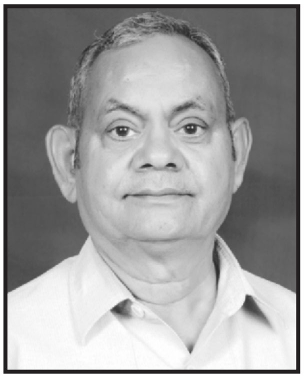
ISSN (Print) : 0975-7139 , ISSN (Online) : 2394-9309

ARYABHATTA JOURNAL OF MATHEMATICS & INFORMATICS

Volume 15 | Number 1 | JAN.-JUNE, 2023

CONTENTS



Dr. T.P. Singh
Chief Editor & Professor in Maths & O.R.

FOREWORD

It gives me an immense pleasure in writing this foreword for **'Aryabhatta Journal of Mathematics & Informatics' (AJMI) Vol.15 issue 1 Jan-June. 2023** published by Aryans Research & Educational Trust. The Journal covers areas of mathematical and statistical sciences, Operational Research, data based managerial & economical issues and information sciences. I am pleased to note that research scholars, professors, executives from different parts of country have sent their papers for this issue. The papers are relevant and focus on the futuristic trends and innovations in the related areas. We have received around 43 papers for this issue from which on reviewer's report only 19 have been selected for publication. DOI no. by cross Ref. have been mentioned on each article.

1. Dr. S. Sasikala et.al. Proposed mathematical study to analyze the M/M/c/N Interdependent Inter Arrival Queueing Model with controllable Arrival Rates, reverse Balkingand Impatient Customers.
2 In paper No. 2 Rashi Jaiswal proposed a stride window approach to deal with sensor data augmentation to perform multiclass classification. The study finds its applications in defense for accurate monitoring and controlling.
3. In Paper No. 3 C. Saranya and her scholar presented the special Diophantine triples involving heptagonal pyramidal numbers.
4. Prof. S.K.Pandey produced a counter example in algebra improving one of the results given by Danchev.
5. In paper no.5, P. Shalini et.al. proved some theorems on graph theory and conclude that path, comb, $P_n \odot K_{1,2}$ is a root cube even Mean Labeling graph.
6. In manuscript 6 Vinita Jain investigated various formulae for new integrals involving the product of generalized I-function of two variables, w.r.t. parameters and angle of SINE &COSINE.
7. Dr. Gowri Shankari& her scholarin their study analyzed the trinity cubic equation.
8. Prof. Naresh Kumar & Amrita examined the reliability of fuzzy scale for assessing survey respondents' opinions.
9. In paper no. 9 Dr. P Saranya et.al. demonstrated the key solutions to the transcendental under complex pattern on applying a variety of numerical illustrations.
10. In paper 10 Vandana Saini et.al. modeled a complex bi-tandem queue network with feedback and fixed size batches.
11. The paper 11 by Renu Bala et.al. deals with a dentistry system through a stochastic model.
12. Dr C.Saranya et.al.explored Special Diophantine triples for octagonal pyramidal numbers.
13. S. Saridha and her scholar in their study introduced plus weighted context free Dendrolanguage.
14. Prof. Pandey in his study showed visualizing each group as a quotient group and each ring as a quotient ring and discussed the algebraic implications of this idea.
15. In paper no. 14 P. Shalini and Meena made an effort to introduce Lehmer-4 mean labeling for some path related graphs
16. In paper no. 15 Aarti Saini et.al. presented a priority queue model under fuzzy environment.
17. In her study Dr. P. Shalini explored a new concept skolem odd vertex graceful signed graph.
18. In paper no. 18 Dr. Naresh Kumar searched out a Heuristic technique for real time scheduling in fuzzy sense which includes various practical parameters.
19. In paper 19 Suman et.al. examined an inventory model for the objects which deteriorate over time having a cubic demand and Weibull deterioration rate.
20. In paper 20 Dr. E. Litta et.al. introduce proper colourings in *r* - regular Zagreb index graph in a new way and established new inequalities on chromatic number.

I would like to thank and felicitate the contributors in this issue. Comments, suggestions and feedback from discerning readers, scholars and academicians are always welcome.

DR. T.P. SINGH

Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00001.X
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

THE M/M/c/N INTERDEPENDENT INTER ARRIVAL QUEUEING MODEL WITH CONTROLLABLE ARRIVAL RATES, REVERSE BALKING AND IMPATIENT CUSTOMERS

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ABSTRACT
In this paper, The M/M/c/N interdependent inter arrival queueing model with controllable arrival rates reverse balking and impatient customers is considered. This model is much useful in analyzing the particular situations arising at the places like a data voice transmission, computer communication system etc. some importance measures of performance are derived. The steady state solution and the system characteristics are derived for this model. The analytical results are numerically illustrated and the effects of the nodal parameters on the system characteristics are studied.
Keywords : Reverse Balking, Impatient Customers, Steady-State Analysis, Multi-server Queuing System.

INTRODUCTION

Queue is an unavoidable phenomenon of modern life that we encounter at every step in our daily life. In real practice, it is often likely that an arrival become discouraged when queue is long and may not wish to enter the queue. This type of arrival is called balking. Jain and Rakesh Kumar [1] have studied M/M/1/N queuing system with reverse balking. On the other hand, a customer may enter the queue, but after a time lose patience and decide to leave the queue. This type of arrival is called impatient customers. Rakesh Kumar and Bhupender Kumar Som [2] have studied an M/M/1/N queuing system with balking and reneging. Rakesh Kumar and Bhupender Kumar Som [3] have studied a multi-server queue with reverse balking and impatient customers.

However, in many particular situations, it is necessary to consider that the arrival and services processes and interdependent. A queuing model in which arrivals and services are correlated is known as interdependent queuing model. Much work has been reported in the literature regarding interdependent standard queuing model with controllable arrival rates. Srinivasa Rao. K and Shobha. T [4] have discussed M/M/1/∞ interdependent queuing model with controllable arrival rates. A. Srinivasa and M. Thiagarajan [5], have analysed M/M/1/K Interdependent queuing model with controllable arrival rates. S. Sasikala and M. Thiagarajan [6-11] they analysed the concept of controllable arrival rates with reverse balking and reverse reneging. S. Sasikala and V.Surthi [12-13] have studied Retention of Reneged customers with controllable arrival rates.

In this paper, an M/M/c/N Interdependent Inter-arrival Queueing Model with controllable arrival rates, reverse balking and impatient customers is considered with the assumption that the arrival and service processes of the system are correlated. Here the arrival rate is considered as λ_0- a faster rate of arrival and λ_1- a slower rate of arrival. Whenever the queue size reaches a certain prescribed number R, the arrival rate reduces from λ_0 to λ_1 and it continues with that rate as long as the content in the queue is greater than some other prescribed integer r $(r \geq 0)$ and $(r < R)$. When the content reaches r, the arrival rate changes back to λ_0 and the same process is repeated. The description of the model is given stating the relevant postulates. The steady state equations are obtained. The characteristics of the model are derived. The analytical results are numerically illustrated.

DESCRIPTION OF THE MODEL

I. The arrival process is Poisson with parameterλ.

II. There are multiple-servers, say c. The service times follow exponential distribution with parameter μ such as $\mu_n = n\mu$ when $n < c$ and $\mu_n = c\mu$ when $n \geq c$.

III. The system capacity is taken as finite, say N.

IV. The queue discipline is First-Come, First-Served.

(a) When the system is empty, the customers balk with probability q' and may not balk with probability $p' = (1 - q')$.

(b) When the system is not empty, customers balk with a probability $1 - \frac{n}{N-1}$ and do not balk with probability $\frac{n}{N-1}$.

The balking described in (a) and (b) is called reverse balking.

V. Each customer upon joining the queue waits for some time for his service to begin. If he does not receive service by then, he leaves the queue without getting service (i.e. reneged). The reneging times follow the exponentially distribution with parameter ξ.

THE STEADY STATE EQUATIONS

Let $P_0(0)$ be the steady state probability that there are n customers in the system when the arrival rate is λ_1 and $P_0(1)$ be the steady state probability that there n customers in the system when the arrival rate is λ_0. We observe that $P_n(0)$ exists when $n = 0,1,2,\dots.r-1,r$ both $P_n(0)\&P_n(1)$ exists when $n = r+1, r+2, \dots R-1$ and $P_n(1)$ exists when$n = R, R+1, \dots K$.further$P_n(0) = P_n(1) = 0$ if $n > K$ with this dependence structure, the steady state equations are

The matrix of the transition densities is given by $A = \left(a_{ij}\right)$

The steady state equations which are written through the matrix of densities are given by

$$0 = -(\lambda_0 - \varepsilon)p'P_0(0) + (\mu - \varepsilon)P_1(0) \quad (1)$$

$$0 = -\left\{\left(\frac{1}{N-1}\right)(\lambda_0 - \varepsilon) + (\mu - \varepsilon)\right\}P_1(0) + 2(\mu - \varepsilon)P_2(0) + (\lambda_0 - \varepsilon)p'P_0(0) \quad (2)$$

$$0 = -\left\{\left(\frac{n}{N-1}\right)(\lambda_0 - \varepsilon) + n(\mu - \varepsilon)\right\}P_n(0) + \left(\frac{n-1}{N-1}\right)(\lambda_0 - \varepsilon)P_{n-1}(0)$$
$$+ \{(n+1)(\mu - \varepsilon)\}P_{n+1}(0)(2 \leq n \leq c-1) \quad (3)$$

$$0 = -\left\{\left(\frac{n}{N-1}\right)(\lambda_0 - \varepsilon) + c(\mu - \varepsilon) + (n-c)\xi\right\}P_n(0) + \left(\frac{n-1}{N-1}\right)(\lambda_0 - \varepsilon)P_{n-1}(0)$$
$$+ [c(\mu - \varepsilon) + \{(n+1) - c\}\xi]P_{n+1}(0)(c \leq n \leq r-1) \quad (4)$$

$$0=-\left\{\left(\frac{n}{N-1}\right)(\lambda_0-\varepsilon)+c(\mu-\varepsilon)+(n-c)\xi\right\}P_r(0)+\left(\frac{n-1}{N-1}\right)(\lambda_0-\varepsilon)P_{r-1}(0)$$
$$+[c(\mu-\varepsilon)+\{(n+1)-c\}\xi]P_{r+1}(0)+[c(\mu-\varepsilon)+\{(n+1)-c\}\xi]P_{r+1}(1)$$
$$(n=r) \qquad (5)$$

$$0=-\left\{\left(\frac{n}{N-1}\right)(\lambda_0-\varepsilon)+c(\mu-\varepsilon)+(n-c)\xi\right\}P_n(0)+\left(\frac{n-1}{N-1}\right)(\lambda_0-\varepsilon)P_{n-1}(0)$$
$$+\quad [c(\mu-\varepsilon)+\{(n+1)-c\}\xi]P_{n+1}(0)$$
$$(r+1\le n\le R-2) \qquad (6)$$

$$0=-\left\{\left(\frac{R-1}{N-1}\right)(\lambda_0-\varepsilon)+c(\mu-\varepsilon)+(n-c)\xi\right\}P_{R-1}(0)+\left(\frac{R-2}{N-1}\right)(\lambda_0-\varepsilon)P_{R-2}(0)$$
$$(n=R-1) \qquad (7)$$

$$0=-\left\{\left(\tfrac{n}{N-1}\right)(\lambda_1-\varepsilon)+c(\mu-\varepsilon)+(n-c)\xi\right\}P_{r+1}(1)+[c(\mu-\varepsilon)+\{(n+1)-c\}\xi]P_{r+2}(1)$$
$$(n=r+1) \qquad (8)$$

$$0=-\left\{\left(\frac{n}{N-1}\right)(\lambda_1-\varepsilon)+c(\mu-\varepsilon)+(n-c)\xi\right\}P_n(1)+\left(\frac{n-1}{N-1}\right)(\lambda_1-\varepsilon)P_{n-1}(1)$$
$$+[c(\mu-\varepsilon)+\{(n+1)-c\}\xi]P_{n+1}(1)$$
$$(r+2\le n\le R-1) \qquad (9)$$

$$0=-\left\{\left(\frac{n}{N-1}\right)(\lambda_1-\varepsilon)+c(\mu-\varepsilon)+(n-c)\xi\right\}P_R(1)+[c(\mu-\varepsilon)+\{(n+1)-c\}\xi]P_{R+1}(1)$$
$$+\left(\frac{n-1}{N-1}\right)(\lambda_0-\varepsilon)P_{R-1}(1)+\left(\frac{n-1}{N-1}\right)(\lambda_0-\varepsilon)P_{R-1}(0)$$
$$(n=R) \qquad (10)$$

$$0=-\left\{\left(\frac{n}{N-1}\right)(\lambda_1-\varepsilon)+c(\mu-\varepsilon)+(n-c)\xi\right\}P_n(1)+\left(\frac{n-1}{N-1}\right)(\lambda_1-\varepsilon)P_{n-1}(1)$$
$$+[c(\mu-\varepsilon)+\{(n+1)-c\}\xi]P_{n+1}(1)(R+1\le n\le N-1) \qquad (11)$$

$$0=\left(\tfrac{N-1}{N-1}\right)(\lambda_1-\varepsilon)P_{N-1}(t)-[c(\mu-\varepsilon)+(N-C)\xi]P_N(t)(\,n=N) \qquad (12)$$

