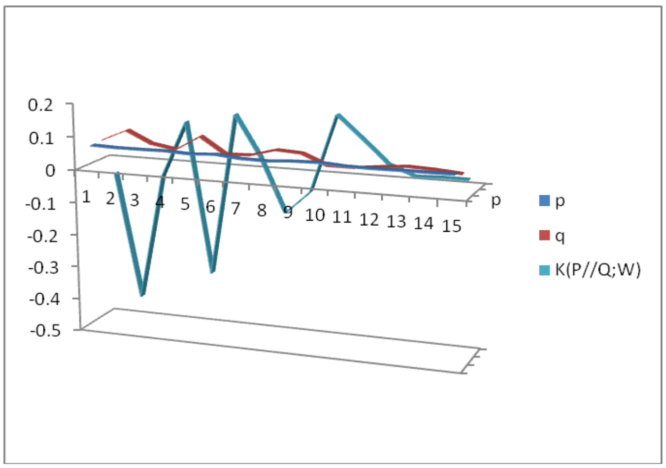
The graphicalrepresentation is asymptotic as weighted KL – Divergence is Non-symmetric.

	Research	Industry Income 2.5				
S. No	**p**	**q**	**w**	**p/q**	**log(p/q)**	**K(P//Q:W)**
1	0.071392551	0.074395905	15	0.9596301	-0.0178961	-0.01916476
2	0.06894316	0.108082371	14	0.6378761	-0.1952637	-0.18846933
3	0.069087242	0.07249137	13	0.9530409	-0.0208885	-0.01876063
4	0.070023774	0.059992858	12	1.1672018	0.067146	0.05642176
5	0.066493768	0.105225568	11	0.6319165	-0.1993403	-0.14580379
6	0.070816224	0.056302821	10	1.257774	0.0996026	0.07053481
7	0.063684173	0.059397691	9	1.0721658	0.030262	0.01734487

8	0.062387436	0.080347578	8	0.7764694	-0.1098757	-0.05483888
9	0.067502341	0.075824307	7	0.8902467	-0.0504896	-0.02385717
10	0.069231323	0.044756577	6	1.5468413	0.1894458	0.07869349
11	0.064188459	0.04487561	5	1.430364	0.1554466	0.04988938
12	0.063251927	0.052969885	4	1.1941111	0.0770447	0.01949291
13	0.064044377	0.059397691	3	1.0782301	0.0327114	0.00628495
14	0.064116418	0.056064754	2	1.1436137	0.0582793	0.00747332
15	0.064836827	0.049875015	1	1.2999861	0.1139387	0.00738742
	1	**1**				**-0.137372**

Table- 6.3

GRAPHICAL REPRESENTATION OF K(P//Q;W):



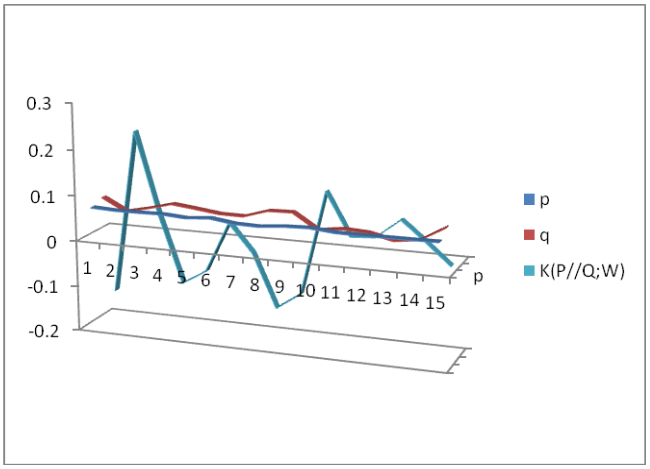
The graphicalrepresentation is asymptotic as weighted
KL – Divergence is Non-symmetric.

	Research	International Outlook 7.5				
S. No	**p**	**q**	**w**	**p/q**	**log(p/q)**	**K(P//Q:W)**
1	0.071392551	0.081733264	15	0.8734822	-0.0587459	-0.06291034
2	0.06894316	0.054834804	14	1.2572883	0.0994349	0.09597498
3	0.069087242	0.066165023	13	1.0441656	0.0187694	0.01685743
4	0.070023774	0.079916969	12	0.8762066	-0.0573935	-0.04822691
5	0.066493768	0.074035634	11	0.8981319	-0.0466599	-0.03412849
6	0.070816224	0.067375887	10	1.0510618	0.0216283	0.01531632
7	0.063684173	0.066770455	9	0.9537777	-0.0205528	-0.01178001
8	0.062387436	0.083463069	8	0.7474855	-0.1263972	-0.06308478

9	0.067502341	0.084846912	7	0.7955781	-0.0993172	-0.04692901
10	0.069231323	0.051548175	6	1.3430412	0.1280893	0.05320677
11	0.064188459	0.058640374	5	1.094612	0.0392602	0.01260026
12	0.063251927	0.055613216	4	1.1373542	0.0558958	0.01414206
13	0.064044377	0.043331604	3	1.4780061	0.1696762	0.03260043
14	0.064116418	0.050164331	2	1.2781276	0.1065742	0.01366632
15	0.064836827	0.081560284	1	0.7949559	-0.099657	-0.00646144
	13.881	**1**				**-0.019156**

Table- 6.4

GRAPHICAL REPRESENTATION OF K(P//Q;W):



The graphical representation is asymptotic as weighted KL – Divergence is Non-symmetric.