From (1), we get

$$P_1(0)=\frac{(\lambda_0-\varepsilon)}{(\mu-\varepsilon)}p'P_0(0)$$

Using the above result in (2), we get

$$P_2(0)=\left(\frac{1}{N-1}\right)\frac{(\lambda_0-\varepsilon)^2}{2(\mu-\varepsilon)}p'P_0(0)$$

And hence, we recursively derive,

$$P_n(0)=\frac{(n-1)!}{(N-1)^{n-1}}\prod_{l=1}^{n}\frac{(\lambda_0-\varepsilon)}{l(\mu-\varepsilon)}p'P_0(0),n=1,2,\ldots.c \qquad (13)$$

And hence, we recursively derive,

$$P_n(0)=\frac{(n-1)!}{(N-1)^{n-1}}\prod_{l=1}^{c}\frac{(\lambda_0-\varepsilon)}{l(\mu-\varepsilon)}\prod_{t=c+1}^{n}\frac{(\lambda_0-\varepsilon)}{[c(\mu-\varepsilon)+(t-c)\xi]}p'P_0(0),$$
$$n=c+1,c+2,\ldots.r \qquad (14)$$

Using (14) in (5), we get

$$P_{r+1}(0) \quad = \frac{r!}{(N-1)^r} \prod_{l=1}^{c} \frac{(\lambda_0-\varepsilon)}{l(\mu-\varepsilon)} \prod_{t=c+1}^{r+1} \frac{(\lambda_0-\varepsilon)}{[c(\mu-\varepsilon)+(t-c)\xi]} p' P_0(0) - P_{r+1}(1)$$

Using the above result in (6), we recursively derive

$$\begin{aligned}
P_n(0) = &\frac{(n-1)!}{(N-1)^{n-1}} \prod_{l=1}^{c} \frac{(\lambda_0-\varepsilon)}{l(\mu-\varepsilon)} \prod_{t=c+1}^{n} \frac{(\lambda_0-\varepsilon)}{[c(\mu-\varepsilon)+(t-c)\xi]} p' P_0(0) \\
&-\left\{\frac{1}{\prod_{t=r+2}^{n}[c(\mu-\varepsilon)+(t-c)\xi]}\left[\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{n-(r+1)} (n-1)P_{n-(r+1)} \right.\right. \\
&+\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{n-(r+2)} (n-1)P_{n-(r+2)}(c(\mu-\varepsilon)+(r+1-c)\xi) \\
&+\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{n-(r+3)} (n-1)P_{n-(r+3)}(c(\mu-\varepsilon)+(r+1-c)\xi)(c(\mu-\varepsilon)+(r+2-c)\xi) \\
&+\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{n-(n-1)} (n-1)P_{n-(r+4)}(c(\mu-\varepsilon)+(r+1-c)\xi)(c(\mu-\varepsilon) \\
&+(r+2-c)\xi)(c(\mu-\varepsilon)+(r+3-c)\xi)+\cdots \\
&\left.\left.+\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{n-n} \prod_{l=r+1}^{n-1}[c(\mu-\varepsilon)+(l-c)\,\xi]\right]\right\} P_{r+1}(1)
\end{aligned}$$

$$n=(r+1, r+2, \dots.R-1) \tag{15}$$

Put $n=R-2$ in (15), we get

$$\begin{aligned}
P_{R-1}(0) = &\frac{(R-2)!}{(N-1)^{R-2}} \prod_{l=1}^{c} \frac{(\lambda_0-\varepsilon)}{l(\mu-\varepsilon)} \prod_{t=c+1}^{R-1} \frac{(\lambda_0-\varepsilon)}{[c(\mu-\varepsilon)+(t-c)\xi]} p' P_0(0) \\
&-\left\{\frac{1}{\prod_{t=r+2}^{R-1}[c(\mu-\varepsilon)+(t-c)\xi]}\left[\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{R-r-2} \frac{(R-2)!}{r!} \right.\right. \\
&+\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{R-r-3} \frac{(R-2)!}{(r+1)!}(c(\mu-\varepsilon)+(r+1-c)\xi) \\
&+\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{R-r-4} \frac{(R-2)!}{(r+2)!}(c(\mu-\varepsilon)+(r+1-c)\xi)(c(\mu-\varepsilon)+(r+2-c)\xi) \\
&+\left(\frac{\lambda_0-\varepsilon}{N-1}\right) \frac{(R-2)!}{(r+3)!}(c(\mu-\varepsilon)+(r+1-c)\xi)(c(\mu-\varepsilon)+(r+2-c)\xi)(c(\mu-\varepsilon) \\
&\left.\left.+(r+3-c)\xi)+\cdots+\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{R-R} \prod_{l=r+1}^{R-2}[c(\mu-\varepsilon)+(l-c)\,\xi]\right]\right\} P_{r+1}(1)
\end{aligned}$$

Consider the equation (8), we get

$$P_{r+2}(1) = \frac{\left[\left(\frac{r+1}{N-1}\right)(\lambda_1-\varepsilon)+c(\mu-\varepsilon)+(r+1-c)\xi\right]}{[c(\mu-\varepsilon)+\{(r+2)-c\}\xi]} P_{r+1}(1) \tag{16}$$

And hence, we recursively derive,

$$P_n(1) = \frac{P_{r+1}(1)}{\prod_{t=r+2}^{n}[c(\mu-\varepsilon)+(t-c)\xi]}\Bigg[\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{n-(r+1)}(n-1)P_{n-(r+1)}$$
$$+\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{n-(r+2)}(n-1)P_{n-(r+2)}(c(\mu-\varepsilon)+(r+1-c)\xi)$$
$$+\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{n-(r+3)}(n-1)P_{n-(r+3)}(c(\mu-\varepsilon)+(r+1-c)\xi)(c(\mu-\varepsilon)+(r+2-c)\xi)$$
$$+\cdots+\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{n-n}\prod_{l=r+1}^{R-2}[c(\mu-\varepsilon)+(l-c)\,\xi]\Bigg]$$

$(n = r+1, r+2, \ldots\ldots, R-2)$ (17)

Put $n = R$ in (10), we get

$$0 = -\left\{\left(\frac{R}{N-1}\right)(\lambda_1-\varepsilon)+c(\mu-\varepsilon)+(R-c)\xi\right\}P_R(1)$$
$$+[c(\mu-\varepsilon)+\{(R+1)-c\}\xi]P_{R+1}(1)+\left(\frac{R-1}{N-1}\right)(\lambda_1-\varepsilon)P_{R-1}(1)+\left(\frac{R-1}{N-1}\right)(\lambda_0-\varepsilon)P_{R-1}(0)$$

Using the above result and (15), we get

Substituting the value $P_0(0)$, we get

$$P_{R+1}(1) = \frac{P_{r+1}(1)}{\prod_{t=r+2}^{R+1}[c(\mu-\varepsilon)+(t-c)\xi]}\Bigg[\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{R-r}\frac{R!}{r!}$$
$$+\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{R-r-1}\frac{R!}{(r+1)!}(c(\mu-\varepsilon)+(r+1-c)\xi)$$
$$+\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{R-r-2}\frac{R!}{(r+2)!}(c(\mu-\varepsilon)+(r+1-c)\xi)(c(\mu-\varepsilon)+(r+2-c)\xi)+\cdots$$
$$+\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{R-R}\prod_{l=r+1}^{R-2}[c(\mu-\varepsilon)+(l-c)\,\xi]\Bigg] \quad (18)$$

Using the above result in (12), we get

$$P_N(1) = \frac{P_{r+1}(1)}{\prod_{t=r+2}^{N}[c(\mu-\varepsilon)+(t-c)\xi]}\Bigg[\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{N-r+3}\frac{(N-1)!}{r!}$$
$$+\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{N-r+4}\frac{(N-1)!}{(r+1)!}(c(\mu-\varepsilon)+(r+1-c)\xi)+\cdots$$
$$+\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{N-N}\prod_{l=r+1}^{N-2}[c(\mu-\varepsilon)+(l-c)\,\xi]\Bigg]$$

$(n = R+1, R+2, \ldots. N)$ (19)

Where$P_{R+1}(1)$ is given by (16),

Thus from (13) to (19), we find that all the steady state probabilities are expressed in terms of $P_0(0)$

CHARACTERISTICS OF THE MODEL

The following system characteristics are considered and their analytical results are derived in this section.

I. The probability $P(0)$ that the system is in faster rate of arrivals

II. The probability $P(1)$ that the system is in slower rate of arrivals

III. The probability $P_0(0)$ that the system is empty

IV. The expected number of customers in the system L_{s_0}, when the system is in the faster rate of arrivals

V. The expected number of customers in the system L_{s_1}, when the system is in the slower rate of arrivals
VI. The expected waiting time of the customers in the system W_s
The probability that the system is in faster rate of arrivals is

$$P(0) = \sum_{n=0}^{N} P_n(0)$$

$$P(0) = \sum_{n=0}^{r} P_n(0) + \sum_{n=r+1}^{R-1} P_n(0) + \sum_{n=R}^{N} P_n(1)$$

Since $P_0(0)$ exists only $n = 0,1,2, \dots r-1, r+1, r+2, \dots . R-2, R-1$,We get

$$P(0) = \sum_{n=0}^{r} P_n(0) + \sum_{n=r+1}^{R-1} P_n(0) \tag{20}$$

$$\begin{aligned} P(0) = [1 + \sum_{n=1}^{R-1} \frac{(n-1)!}{(N-1)^{n-1}} \prod_{l=1}^{c} \frac{(\lambda_0 - \varepsilon)}{l(\mu - \varepsilon)} \prod_{t=c+1}^{n} \frac{(\lambda_0 - \varepsilon)}{[c(\mu-\varepsilon) + (t-c)\xi]} p' \\ - \sum_{n=r+1}^{R-1} \Bigg\{ \frac{1}{\prod_{t=r+2}^{n} [c(\mu-\varepsilon) + (t-c)\xi]} \Bigg[\left(\frac{\lambda_0 - \varepsilon}{N-1}\right)^{n-(r+1)} (n-1) P_{n-(r+1)} \\ + \left(\frac{\lambda_0 - \varepsilon}{N-1}\right)^{n-(r+2)} (n-1) P_{n-(r+2)} (c(\mu-\varepsilon) + (r+1-c)\xi) \\ + \left(\frac{\lambda_0 - \varepsilon}{N-1}\right)^{n-(r+3)} (n-1) P_{n-(r+3)} (c(\mu-\varepsilon) + (r+1-c)\xi)(c(\mu-\varepsilon) + (r+2-c)\xi) \\ + \left(\frac{\lambda_0 - \varepsilon}{N-1}\right)^{n-(n-1)} (n-1) P_{n-(r+4)} (c(\mu-\varepsilon) + (r+1-c)\xi)(c(\mu-\varepsilon) \\ + (r+2-c)\xi)(c(\mu-\varepsilon) + (r+3-c)\xi) + \cdots \\ + \left(\frac{\lambda_0 - \varepsilon}{N-1}\right)^{n-n} \prod_{l=r+1}^{n-1} [c(\mu-\varepsilon) + (l-c)\,\xi] \Bigg] \Bigg\} P_{r+1}(1)] P_0(0) \end{aligned} \tag{21}$$

$$A = \frac{\sum_{n=1}^{R-1} \frac{(n-1)!}{(N-1)^{n-1}} \prod_{l=1}^{c} \frac{(\lambda_0-\varepsilon)}{l(\mu-\varepsilon)} \prod_{t=c+1}^{n} \frac{(\lambda_0-\varepsilon)}{(c(\mu-\varepsilon)+(t-c)\xi)}}{\sum_{n=r+1}^{R-1} \prod_{t=r+2}^{n} (c(\mu-\varepsilon)+(t-c)\xi)}$$

$$[\left(\frac{\lambda_0 - \varepsilon}{N-1}\right)^{n-(r+1)} (n-1) P_{n-(r+1)}$$

$$+ \left(\frac{\lambda_0 - \varepsilon}{N-1}\right)^{n-(r+2)} (n-1) P_{n-(r+2)} (c(\mu-\varepsilon) + (t-c)\xi)$$

$$+ \cdots + \left(\frac{\lambda_0 - \varepsilon}{N-1}\right)^{n-n} \prod_{l=r+1}^{n-1} [c(\mu-\varepsilon) + (l-c)\xi]\, P_0(0)$$

The probability that the system is in slower rate of arrival is

$$P(1) = \sum_{n=0}^{N} P_n(1)$$

$$= \sum_{n=0}^{N} P_n(1) + \sum_{n=r+1}^{R-1} P_n(1) + \sum_{n=R+1}^{N} P_n(1) \tag{22}$$

Since $P_n(1)$ exists only when $n = r+1, r+2, \dots . R-2, R-1, \dots N$, We get

$$P(1) = \sum_{n=r+1}^{R} P_n(1) + \sum_{n=R+1}^{N} P_n(1)$$

$$P(1) = [\sum_{n=r+1}^{R} \frac{1}{\prod_{t=r+2}^{n}[c(\mu-\varepsilon)+(t-c)\xi]}$$
$$\left[\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{n-(r+1)}(n-1)P_{n-(r+1)} + \left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{n-(r+2)}(n-1)P_{n-(r+2)}(c(\mu-\varepsilon)+(r+1-c)\xi) + \cdots +\right.$$
$$\left.\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{n-n}\prod_{l=r+1}^{R-2}[c(\mu-\varepsilon)+(l-c)\,\xi]\right] + \sum_{n=R+1}^{N}\frac{1}{\prod_{t=r+2}^{N}[c(\mu-\varepsilon)+(t-c)\xi]}\left[\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{N-r+3}(N-1)P_{N-r+3} +\right.$$
$$\left.\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{N-r+4}(N-1)P_{N-r+4}(c(\mu-\varepsilon)+(r+1-c)\xi) + \cdots + \left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{n-n}\prod_{l=r+1}^{N-2}[c(\mu-\varepsilon)+(l-c)\,\xi]\right]]\,P_{r+1}(1)$$

(23)

The probability $P_0(0)$ that the system is empty, can be calculated from the normalizing condition

$$P(0) + P(1) = 1 \tag{24}$$

$$P_0(0) = \frac{1}{\begin{array}{c}[[1+\sum_{n=1}^{R-1}\frac{(n-1)!}{(N-1)^{n-1}}\prod_{l=1}^{c}\frac{(\lambda_0-\varepsilon)}{l(\mu-\varepsilon)}\prod_{t=c+1}^{n}\frac{(\lambda_0-\varepsilon)}{[c(\mu-\varepsilon)+(t-c)\xi]}p' \\ -\sum_{n=r+1}^{R-1}[\frac{1}{\prod_{t=r+2}^{n}(c(\mu-\varepsilon)+(t-c)\xi)} \\ [\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{n-(r+1)}(n-1)P_{n-(r+1)} \\ +\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{n-(r+2)}(n-1)P_{n-(r+2)}(c(\mu-\varepsilon)+(r+1-c)\xi) \\ +\cdots+\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{n-n}\prod_{l=r+1}^{n-1}(c(\mu-\varepsilon)+(l-c)\xi)] \\ +\sum_{n=r+1}^{R}[\frac{1}{\prod_{t=r+2}^{n}(c(\mu-\varepsilon)+(t-c)\xi)} \\ [\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{n-(r+1)}(n-1)P_{n-(r+1)} \\ +\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{n-(r+2)}(n-1)P_{n-(r+2)}(c(\mu-\varepsilon)+(r+1-c)\xi) \\ +\cdots+\left(\frac{\lambda_0-\varepsilon}{N-1}\right)^{n-n}\prod_{l=r+1}^{R-2}(c(\mu-\varepsilon)+(l-c)\xi)] \\ +\sum_{n=R+1}^{N}[\frac{1}{\prod_{t=r+2}^{N}(c(\mu-\varepsilon)+(t-c)\xi)} \\ [\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{N-r+3}(N-1)P_{N-(r+3)} \\ +\left(\frac{\lambda_1-\varepsilon}{N-1}\right)^{N-r+4}(N-1)P_{N-(r+4)}(c(\mu-\varepsilon)+(r+1-c)\xi) \\ \left(\frac{\lambda_1-\varepsilon}{N-1}\right)(N-1)\prod_{l=r+1}^{N-2}(c(\mu-\varepsilon)+(r+1-c)\xi)]]A\end{array}}$$

(25)

Now we calculate the expected number of customers in the system.

Let $L_s$denote the average number of customers in the system, then We have,

$$L_s = L_{s_0} + L_{s_1} \tag{26}$$

Where

$$L_{s_0} = \sum_{n=0}^{r} nP_n(0) + \sum_{n=r+1}^{R-1} nP_n\,(0) \qquad (27)$$

And

$$L_{s_1} = \sum_{n=r+1}^{R-1} nP_n(1) + \sum_{n=R}^{N} nP_n\,(1) \qquad (28)$$

Using Little's formula, the expected waiting time of the customers in the system is calculates as

$$W_s = \frac{L_s}{\bar{\lambda}}$$

Where $\bar{\lambda} = \lambda_0 P(0) + \lambda_1 P(1)$ (29)

NUMERICAL ILLUSTRATION

For various values of λ_0, λ_1, μ, $\varepsilon, r, R, P, p, \boldsymbol{\xi}$ the values of $P_0(0), P(0), P(1), L_s$ and W_s are computed and tabulates the following Table.

Table 1

$\boldsymbol{\lambda_0}$	$\boldsymbol{\lambda_1}$	μ	**r**	**R**	**N**	$\boldsymbol{p'}$	$\boldsymbol{\varepsilon}$	$\boldsymbol{\xi}$	$\boldsymbol{P_0(0)}$	$\boldsymbol{P(0)}$	$\boldsymbol{P(1)}$	$\boldsymbol{L_S}$	$\boldsymbol{W_S}$
3	3	4	2	5	8	0.2	0.5	0.1	0.886014	0.999485	0.000515	0.128095	0.042698
3	3	4	2	5	8	1.0	0.5	0.1	0.608921	0.998233	0.001767	0.437727	0.145162
3	3	4	2	5	8	1.0	0.5	0.2	0.652130	0.999064	0.000936	0.38321	0.127744
3	3	5	2	5	8	0.8	0.0	0.1	0.589081	0.998045	0.001955	0.470818	0.117705
4	4	4	2	5	8	1.0	0.5	0.1	0.512485	0.990528	0.009472	0.601150	0.150288
4	3	4	2	5	8	1.0	0.5	0.1	0.513809	0.993088	0.006912	0.58545	0.146616
4	4	5	2	5	8	0.0	0.5	0.1	1.000000	1.000000	0.000000	0.000000	0.000000
4	4	5	2	5	8	0.8	1.0	0.1	0.589081	0.998045	0.001955	0.470818	0.117705
4	4	5	2	5	8	1.0	0.5	0.1	0.573634	0.996873	0.003127	0.492510	0.123143
4	4	5	2	5	8	0.8	1.0	0.1	0.627110	0.997267	0.002733	0.4307	0.107698

CONCLUSIONS

The observations made from the table 1 are:

1. When λ_0 increases keeping other parameters fixed, $P_0(0)$ and$P(0)$ decrease but $P(1)$ and $L_s$increase. It is also observed that the expected system size is zero when $1 - p^{'}$ is
2. When λ_0 increases keeping other parameters fixed,$P_0(0)$and $P(0)$ decrease but $P(1)$ and L_s decrease.
3. When the mean dependence rate increases and the other parameters are kept fixed,$P_0(0)$ and $P(0)$ decrease andL_s increase.
4. When the balking rate decreases and other parameters are kept fixed,$P_0(0)$ and $P(0)$ decrease, $L_s$increase regularly and attains maximum when $1 - p^{'}$is zero.
5. When ξ increases and other parameters are kept constant,$P_0(0)$and $P(0)$ increase but $P(1)$,$L_s$and W_s decrease.

GRAPHICAL REPRESENTATION

The Average number of customers in the system (L_s) and the expected waiting time of the customers in the system (W_s) by varying ξ keeping other parameters fixed.

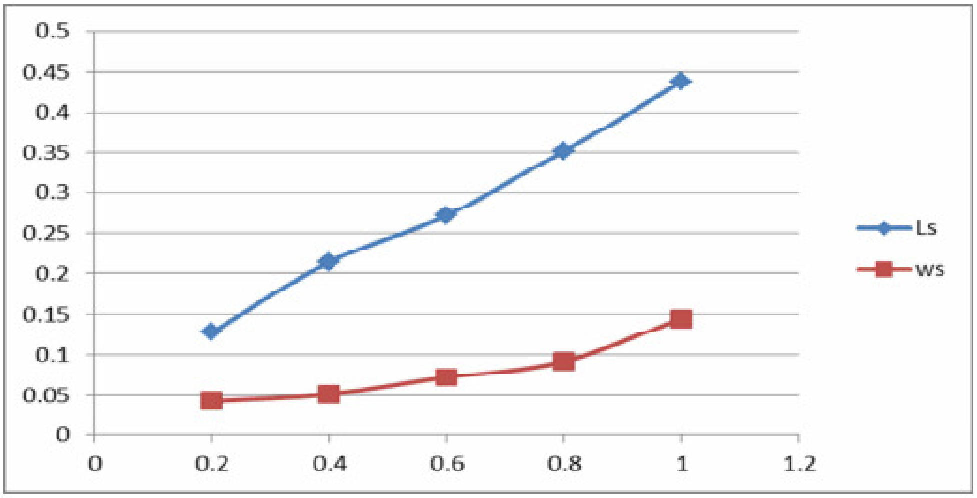

Figure 1

The Average number of customers in the system (L_s) and the expected waiting time of the customers in the system (W_s) by varying ξ keeping other parameters fixed.

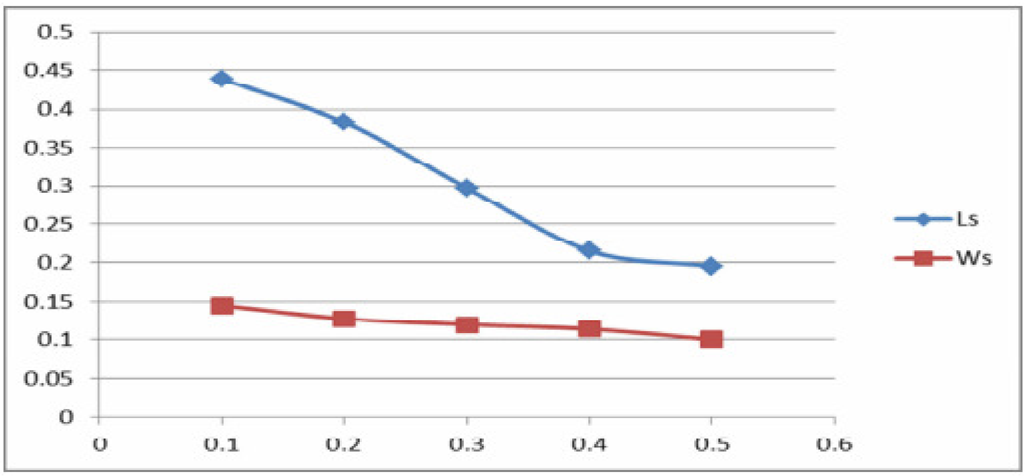

Figure 2

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00002.1
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

STRIDE WINDOW APPROACH IN MULTICLASS CLASSIFICATION FOR TEMPORAL SENSOR DATA AUGMENTATION

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ABSTRACT

In the real world, the sensors are used to fetch the data to perform different tasks such as analysis, sense to make a decision, and prediction purposes. Existing methods for sensor data are used to perform various tasks such as classification and regression. Various methods are available to compute the temporal sensor data with data augmentation as the sequential data. However, existing methods augmented a lot of messy and biased data that affects the outcome. In this paper, we propose a stride window approach for dealing with sensor data augmentation to perform multiclass classification. The stride window approach reduces the biased data and controls the overlapping in data augmentation and preprocessing. Illustrate the proposed method and justified by visualization and quantitative results on an open-source Human activity recognition dataset. The performance analysis has been done for multi-class classification by measuring the Accuracy and F1-Score, the results show that the proposed stride window approach outperforms with approx. 1%-10% more performance score than the resembling and time slice approaches for multiclass classification. The proposed approach can be used to generate augmented data concerning sensor data in defense applications for accurate monitoring and controlling.

Keywords: Stride window Approach, Multiclass Classification, Data Augmentation, Temporal Data, Sensor Data, and Human Activity Recognition.