SECTION 7

APPLICATION OF WEIGHTED J. DIVERGENCE

In this section, we utilized the other information theoretic measure – the Weighted J-divergence to support the application of information theoretic weighted J-divergence for ranking problems. As in case of Correlation and Weighted KL-divergence, the results indicate that the best combination, correlation of Research and citation or research and peer review raises the rank 1of any University, the same is the pattern of weighted J-divergence for the support of application of Weighted J-divergence.

The weighed J-divergence has been defined with ways

$$J(P\|Q:W)=\sum_{i=1}^{n} w_i(p_i-q_i)\log\frac{p_i}{q_i} \tag{7.1}$$

and in term fo K-weighted KL-divergence,

$$J(P\|Q:W)=K(P\|Q;W)+K(Q\|P;W) \tag{7.2}$$

Here in the next tables we use (7.1):

	Teaching	Research				
S. No	**p**	**q**	**w**	**p-q**	**log(p/q)**	**J(P//Q:W)**
1	0.0684649	0.0713926	15	-0.002927662	-0.01818499	0.000798593
2	0.0729732	0.0689432	14	0.004030019	0.024672081	0.001392005
3	0.0707572	0.0690872	13	0.001669998	0.010373028	0.000225198
4	0.069229	0.0700238	12	-0.000794768	-0.00495742	4.728E-05
5	0.0689998	0.0664938	11	0.002506003	0.016066706	0.000442895
6	0.0668602	0.0708162	10	-0.003955981	-0.02496482	0.000987603
7	0.0683885	0.0636842	9	0.004704304	0.030951419	0.001310444
8	0.0660197	0.0623874	8	0.003632278	0.024576501	0.000714149
9	0.0622755	0.0675023	7	-0.0052268	-0.03500133	0.001280614
10	0.0629632	0.0692313	6	-0.006268077	-0.04121552	0.001550052
11	0.0673187	0.0641885	5	0.003130254	0.020678855	0.00032365
12	0.0676244	0.0632519	4	0.004372433	0.029029407	0.000507717
13	0.0656377	0.0640444	3	0.001593279	0.010672058	5.10107E-05
14	0.0633453	0.0641164	2	-0.000771113	-0.00525482	8.10412E-06
15	0.0591427	0.0648368	1	-0.005694166	-0.03992089	0.000227316
	1	1				**0.00986663**

Table 6.5

	Research	Industry Income 2.5				
S. No	**p**	**q**	**w**	**p-q**	**log(p/q)**	**J(P//Q:W)**
1	0.0713926	0.0743959	15	-0.003003354	-0.01789613	0.000806226
2	0.0689432	0.1080824	14	-0.039139211	-0.19526368	0.106994528
3	0.0690872	0.0724914	13	-0.003404128	-0.02088845	0.000924391
4	0.0700238	0.0599929	12	0.010030916	0.067145962	0.008082426
5	0.0664938	0.1052256	11	-0.0387318	-0.19934033	0.08492891
6	0.0708162	0.0563028	10	0.014513403	0.099602611	0.014455728
7	0.0636842	0.0593977	9	0.004286482	0.030261951	0.001167456
8	0.0623874	0.0803476	8	-0.017960142	-0.10987565	0.015787059
9	0.0675023	0.0758243	7	-0.008321966	-0.05048962	0.00294121
10	0.0692313	0.0447566	6	0.024474746	0.189445767	0.027819822
11	0.0641885	0.0448756	5	0.019312849	0.155446584	0.015010582
12	0.0632519	0.0529699	4	0.010282042	0.077044731	0.003168709
13	0.0640444	0.0593977	3	0.004646686	0.032711443	0.000455999
14	0.0641164	0.0560648	2	0.008051664	0.058279331	0.000938491

15	0.0648368	0.049875	1	0.014961812	0.113938714	0.00170473
	1	**1**				**0.28518627**

Table 6.6

	Research	International Outlook 7.5				
S. No	**p**	**q**	**w**	**p-q**	**log(p/q)**	**J(P//Q:W)**
1	0.0713926	0.0817333	15	-0.010340713	-0.05874594	0.009112124
2	0.0689432	0.0548348	14	0.014108356	0.09943489	0.01964008
3	0.0690872	0.066165	13	0.002922219	0.018769388	0.000713027
4	0.0700238	0.079917	12	-0.009893195	-0.05739349	0.00681366
5	0.0664938	0.0740356	11	-0.007541866	-0.04665986	0.003870926
6	0.0708162	0.0673759	10	0.003440337	0.02162827	0.000744085
7	0.0636842	0.0667705	9	-0.003086282	-0.02055282	0.000570886
8	0.0623874	0.0834631	8	-0.021075633	-0.12639721	0.02131121
9	0.0675023	0.0848469	7	-0.017344571	-0.09931721	0.0120583
10	0.0692313	0.0515482	6	0.017683148	0.128089337	0.013590136
11	0.0641885	0.0586404	5	0.005548085	0.039260218	0.001089095
12	0.0632519	0.0556132	4	0.007638711	0.055895751	0.001707886
13	0.0640444	0.0433316	3	0.020712773	0.16967624	0.010543396
14	0.0641164	0.0501643	2	0	0.106574227	0
15	0.0648368	0.0815603	1	-0.016723457	-0.09965698	0.001666609
	1	**1**				**0.10343142**

Table 6.7

All the universities have positive weighted J-divergence, implies combination of teaching and research is to raise the rank.

CONCLUSION: The world ranking systems QS and THE both emphasize that research and citation, combination or research and peer review is highly preferred from the angles of correlation, K-Divergence and J-Divergence.

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