INTRODUCTION

In the real world, the sensors are used for data acquisition from the environment and perform the next action or decision-making accordingly[1].Application-oriented tasks such as Human Activity Recognition[2], Suspicious Activity Detection[3], Fault Detection, Clinical Labs[4], Disease Monitoring[5], exercise motion monitoring[6], Health checkup and Treatment[7-8], Future data Prediction[9], and many other tasks perform based on sensor data in different domains (Defense, Finance, Banking, Marketing, Health care, etc.). The temporal sensor data are complex to preprocess due to time dependency and less data, so we have to handle it with different augmentation techniques[10]. The multiple sensors generate the different massive data with multi-classes where the classes indicate the multiple tasks performed at the same object. Therefore, the multiclass classification technique applies to classify the different classes at the same timestamp for specific purposes[11]. There are three general techniques are available for temporal sensor data augmentation: The re-sampling technique, the Sliding window approach, and the Time slice window approach[10]. There are some limitations with the above techniques are as overlapping of the data that compute the data with biased results and time taken in computation due to the huge amount of data augmented.

This paper proposed a new stride window approach for the data augmentation in multiclass classification on temporal sensor data to reduce the bias and repeated augmented data. The proposed approach has been illustrated in the human activity recognition dataset. The LSTM and the GRU models have been selected for the multiclass classification in experiments. The performance analysis based on experimental results shows that the stride window outperforms the sensor data for the multiclass classification task.

The paper has been organized into six sections: Section I provides the introduction of the paper and discuss the importance of the sensor data in multiclass classification. Section II discussed the generalized approaches of data augmentation and preprocessing of temporal sensor data. The proposed stride window approach and its importance in its application discuss in Section III. The illustration of the proposed stride window is elaborate in Section IV. Section V discussed the results obtained from the experiments and provides the applications of the proposed research. Section VI concludes the paper.

GENERALIZED APPROACHES

This paper uses acquired temporal sensor data for the multiple subjects at the same time and environment. The collected data have variable lengths concerning the time. That causes difficulties in the preprocessing to prepare and train the model. It required the fixed time interval samples from the sensor data. Data augmentation requires creating new samples at the new points from real data. Following techniques for preprocessing and data augmentation of temporal sensor data are available in the literature. In this Section, we provide the pros and cons of existing methods.

RESAMPLING

Re-sampling methods were introduced in the 1950s, which become the traditional statistical term and evergreen methodology[12]. Re-sampling is the technique that is used to create or generate repeated samples from the original data. It is used to make the data samples to quantify the uncertainty of population parameters to improve the accuracy of the results. There is two way to perform the re-sampling: up-sampling and down sampling. In sequential data, if we want to compress the data based on its frequency then have to perform the re-sampling, which has shown in Figure 1. If the data is temporal then we can resample (down sampled) the data based on their frequency of time, such as month to year and days to a month. Due to time dependency, up-sampling creates the noisy random temporal sequential data while down sampling creates data abstraction. The main disadvantage in re-sampling of the temporal sequential datais the data abstraction and noisy data that affects the temporal data as biased outcomes. The complete process for the re-sampling technique has provided in Algorithm 1.

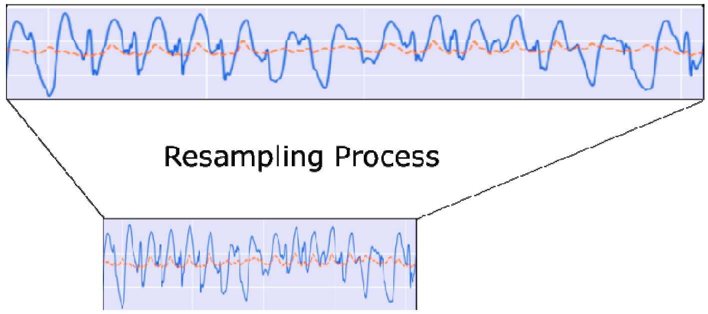


Figure 1 Re-sampling Approach

Algorithm 1: Re-sampling Technique

Input:
Temporal Sensor data (Tsd = {T1, T2, …, Tn}, time_interval

Procedure:
For each (Ti) in Tsd:
    data = read(Ti)
    Remove index column in data
    For each column (Ci) in the data:
        Ri = Resample Ci for the length of time_interval
(Each point of Ci)*length of time_interval/length of Ci
    Join all [R1, R2, …, Rk] as RTi
Append all [RT1, RT2, ….RTn] as RTsd

Output: Re-sampled Temporal Sensor data (RTsd)

Time Slice window

Time Slice is the approach used for temporal data preprocessing tasks, where the data is focused and selected for computation based on specified timestamps from the holistic data[13]. This becomes a popular statistical method in the 1950s in time series forecasting also called as time lag method. The time slice approach is useful for classification tasks to perform on temporal sequential data. Here, the windows are created based on time slices and can be separately analyzed to classify the activity in different time slices. Figure 2 shows the Time Slice window process. The fixed time window approach has the limitation of fixing the window size. The common drawback of this approach is the selection bias and sampling error. Algorithm 2 has described the process for the Time slice window approach in temporal data.

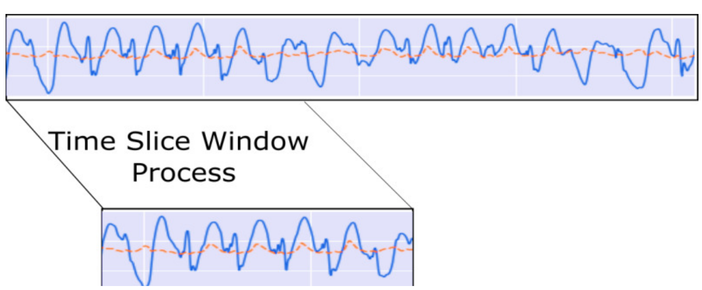


Figure 2 Time Slice Window Approach

Algorithm 2: Time Slice window

```
Input:
Temporal Sensor data (Tsd = {T1, T2, …, Tn}, time_interval
Procedure:
For each (Ti) in Tsd:
    data = read(Ti)
    Remove index column in data
    For each column (Ci) in data:
        Si = slice Ci for length of time_interval
            Si = Ci[0:time_interval]
    Join all [S1, S2, …, Sk] as STi
Append all [ST1, ST2, ….STn] as STsd
Output: Time Slice Temporal Sensor Data (STsd)
```

Sliding window

The sliding window approach was introduced by Datar et al. for data streams that are used to slide the window continuously to fetch the sequential data as a subset of data with a fixed window size[14]. The window size can shrink and expand as per certain conditions. The sliding window automatically shifts forward over time. The initial window starts from the starting point and then slides according to condition. Figure 3 visualize the sliding window process. The main drawback of this approach for augmentation is that the windows overlapped by sliding with repeated time interval data. That creates unwanted repeated data with biases of the data concerning the subject. The procedure for the sliding window approach in temporal data augmentation has given in Algorithm 3.

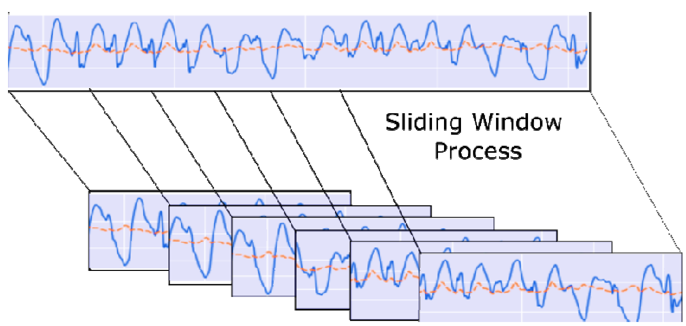


Figure 3 Sliding Window Approach

Algorithm 3: Sliding Window

```
Input:
Temporal Sensor data (Tsd = {T1, T2, …, Tn}, time_interval
Procedure:
TW =[ ]
For each (Ti) in Tsd:
    data = read(Ti)
    Remove index column in data
        temp_data = []
    for i in range(0, len(data) – time_interval):
        x = data[i:i + time_interval]
        temp_data.append([x, j])
    TW.append(temp_data)
Output:Augmented Temporal Sensor Data (TW)
```

PROPOSED STRIDE TIME WINDOW APPROACH FOR TEMPORAL SENSOR DATA

Generalized approaches have the limitation with temporal sensor data augmentation and preparation by augmenting the bias and repeated data. To overcome the drawback of the above approaches, we proposed a stride window approach. The stride window performs the less-repeated augmentation by skipping the time interval to the next window. The stride is the distance that the window covers in each step. We need to specify the dimensions for the stride to move in specific directions as per the dimension of the data. Generally, the stride window concept uses in the image datasets in two dimensions: one for moving left to right and another one is top to bottom. Here, we are concentrating on the sequential data in one dimension. The value of stride should be in an interval of half to the full-time window. If the stride value is the same as the length of the time window, it will generate the non-overlapped/repeated sub-samples and if the stride value is less than the length of the time window, it will generate the overlapped sub-sample. The ratio of overlapping depends on the formula (1 - the ratio of stride value and time window length). Algorithm 4 provides the complete process for data augmentation through the proposed stride window approach.

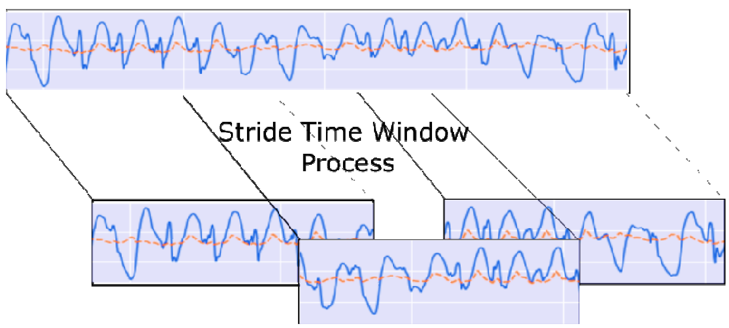


Figure 4 Stride Window Approach

Algorithm 4: Proposed Stride Window Approach

```
Input:
Temporal Sensor data (Tsd = {T1, T2, …, Tn}, time_interval, stride
Procedure:
STW =[ ]
For each (Ti) in Tsd:
    data = read(Ti)
    Remove index column in data
        temp_data = []
    for i in range(0, len(data) – time_interval, stride):
        x = data[i:i + time_interval]
        temp_data.append([x, j])
    STW.append(temp_data)
Output: Augmented Temporal Sensor Data (STW)
```

The proposed stride window approach for temporal sensor data has been illustrated in the next section.

EXPERIMENTS

The experiments have been performed on the proposed design-based setup with hardware requirements as a window operating system, an i5 processor with 12 GB RAM, and the Software requirements as a python programming language with its rich libraries, Tensor flow (Keras), Pandas, SKlearn, matplotlip/sea born for data visualization. The illustration of the proposed stride window approach has been performed on selected sensor data for multiclass classification by using selected recurrent classifiers. The details have been discussed in subsections.

HAR Dataset (Motion Sense)

This paper selected the open-source Human Activity Recognition Dataset[15] for illustration of the stride window approach to compare and analyze with the other data augmentation methods. There are 6 human activities analyzed in the selected HAR dataset as Upward sitting, downward sitting, jogging, sitting, standing, and walking positions. Each activity (subject) was noted based on different sensors at a different time interval. The dataset has a different number of instances for each activity. So, there preprocessing of the data need to perform for better results with fewer biased data. The proposed stride window approach illustrates this multiclass data with other existing approaches for data augmentation and preprocessing. To evaluate the proposed approach we have implemented it on all approaches with selected classifiers. The details of the selected classifiers have discussed in the next subsection.

Selected Classifiers

The selected classifiers for the evaluation of the proposed approach for multiclass classification on sensor data are LSTM and the GRU, which has selected based on their computational capability and sequential data handing property[16]. The LSTM and the GRU are recurrent neural networks with fast computation with long-term memory utilization advantages. The selected LSTM and GRU classifiers were used for performance evaluation on the selected dataset to justify the proposed stride window approach to sensor data classification. The performance of the proposed approach-based multiclass classification has been evaluated by using the performance metrics of classifiers. The graphical representation of the proposed framework has shown in Figure 5, where the data augmentation approach

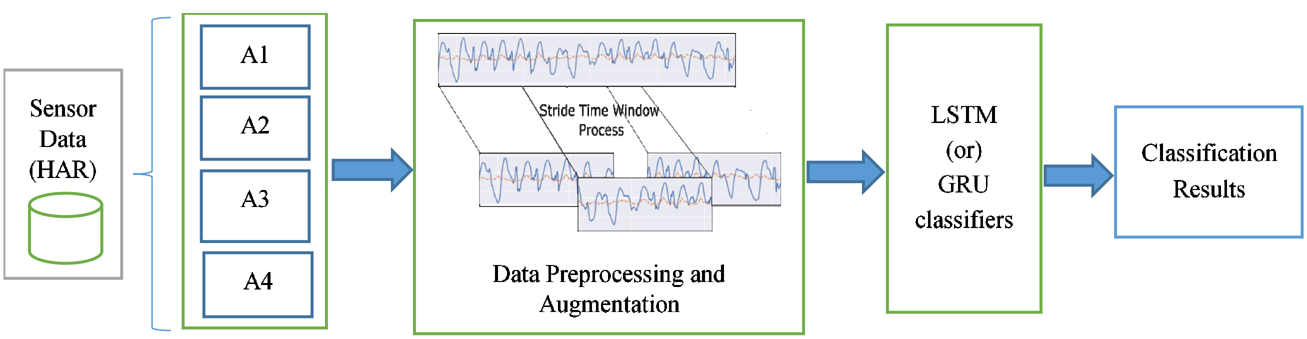


Figure 5. Proposed Framework for Multiclass Classification with Data Augmentation

PERFORMANCE EVALUATION

This paper evaluated the performance of the stride approach with other generalized approaches by using performance metrics of classifiers that are Accuracy and the F1-Score[17].Where the accuracy indicates how accurate the model works or the accuracy of the results with a specific approach and the F1 score indicates the harmonic mean of the precision and the recall score that shows the exactness and completeness of the model respectively. These performance metrics have been used to evaluate the performance of the classifiers for the performance analysis of the exiting and the proposed approach-based multiclass classification. The performance scores have shown in Table 1 concerning three data augmentation techniques (Resampling, Time slice, and Stride window) on two different classifiers LSTM and GRU.This paper does not consider the sliding window approach in performance evaluation because it creates the massive repeated augmented data which required a large amount of time and space to train the classifiers.

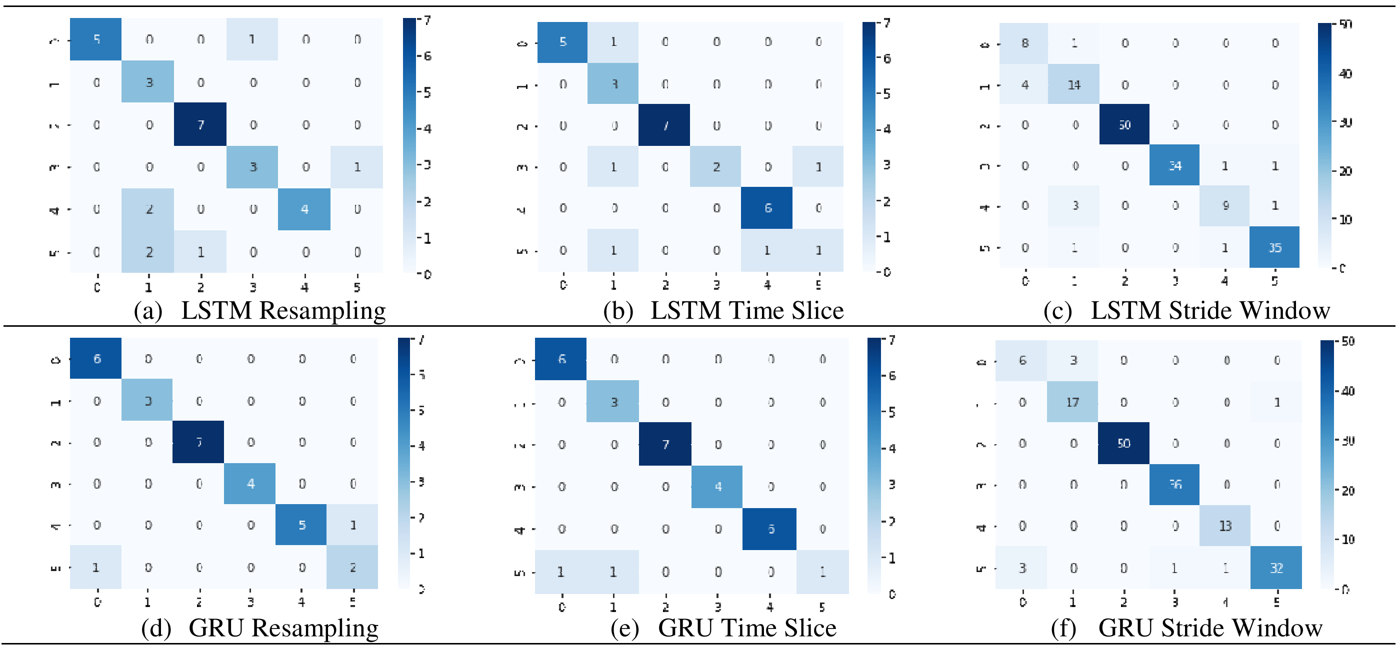
Table 1. Performance Score on Human Activity Recognition Dataset on different Approaches

Approaches	Accuracy		F1-Score	
	LSTM	GRU	LSTM	GRU
Resampling	0.758621	0.931034	0.744410	0.930793
Time Slice Window	0.827586	0.931034	0.822747	0.917582
Stride Time Window	**0.920245**	**0.944785**	**0.921474**	**0.944535**

There are two-step evaluation has been also performed that help to validate the experimental results and the proposed approach for sensor data classification. The first one is the confusion metrics-based evaluation and the second is performance metrics-based evaluation.

4. RESULTS VISUALIZATION

This paper also used the graphical visualization of the class-wise classification results between actual classes and predicted classes by confusion matrix[18]whichis shown in Figure 6. In the confusion matrix, actual classes have shown on the y-axis and predicted classes on the x-axis. The depth of color in the heatmap shows the density of predicted classes. Further, the final results are also visualized through the bar chart for the model performance evaluation based on Accuracy and F1 score in Figure 7.



(a) LSTM Resampling (b) LSTM Time Slice (c) LSTM Stride Window

(d) GRU Resampling (e) GRU Time Slice (f) GRU Stride Window

Figure 6. Confusion Matrix Visualization of different Augmentation approaches for LSTM and GRU classifiers

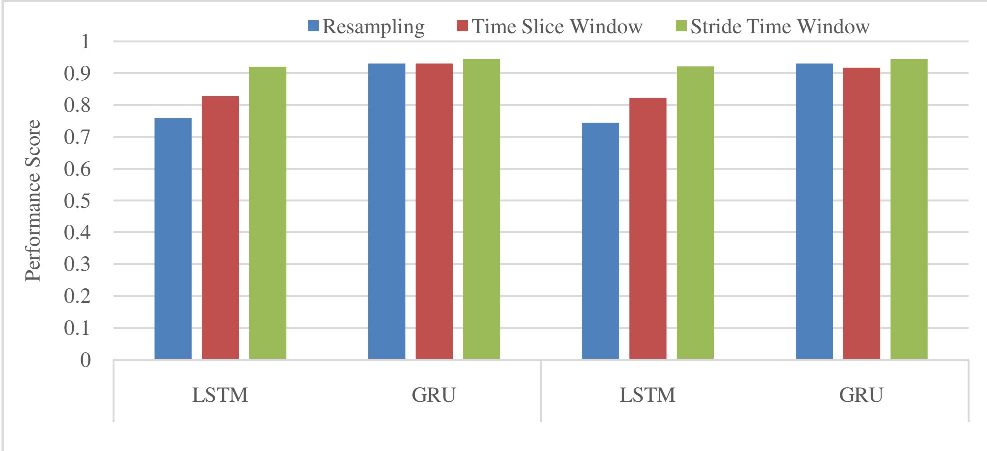


Figure 7. Performance Analysis of Stride Window Approach on LSTM and GRU Classifiers

RESULTS, DISCUSSION, AND APPLICATIONS

The experiments show that the original data has different data instances for different subjects of the HAR sensor dataset. This paper preprocessed and augmented the data before performing the classification task on it. And then, implement the generalized approaches (resampling and the time slice window). The proposed stride window approach uses for the preprocessing and augmentation of the data. In this paper, we illustrated the proposed stride window approach to temporal sensor data (HAR dataset). The time–interval plays a vital role in the augmentation process of temporal sensor data due to time dependency. We have done experiments only on re-sampling, time slice, and stride window, and have not taken the sliding window because it generates mostly massive repeated augmented data. Results obtained from the experiments show that the stride window approach outperforms without bias results.

The different augmentation approach-based classification results on the HAR dataset has shown in Figure 6 by using a confusion matrix for LSTM and GRU classifiers. The confusion matrix shows the actual (Y-axis) and the predicted value (X-axis) difference representation that is shown through Heat Map. Where it is easy to understand the stride window approach has augmented more data in comparison to re-sampling and the time-slicing approach with maximum true positive results. Whereas the re-sampling and time slice window with false positive and false negative values (upper and lower diagonal respectively).

This paper selects the LSTM and GRU classifiers for the multiclass classification task. The performance evaluation of classifiers has been done by using performance metrics of classifier's Accuracy and F1-Score, which are visualized in Figure 7showing the accuracy and F1-score on LSTM and GRU classifiers. Where we obtained that the highest performance score of stride window-based multiclass classification outperforms on selected (HAR) sensor data.

The experiments based on comparative analysis of different augmented approaches for multiclass classification also show that the generalized preprocessing and augmentation approaches such as re-sampling, time slice window, and sliding window have the limitations of data abstraction / noisy data, selection bias error, and massive repeated sample respectively. However, the proposed stride window approach overcomes these above-mentioned drawbacks of the existing generalized approach. Overall, our proposed stride window performs more accurately with less bias augmented data than the generalized approach in multiclass classification on temporal sensor data.

Applications

Various sensor-based approaches are used for Human activity recognition such as wearable sensors (human carry a sensor), objects tagged (sensor tag with object), and dense sensing (Environment carry sensor). At present, Four main sensors are used Accelerometer, Magnetometer, Motion Sensor, and Proximity Sensor in different domains (Defense, Healthcare, Finance, etc.) for human activity recognition. These sensors are used for different action-based activities such as Gesture recognition, Posture Recognition, Behavior Recognition, and Fall Detection. The sensors-based activity monitoring and controlling applications are used in defense to maintain discipline, safety, and security as intelligent defense systems. Where sensors are used to collect the data where sometimes data is noisy due to environmental effects. Data augmentation is required there to cover up the noisy and missing data by adorning the augmented data. Some sensor-based defense applications are given as follows:

1. There is a different kinds of Behavior analyses of the armed forces during combat and military operations where the sensor data used to deal with the activity recognition of the warrior.
2. The different sensors can assign to different parts of the agent that observes the activity of the focused element or object.
3. In defense, mock drill operations are performed by using wearable sensors during the training period to monitor the candidate's performance.
4. During the military secret operation, tracking the person or an object for monitoring and tracking the location.
5. Motion Detection task performed in military camp to detect the nearby people or objects to alert the security system by sending the signal to the alarm panel.
6. Creating a smart environment by implementing the sensor-based intelligent system to maintain confidentiality in defense offices to identify the suspicious person.

The proposed model is used to generate the augmented data for the above defense applications for multiclass classification. The data augmentation technique helps to obtain complete and accurate information as well as increase the performance of the system or applications.

In the future, we can extend this study by developing a new approach to data augmentation for temporal sensor data. Intelligent defense systems can develop for monitoring and controlling the object and the person with advanced operations. More advancement can be done in the military camps, airbase stations, and defense mobile ships by using temporal sensors.

CONCLUSION

This paper proposed anew stride window approach for the temporal sensor data preprocessing and data augmentation for multiclass classification with less bias and repeated samples. The illustration of the proposed approach has been performed through experiments by using LSTM and GRU classifiers on the human activity recognition dataset. Performance analysis based on results shows that the stride window approach outperforms multiclass classification on sensor data with 1-10% more accuracy than the existing data augmentation approaches. The proposed approach for data augmentation can be useful in defense applications concerning temporal sensor data.

Conflict of Interest

The author declares no conflict of interest.

ACKNOWLEDGEMENT

I am thankful to cited authors and their research work.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00003.3
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

SOME NON-EXTENDABLE SPECIAL DIOPHANTINE TRIPLES INVOLVING HEPTAGONAL PYRAMIDAL NUMBERS

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ABSTRACT

In this paper, we look for three specific polynomials with integer coefficients to such an extent that the product of any two numbers added by a non-zero number (or polynomials with integer coefficients) is a perfect square.

KEYWORDS: Special Diophantine triples, heptagonal pyramidal number, Diophantine triples, triples, perfect square, pyramidal number.

NOTATION:

$P_n^7 = \frac{n(n+1)}{6}(5n-2)$***: heptagonal pyramidal number of rank n.***

INTRODUCTION:

A Diophantine equation is a polynomial equation in number theory where only the integer solutions are considered or searched for, typically with two or more unknowns [1-4]. The term "Diophantine" refers to the Greek mathematician Diaphanous of Alexandria, who investigated similar situations and was one of the pioneers in introducing symbolism to variable-based mathematics in the third century.

The problem of the occurrence of Dio triples and quadruples with the property *D(n)* for any integer n as well as for any linear polynomial in *n* was studied by a number of mathematicians [5-8]. In this particular situation, one may turn to [9–16] for a thorough analysis of various difficulties on Diophantine triples. Half companion Diophantine triple sequences were studied in [17]. These results motivated us to examine for Diophantine triples with involving heptagonal pyramidal numbers. This paper aims at constructing special Diophantine triples where the product of any two members of the triple with the addition of a non-zero integer or a polynomial with integer coefficients satisfies the required property. Also, we present three sections where in each of which we find the special Diophantine triples from heptagonal pyramidal number of different ranks with their corresponding properties.

BASIC DEFINITION:

A set of three distinct polynomials with integer coefficients (a_1, a_2, a_3) is said to be Special Diophantine triple with property $D(n)$ if $a_i * a_j + a_i + a_j + n$ is a perfect square for all $1 \le i < j \le 3$, where n may be non zero or polynomial with integer coefficients.

METHOD OF ANALYSIS:

Section-A:

Formation of some non-extendable special Diophantine triples involving heptagonal pyramidal number of rank n and n-1

Let $a = 6\,p_n^7$ and $b = 6\,p_{n-1}^7$ be heptagonal pyramidal numbers of rank n and n-1 respectively.

$a = 6\,p_n^7$ and $b = 6\,p_{n-1}^7$

$$ab + (a+b) = 25n^6 - 45n^5 - 11n^4 + 55n^3 - 23n^2 + 5n$$

$$ab + (a+b) + \left(-5n^5 - 4n^4 + 5n^3 + 19n^2 - 21n + 4\right) = \left(5n^3 - 5n^2 - 4n + 2\right)^2$$

$$ab + (a+b) + \left(-5n^5 - 4n^4 + 5n^3 + 19n^2 - 21n + 4\right) = \alpha^2 \quad \text{(say)} \qquad (1)$$

Equation (1) is a perfect square.

$$bc + (b+c) + \left(-5n^5 - 4n^4 + 5n^3 + 19n^2 - 21n + 4\right) = \beta^2 \qquad (2)$$

$$ca + (c+a) + \left(-5n^5 - 4n^4 + 5n^3 + 19n^2 - 21n + 4\right) = \gamma^2 \qquad (3)$$

Solving (2) and (3) we get,

$$c(a-b) + (a-b)\left(-5n^5 - 4n^4 + 5n^3 + 19n^2 - 21n + 4\right) = a\beta^2 - b\gamma^2 \qquad (4)$$

The equation (3)-(2) we get.

$$c(b-a) + a - b = \gamma^2 - \beta^2 \qquad (5)$$

Substituting the equation (5) in (4).

$$(a-b)\left(-5n^5 - 4n^4 + 5n^3 + 19n^2 - 12n + 3\right) = (a+1)\beta^2 - (b+1)\gamma^2 \qquad (6)$$

Assume $\beta = x + (b+1)y$ and $\gamma = x + (a+1)y$, and it reduces to,

$$x^2 = (ab + a + b + 1)y^2 - 5n^5 - 4n^4 + 5n^3 + 19n^2 - 12n + 3 \qquad (7)$$

The initial solution of the equation (7) is given by,

$$y_0 = 1,\ x_0 = 5n^3 - 5n^2 - 4n + 2$$

Therefore, $\beta = 10n^3 - 17n^2 + 3n + 3$

Substituting the values in equation (2) we get.

$$c = 20n^3 - 19n^2 - 3n + 5$$

$$c = 4\left(6\,p_n^7\right) - 31n^2 + 5n + 5$$

Therefore, the triple

$\{a,b,c\} = \left\{6\,p_n^7, 6\,p_{n-1}^7, 4\left(6\,p_n^7\right) - 31n^2 + 5n + 5\right\}$ is a Diophantine triple with the property $D\left(-5n^5 - 4n^4 + 5n^3 + 19n^2 - 21n + 4\right)$

Some numerical example are given below in the following table.

TABLE 3.1

s.no	n	(a,b,c)	Property
1	0	(0,0,5)	4
2	1	(6,0,3)	-2
3	2	(48,6,83)	-146
4	3	(156,48,365)	-1292
5	4	(360,156,969)	-5600

NON-EXTENDABILITY:

Let us now show that the special Diophantine triples cannot be extended to the special Diophantine Quadruples.

Consider,

$$ad+(a+d)+\left(-5n^5-4n^4+5n^3+19n^2-21n+4\right)=\mu^2 \quad (8)$$

$$bd+(b+d)+\left(-5n^5-4n^4+5n^3+19n^2-21n+4\right)=\lambda^2 \quad (9)$$

$$cd+(c+d)+\left(-5n^5-4n^4+5n^3+19n^2-21n+4\right)=\omega^2 \quad (10)$$

Solving (9) and (10) we get,

$$d(c-b)+(c-b)\left(-5n^5-4n^4+5n^3+19n^2-21n+4\right)=c\lambda^2-b\omega^2 \quad (11)$$

The equation (3)-(2) we get,

$$d(c-b)+(c-b)=\omega^2-\lambda^2 \quad (12)$$

Substituting the equation (5) in (4).

$$(c-b)\left(-5n^5-4n^4+5n^3+19n^2-21n+3\right)=\lambda^2(c+1)-\omega^2(b+1) \quad (13)$$

Assuming $\lambda=x+(b+1)y$ and $\omega=x+(c+1)y$ and it reduces to,

$$x^2=(bc+b+c+1)y^2-5n^5-4n^4+5n^3+19n^2-21n+3 \quad (14)$$

The initial solution of the equation (14) is given by

$$y_0=1,\, x_0=10n^3-17n^2+3n+3$$

Therefore, $\lambda=15n^3-29n^2+10n+4$

Substituting the values of λ and 'b' in equation (9), we get

$$d=45n^3-65n^2+10n+12$$

Substituting the above value in equation (8), we get

$$ad+(a+d)+\left(-5n^5-4n^4+5n^3+19n^2-21n+4\right)=225n^6-195n^5-239n^4+275n^3-27n^2+11n+16$$

This is not a perfect square.

Hence, the triple cannot be extended to a quadruple.

Section-B:

Formation of some non-extendable special Diophantine triples involving heptagonal pyramidal number of rank n and n-2

Let $a=6\,p_n^7$ and $b=6\,p_{n-2}^7$ be heptagonal pyramidal numbers of rank n and n-2 respectively.

$$ab+(a+b)+\left(-5n^4+2n^3+45n^2-104n+60\right)=\left(5n^3-12n^2-n+6\right)^2$$

$$ab+(a+b)+\left(-5n^4+2n^3+45n^2-104n+60\right)=\alpha^2 \qquad \text{(say)} \qquad (15)$$

Equation (15) is a perfect square.

$$bc+(b+c)+\left(-5n^4+2n^3+45n^2-104n+60\right)=\beta^2 \qquad (16)$$

$$ca+(c+a)+\left(-5n^4+2n^3+45n^2-104n+60\right)=\gamma^2 \qquad (17)$$

Solving (16) and (17) we get,

$$c(a-b)+(a-b)\left(-5n^4+2n^3+45n^2-104n+60\right)=a\beta^2-b\gamma^2 \qquad (18)$$

The equation (17)-(16) we get.

$$c(b-a)+a-b=\gamma^2-\beta^2 \qquad (19)$$

Substituting the equation (19) in (18).

$$(a-b)\left(-5n^4+2n^3+45n^2-104n+59\right)=(a+1)\beta^2-(b+1)\gamma^2 \qquad (20)$$

Assume $\beta=x+(b+1)y$ and $\gamma=x+(a+1)y$, and it reduces to,

$$x^2=(ab+a+b+1)y^2-5n^4+2n^3+45n^2-104n+59 \qquad (21)$$

The initial solution of the equation (21) is given by,

$$y_0=1,\ x_0=5n^3-12n^2-n+6$$

Therefore, $\beta=10n^3-39n^2+45n-17$

Substituting the values in equation (16) we get.

$$c=20n^3-48n^2+42n-11$$

$$c=4\left(6\,p_n^7\right)-60n^2+54n-11$$

Therefore, the triple

$\{a,b,c\}=\left\{6\,p_n^7, 6\,p_{n-2}^7, 4\left(6\,p_n^7\right)-60n^2+54n-11\right\}$ is a Diophantine triple with the property $D\left(-5n^4+2n^3+45n^2-104n+60\right)$

Some numerical example are given below in the following table.

TABLE 3.2

s.no	N	(a,b,c)	Property
1	0	(0,-24,-11)	60
2	1	(6,0,3)	-2
3	2	(48,0,41)	-32
4	3	(156,6,223)	-198
5	4	(360,48,669)	-788

NON-EXTENDABILITY:

Let us now show that the special Diophantine triple cannot be extended to the special Diophantine Quadruple.

Consider,

$$ad+(a+d)+(-5n^4+2n^3+45n^2-104n+60)=\mu^2 \qquad (22)$$
$$bd+(b+d)+(-5n^4+2n^3+45n^2-104n+60)=\lambda^2 \qquad (23)$$
$$cd+(c+d)+(-5n^4+2n^3+45n^2-104n+60)=\omega^2 \qquad (24)$$

Solving (23) and (24) we get,

$$d(c-b)+(c-b)(-5n^4+2n^3+45n^2-104n+60)=c\lambda^2-b\omega^2 \qquad (25)$$

The equation (24)-(23) we get,

$$d(c-b)+(c-b)=\omega^2-\lambda^2 \qquad (26)$$

Substituting the equation (26) in (25),

$$(c-b)(-5n^4+2n^3+45n^2-104n+59)=\lambda^2(c+1)-\omega^2(b+1) \qquad (27)$$

Assuming $\lambda=x+(b+1)y$ and $\omega=x+(c+1)y$ and it reduces to,

$$x^2=(bc+b+c+1)y^2-5n^4+2n^3+45n^2-104n+59 \qquad (28)$$

The initial solution of the equation (28) is given by

$$y_0=1, x_0=10n^3-39n^2-45n-17$$

Therefore, $\lambda=15n^3-66n^2+91n-40$

Substituting the values of λ and 'b' in equation (23), we get

$$d=45n^3-153n^2+178n-68$$

Substituting the above value in equation (22), we get

$ad+(a+d)+(-5n^4+2n^3+45n^2-104n+60)=225n^6-630n^5+336n^4+552n^3-665n^2+208n-8$ This is not a perfect square.

Hence, the triple cannot be extended to a quadruple.

Section-C:

Formation of some non-extendable special Diophantine triples involving heptagonal pyramidal number of rank n and n-3

Let $a=6\,p_n^7$ and $b=6\,p_{n-3}^7$ be heptagonal pyramidal numbers of rank n and n-3 respectively.

$$ab+(a+b)+(-5n^5+n^4+n^3+231n^2-245n+183)=(5n^3-20n^2+4n+9)^2$$
$$ab+(a+b)+(-5n^5+n^4+n^3+231n^2-245n+183)=\alpha^2 \quad \text{(say)} \qquad (29)$$

Equation (29) is a perfect square.

$$bc+(b+c)+(-5n^5+n^4+n^3+231n^2-245n+183)=\beta^2 \qquad (30)$$
$$ca+(c+a)+(-5n^5+n^4+n^3+231n^2-245n+183)=\gamma^2 \qquad (31)$$

Solving (30) and (31) we get.

$$c(a-b)+(a-b)(-5n^5+n^4+n^3+231n^2-245n+183)=a\beta^2-b\gamma^2 \qquad (32)$$

The equation (3.31)-(3.30) we get.

$$c(b-a)+a-b=\gamma^2-\beta^2 \qquad (33)$$

Substituting the equation (33) in (32).

$$(a-b)\left(-5n^5+n^4+n^3+231n^2-245n+182\right)=(a+1)\beta^2-(b+1)\gamma^2 \tag{34}$$

Assume $\beta = x+(b+1)y$ and $\gamma = x+(a+1)y$, and it reduces to,

$$x^2=(ab+a+b+1)y^2-5n^5+n^4+n^3+231n^2-245n+182 \tag{35}$$

The initial solution of the equation (35) is given by,

$$y_0=1,\ x_0=5n^3-20n^2+4n+9$$

Therefore, $\beta = 10n^3-62n^2+119n-92$

Substituting the values in equation (30) we get.

$$c=20n^3-79n^2+121n+183$$

$$c=4\left(6\,p_n^7\right)-91n^2+129n-83$$

Therefore, the triples

$\{a,b,c\}=\left\{6\,p_n^7, 6\,p_{n-3}^7, 4\left(6\,p_n^7\right)-91n^2+129n-83\right\}$ is a Diophantine triple with the property $D\left(-5n^5+n^4+n^3+231n^2-245n+183\right)$

Some numerical example are given below in the following table.

TABLE 3.3

s.no	n	(a,b,c)	Property
1	0	(0,-102,-83)	183
2	1	(6,-24,-21)	166
3	2	(48,0,3)	481
4	3	(156,0,109)	420
5	4	(360,6,417)	-1901

NON-EXTENDABILITY:

Let us now show that the special Diophantine triple cannot be extended to the special Diophantine Quadruple.

Consider,

$$ad+(a+d)+\left(-5n^5+n^4+n^3+231n^2-245n+183\right)=\mu^2 \tag{36}$$

$$bd+(b+d)+\left(-5n^5+n^4+n^3+231n^2-245n+183\right)=\lambda^2 \tag{37}$$

$$cd+(c+d)+\left(-5n^5+n^4+n^3+231n^2-245n+183\right)=\omega^2 \tag{38 }$$

Solving (37) and (38) we get,

$$d(c-b)+(c-b)\left(-5n^5+n^4+n^3+231n^2-245n+183\right)=c\lambda^2-b\omega^2 \tag{39}$$

The equation (38)-(37) we get.

$$d(c-b)+(c-b)=\omega^2-\lambda^2 \tag{40}$$

Substituting the equation (40) in (39).

$$(c-b)\left(-5n^5+n^4+n^3+231n^2-245n+182\right)=\lambda^2(c+1)-\omega^2(b+1) \tag{41}$$

Assuming $\lambda = x+(b+1)y$ and $\omega = x+(c+1)y$ and it reduces to,

$$x^2=(bc+b+c+1)y^2-5n^5+n^4+n^3+231n^2-245n+182 \tag{42}$$

The initial solution of the equation (42) is given by

$$y_0 = 1, x_0 = 10n^3 - 62n^2 + 119n - 92$$

Therefore, $\lambda = 15n^3 - 104n^2 + 234n - 193$

Substituting the values of λ and 'b' in equation (37), we get

$$d = 45n^3 - 245n^2 + 474n - 368$$

Substituting the above value in equation (36), we get

$$ad + (a+d) + \left(-5n^5 + n^4 + n^3 + 231n^2 - 245n + 183\right) = 225n^6 - 1095n^5 + 1546n^4 + 123n^3 - 2063n^2 + 963n - 185$$

This is not a perfect square.

Hence, the triple cannot be extended to a quadruple.

CONCLUSION:

We have presented the Special Diophantine triples involving heptagonal pyramidal numbers. To conclude one may look for triples or quadruples for different numbers with their relating properties.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00004.5
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

A COUNTER EXAMPLE IN ALGEBRA

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ABSTRACT

Danchev [7] studied in a comprehensive manner the class of locally invo-regular rings and using his results he has refined the classification of weakly tripotent rings. In this paper we produce a counter example in algebra which improves one of the results given by Danchev [7].

Keywords: Boolean ring, idempotent, Jacobson radical, locally invo-regualr ring.

MSC 2020: 16U60, 16D60.

1. INTRODUCTION

In our previous works [1-3] we have improved some results of well known mathematical literature [4-7] and in this note we take an opportunity to improve a result of [7].

In this note we provide an example of a unital and associative ring R with the following properties.

1. Each element r of the Jacobson radical $J(R)$ of R satisfies $r^2 = 2r$.

2. R is decomposable as the direct product of two rings A and B such that

a) A is an abelian ring and for each element $a \in A$ we have $a = aca$ for some $c \in A$. Here $c^2 = 1$.

b) Each element of $\frac{A}{J(A)}$ is idempotent and $a^4 = 0$ for each $a \in J(A)$.

c) B is a subdirect product of a family of copies of Z_3.

A ring R satisfying above properties is a locally invo-regular ring (refer theorem 1 [7]).

It has been noted in [7, theorem 3 (ii)] that if R is a locally invo-regular ring without central nilpotents of even characteristic less than or equal to eight and each nilpotent of index less than or equal to two in R is a member of $J(R)$ then each element of $\frac{R}{J(R)}$ is idempotent and the characteristic of R must be two.

In this paper we note that in the case of this example given in section 2, each element of $\frac{R}{J(R)}$ is not idempotent and the characteristic of R is not two though R satisfies the conditions given in theorem 3 (ii) [7].

For further details on invo-clean rings one may refer [8] also. In the next section we produce the required counterexample.

2. A COUNTEREXAMPLE IN ALGEBRA

Let $R = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 4 & 4 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 5 & 5 \\ 0 & 0 \end{pmatrix} \right\}$. Here R is a ring under addition and multiplication of matrices modulo six. We discuss in the next section that this example works as a counterexample for theorem 3 (ii) [7].

3. DISCUSSION

It is known that Z_3 is a reduced ring and as per [7], Z_3 is a ring without central nilpotent element. Similarly we note that R is a ring without central nilpotent element because it is also a reduced ring.

One may easily verify that R can be decomposed as the direct product of two rings $A = Z_2$ and $B = Z_3$.

We note that the Jacobson radical $J(R)$ of R is $\{0\}$. Therefore $r^2 = 2r$ holds good for each $r \in J(R)$.

Further we note the following.

a) A is an abelian ring and $J(A)=0$ with $\frac{A}{J(A)} \cong Z_2$. Therefore each element of $\frac{A}{J(A)}$ is idempotent. Also for each $a \in A$ we have $a = aca$ for some $c \in A$ and $c^2 = 1$. This guarantees that A is a locally invo-regular ring as per definition 1 [7].

b) $a^4 = 0$ for each $a \in J(A)$.

c) B is a subdirect product of a family of copies of Z_3.

Hence in the view of theorem 1 [7], R is a locally invo-regular ring. Also, each nilpotent of index less than equal to two in R is a member of $J(R)$ as R is a reduced ring. Clearly the characteristic of R is six and each element of $\frac{R}{J(R)}$ is not idempotent since $J(R)=\{0\}$ and R is not Boolean. Thus we have given a counter example for theorem 3 (ii) [7].

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00005.7
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

ROOT CUBE EVEN MEAN LABELING OF GRAPH

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ABSTRACT

A graph G = (V,E) with p vertices and q edges is said to be a Root Cube Even Mean Graph if it is possible to label the vertices x V with distinct elements f(x) from 1,2......,q+1 in such a way that when each edge e = uv is labeled with f(e = uv) = $\left\lceil \sqrt{\frac{f(u)^3+f(v)^3}{2}} \right\rceil$ ***or*** $\left\lfloor \sqrt{\frac{f(u)^3+f(v)^3}{2}} \right\rfloor$ ***, then the resulting edge labels are distinct. Here f is called a Root Cube Even Mean Labeling of G. In this paper we prove that path, comb,*** $P_n \odot K_{1,2}$ ***is a root cube even Mean Labeling graph.***

1. INTRODUCTION

All Graphs in this paper are finite and undirected. The symbols V(G) and E(G) denote the vertex set and edge set of a graph G. The cardinality of the vertex set is called the order of G denoted by p. The cardinality of the edge set is called the size of G denoted by q edges is called a (p,q) graph. A graph labeling is an assignment of integers to the vertices or edges. Bloom and Hsu [2] extended the notion of graceful labeling to directed graphs. Further this work can be extended in the field of automata theory cited as Saridha , Rajaretnam [13,14,15,16,17] which has a wide range of application in automata theory. Saridha and Haridha Banu [18] discussed a new direction towards plus weighted grammar. Shalini, Paul Dhayabaran introduced different types of labeling [19,20]. Different types of labelings have been proved under connected and disconnected graphs[21,22,23,24,25,26,27,28,29,30]. Shalini.P, S.A. Meena [31] introduced "Lehmer -4 mean labelling of graph ".Shalini.P S.Tamizharasi [32,33] studied "Power -3 Heronian Odd Mean Labeling Graphs". Graph Labeling can be further extended to iner index polynomial is cited as Palani Kumar, Rameshkumar [8,9,10,11,12]

2. PRELIMINARIES:

Definition:2.1

A walk in which vertices are distinct is called a path.A path on a n vertices is denoted by P_n.

Definition:2.2

A closed path is called a cycle. A cycle on vertices is denoted by C_n.

Definition:2.3

The graph obtained by joining a single pendent edge to each vertex of a path is called a comb graph.

Definition:2.4

The graph $P_n \odot K_{1,2}$ is obtained by attaching complete bipartite graph $K_{1,2}$ to each vertex of path P_n

Definition: 2.5

Let G be simple Graphs with vertex set *V(G)* and the Edge set *E(G)* respectively. Vertex set *V(G)* are labeled arbitrary by positive integers and let *E(e)* denoted the edge label such that it is the sum of labels of vertices incident with edge *e*.

3. MAIN RESULTS

Theorem 3.1:

The P_n is Root cube Even Mean labeling Graph

Proof:

Let G be a graph P_n

Define f : V(G) → {2,4,6,.........2n}

$f(u_i) = 2i$, $1 \le i \le n$

Then edge lables are distinct

Hence the path P_n is a root cube Even Mean labeling graph.

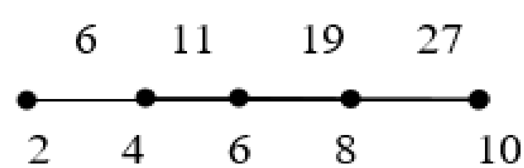

Fig: 3.1

Theorem 3.2:

The comb $P_n \odot K_1$ is a root cube even mean labeling graph.

Proof:

Let G be a graph $P_n \odot K_1$

Let $P_n \odot K_1$ with vertices $u_1, u_2, \ldots\ldots u_n$ and $v_1, v_2, \ldots\ldots, v_n$

Define f: V(G) → {2,4,6,……4n}

f(ui) = 4i-2

f(vi) =4i

Then edge lables are distinct

Hence the path $P_n \odot K_1$ is a root cube even mean labeling graph.

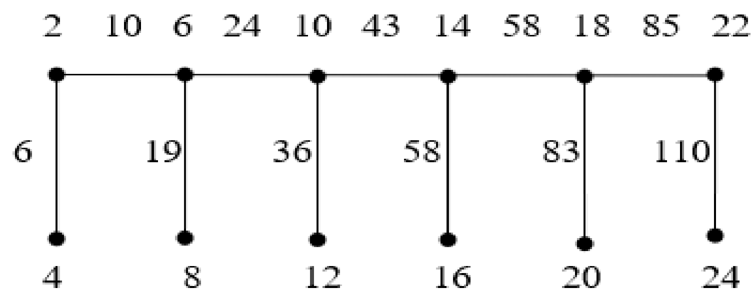

Fig: 3.2 Comb

Theorem 3.3:

$P_n \odot K_{1,2}$is a root cube even mean labeling graph**.**

Proof:

Let G be a graph of $P_n \odot K_{1,2}$

Let $P_n \odot K_{1,2}$ with verticesu$_1$, u_2, ………u_n and v_1, v_2, …………v_n

Define a function f: V(G) → {2,4,6,8,……2n}

f(vi) = 6i-4, $1 \leq i \leq n$

f(wi) = 6i-2, $1 \leq i \leq n$

f(ui) = 6i, $1 \leq i \leq n$

Then the edge labels are distinct

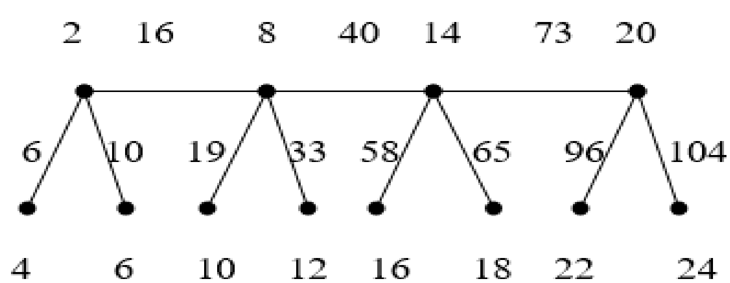

Fig:3.3$P_n \odot K_{1,2}$

4. CONCLUSION

Finally, we conclude that path, comb, $P_n \odot K_{1,2}$is a root cube even Mean Labeling graph.

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DOI No. : 10.5958/2394-9309.2023.00006.9
Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023 ISSN (Print) : 0975-7139
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com ISSN (Online) : 2394-9309

INTEGRALS INVOLVING THE PRODUCT OF SINE AND COSINE W.R.T. THE PARAMETER OF GENERALIZED I-FUNCTION OF TWO VARIABLES WITH APPLICATIONS

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ABSTRACT

In this article, we investigate various formulae for new integrals involving the product of generalized I-function of two variables, w.r.t. parameters and angle of SINE &COSINE. New relation may be obtained as Application of our integral by giving suitable value in parameters.

Keywords : I-function of two variables, H-function of two variables, G-function of two variables etc.

1 INTRODUCTION :

The I-function of two variables represented by means of two contour integrals is defined by Goyal and Agarwal [2,3] and represented in the following manner :

$$I^{m_1,n_1;m_2,n_2;m_3,n_3}_{p,q:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}:r}\left[\begin{matrix} Z_1 \\ Z_2 \end{matrix}\middle| \begin{matrix} [(e_p:E_p,E_p')]:\left[(a_j,\alpha_j)_{1,n_2}\right],\left[(a_{ji},\alpha_{ji})_{n_2+1,p_i^{(1)}}\right];\left[(c_j,\gamma_j)_{1,n_3}\right],\left[(c_{ji},\gamma_{ji})_{n_3+1,p_i^{(2)}}\right] \\ [(f_q:F_q,F_q')]:\left[(b_j,\beta_j)_{1,m_2}\right],\left[(b_{ji},\beta_{ji})_{m_2+1,q_i^{(1)}}\right];\left[(d_j,\delta_j)_{1,m_3}\right],\left[(d_{ji},\delta_{ji})_{m_3+1,q_i^{(2)}}\right] \end{matrix}\right]$$

or

$$I^{m_1,n_1:p}_{p,q:Q,r}\left[\begin{matrix} z_1 \\ z_2 \end{matrix}\middle| \begin{matrix} ((P)):((Q)):((R)),((S)),((T)) \\ ((U)):((V)),((W)),((X)),((Y)) \end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi)\,\phi_2(\eta)\psi(\xi,n)Z_1^{\xi}Z_2^{\eta}d\xi d\eta, \tag{1.1}$$

$$\phi_1(\xi) = \frac{\prod_{j=1}^{m_2}\Gamma(b_j-\beta_j+\alpha_j\xi)\prod_{j=1}^{n_2}(1-a_j+\alpha_j\xi)}{\sum_{i=1}^{r}\left[\prod_{j=m_2+1}^{q_i^{(1)}}\Gamma(1-b_{ji}+\beta_{ji}\xi+\alpha_j\xi)\prod_{j=n_2+1}^{p_i^{(2)}}\Gamma(a_{ji}-\alpha_{ji}\xi)\right]}, \tag{1.2}$$

$$\phi_2(\eta) = \frac{\prod_{j=1}^{m_3}\Gamma(d_j - \delta_j\eta)\prod_{j=1}^{n_3}(1 - c_j + \gamma_{ji}\eta)}{\sum_{i=1}^{r}\left[\prod_{j=m_3+1}^{q_i^{(2)}}\Gamma(1 - d_{ji} + \delta_{ji}\eta)\prod_{j=n_3+1}^{p_i^{(2)}}\Gamma(c_{ji} - \gamma_{ji}\eta)\right]}, \tag{1.3}$$

$$\psi(\xi,\eta) = \frac{\prod_{j=1}^{m_1}\Gamma(f_j - F_j\xi - F_j'\eta)\prod_{j=1}^{n_1}(1 - e_j + E_j\xi + E_j'\eta)}{\sum_{i=1}^{r}\left[\prod_{j=m_1+1}^{q}\Gamma(1 - f_j\xi_j + F_i'\eta)\prod_{j=n_1+1}^{p}\Gamma(e_j - E_j\xi - E_j'\eta)\right]}, \tag{1.4}$$

T_1 and T_2 are sets of parameters given below,

$$T_1 = \left[\left(e_p, E_p, E_p'\right)\right], \left[\left(a_j, \alpha_j\right)_{1,n_2}\right], \left[\left(a_{ji}, \alpha_{ji}\right)_{n_2+1,p_i^{(1)}}\right], \left[\left(c_j, \gamma_j\right)_{1,n_3}\right], \left[\left(c_{ji}, \gamma_{ji}\right)_{n_3+1,p_i^{(2)}}\right] \tag{1.5}$$

$$T_2 = \left[\left(f_q, F_q, F_q'\right)\right], \left[\left(b_j, \beta_j\right)_{1,m_2}\right], \left[\left(b_{ji}, \beta_{ji}\right)_{m_2+1,p_i^{(1)}}\right], \left[\left(d_j, \delta_j\right)_{1,m_3}\right], \left[\left(d_{ji}, \delta_{ji}\right)_{m_3+1,p_i^{(2)}}\right] \tag{1.6}$$

Z_1 and Z_2 are two non-zero complex variables L_1 and L_2 are two Mellin-Barnes type contour integrals, convergence condition are as follows:

$$\left|\arg(z_1)\right| < \frac{\pi A_i}{2}, \tag{1.7}$$

$$\left|\arg(z_2)\right| < \frac{\pi B_i}{2} \tag{1.8}$$

Where

$$A_i = \sum_{j=1}^{n_1} E_j - \sum_{j=n_1+1}^{p} E_j + \sum_{j=1}^{m_1} F_j - \sum_{j=m_1+1}^{q} F_j + \sum_{j=1}^{m_2}\beta_j - \sum_{j=m_2+1}^{q_i^{(1)}}\beta_{ji} + \sum_{j=1}^{n_2}\alpha_j - \sum_{j=n_2+1}^{p_i^{(1)}}\alpha_{ji} > 0 \tag{1.9}$$

$$B_i = \sum_{j=1}^{n_1} E_j' - \sum_{j=n_1+1}^{p} E_j' + \sum_{j=1}^{m_1} F_j' - \sum_{j=m_1+1}^{q} F_j' + \sum_{j=1}^{m_3}\delta_j - \sum_{j=m_3+1}^{q_i^{(2)}}\delta_{ji} + \sum_{j=1}^{n_3}\gamma_j - \sum_{j=n_3+1}^{p_i^{(2)}}\alpha_{ji} > 0 \tag{1.10}$$

Where $\mathbf{m_1}, \mathbf{n_1}; \mathbf{m_2}, \mathbf{n_2}; \mathbf{m_3}, \mathbf{n_3}$ **and** $\mathbf{p}, \mathbf{q} : \mathbf{p_i^{(1)}}, q_i^{(1)} : \mathbf{p_i^{(2)}}, q_i^{(2)}$ are non-zero integers satisfying the conditions $\mathbf{0 \le n_1 \le p}$, $\mathbf{0 \le n_2 \le p_i^{(1)}}$, $\mathbf{0 \le n_3 \le p_i^{(2)}}$, $\mathbf{0 \le m_2} \le q_i^{(1)}, \mathbf{0 \le m_3} \le q_i^{(2)}$, for all i=1,2... ,r where are r is positive integers, for asymptotic expansion and all other conditions, see Goyal Anil and Agrawal R.D. [01]. These condition will be assumed to hold good in the I-Function of too variables occuring in this paper.

2 FORMULA REQUIRED : In this paper we require the following result [04],

$$\int_{-\infty}^{\infty} \frac{\sin(cx)\,dx}{\Gamma(\alpha+x).\Gamma(\beta-x)} = \frac{\left[2\cos\left(\frac{c}{2}\right)\right]^{(\alpha+\beta-2)} \sin\left[\frac{c}{2}(\beta-\alpha)\right]}{\Gamma(\alpha+\beta+1)} \tag{2.1}$$

$$\int_{-\infty}^{\infty} \frac{\cos(cx)\,dx}{\Gamma(\alpha+x).\Gamma(\beta-x)} = \frac{\left[2\sin\left(\frac{c}{2}\right)\right]^{(\alpha+\beta-2)} \cos\left[\frac{c}{2}(\beta-\alpha)\right]}{\Gamma(\alpha+\beta+1)} \tag{2.2}$$

3 INTEGRALS :

In this section, we start with presenting four integrals involving I-Function of two variables as follow-

First Integral –

$$\int_{-\infty}^{\infty} \mathrm{Sin}(cx)\, I^{m,n,P}_{p,q+2,Q:r}\left[\begin{matrix} Z_1 \\ Z_2 \end{matrix}\middle|\begin{matrix} T_1 \\ T_2;\left[(1-\alpha-x);(h_1+h_2),(k_1+k_2)\right],\left[(1-\beta+x);(h_1+h_2),(k_1+k_2)\right] \end{matrix}\right]dx$$

$$=\left[2\mathrm{Cos}\left(\frac{c}{2}\right)\right]^{(\alpha+\beta-2)} \mathrm{Sin}\left[\frac{c}{2}(\beta-\alpha)\right] I^{m,n,P}_{p,q+1,Q:r}\left[\begin{matrix} Z_1\left[2\cos\left(\frac{c}{2}\right)\right]^{2(h_1+h_2)} \\ Z_2\left[2\cos\left(\frac{c}{2}\right)\right]^{2(k_1+k_2)} \end{matrix}\middle|\begin{matrix} T_1 \\ T_2;\left[(-\alpha-\beta);2(h_1+h_2),2(k_1+k_2)\right] \end{matrix}\right]$$

Provided that,

1 $(\alpha+\beta)<1, \alpha>0, \beta>0$

2 $0<c<\pi, h_1>0, h_2>0, k_1>0, k_2>0$

3 $\Delta_1 < \frac{1}{2}\pi A_i, \Delta_2 < \frac{1}{2}\pi B_i, \forall i = 1, 2, ---r$

Where A_i and B_i are given in (1.9) and (1.10) respectively (3.1)

Second Integral –

$$\int_{-\infty}^{\infty} \mathrm{Sin}\,(cx)\, I^{m,n:P}_{p,q+2:Q,r}\left[\begin{matrix} Z_1 \\ Z_2 \end{matrix}\left|\begin{matrix} T_1 \\ T_2;\left[(\alpha+x);\left(h_1-h_2\right),\left(k_1-k_2\right)\right],\left[(\beta-x);\left(h_1-h_2\right),\left(k_1-k_2\right)\right] \end{matrix}\right.\right] dx$$

$$=\left[2\mathrm{Cos}\left(\frac{c}{2}\right)\right]^{(\alpha+\beta-2)} \mathrm{Sin}\left[\frac{c}{2}(\beta-\alpha)\right] I^{m,n:P}_{p,q+1:Q,r}\left[\begin{matrix} Z_1\left[2\cos\left(\frac{c}{2}\right)\right]^{-2\left(h_1-h_2\right)} \\ Z_2\left[2\cos\left(\frac{c}{2}\right)\right]^{-2\left(k_1-k_2\right)} \end{matrix}\left|\begin{matrix} T_1 \\ T_2;\left[(\alpha+\beta+1);-2\left(h_1-h_2\right),2\left(k_1-k_2\right)\right] \end{matrix}\right.\right]$$

Provided that,

1 $\left(\alpha+\beta\right)<1, \alpha>0, \beta>0$

2 $0<c<\pi, h_1>0, h_2>0, k_1>0, k_2>0$

3 $\Delta_1<\frac{1}{2}\pi A_i, \Delta_2<\frac{1}{2}\pi B_i, \forall i=1,2,---r$

Where A_i and B_i are given in (1.9) and (1.10) respectively (3.2)

Third Integral –

$$\int_{-\infty}^{\infty} \mathrm{Cos}\,(cx)\, I^{m,n:P}_{p,q+2:Q,r}\left[\begin{matrix} Z_1 \\ Z_2 \end{matrix}\left|\begin{matrix} T_1 \\ T_2;\left[(1-\alpha-x);\left(h_1+h_2\right),\left(k_1+k_2\right)\right],\left[(1-\beta+x);\left(h_1+h_2\right),\left(k_1+k_2\right)\right] \end{matrix}\right.\right] dx$$

$$=\left[2\mathrm{Sin}\left(\frac{c}{2}\right)\right]^{(\alpha+\beta-2)} \mathrm{Cos}\left[\frac{c}{2}(\beta-\alpha)\right]$$

$$I^{m,n:P}_{p,q+1:Q,r}\left[\begin{matrix} Z_1\left[2\mathrm{Sin}\left(\frac{c}{2}\right)\right]^{2\left(h_1+h_2\right)} \\ Z_2\left[2\mathrm{Sin}\left(\frac{c}{2}\right)\right]^{2\left(k_1+k_2\right)} \end{matrix}\left|\begin{matrix} T_1 \\ T_2;\left[(-\alpha-\beta);2\left(h_1+h_2\right),2\left(k_1+k_2\right)\right] \end{matrix}\right.\right]$$

Provided that,

1 $\left(\alpha+\beta\right)<1, \alpha>0, \beta>0$

2 $0<c<\pi, h_1>0, h_2>0, k_1>0, k_2>0$

3 $\Delta_1<\frac{1}{2}\pi A_i, \Delta_2<\frac{1}{2}\pi B_i, \forall i=1,2,---r$

Where A_i and B_i are given in (1.9) and (1.10) respectively (3.3)

Fourth Integral –

$$\int_{-\infty}^{\infty} \mathrm{Cos}(cx)\, I^{m,n:P}_{p+2,q:Q,r}\left[\begin{matrix} Z_1 \\ Z_2 \end{matrix}\middle|\begin{matrix} T_1 \\ T_2;\left[(\alpha+x);(h_1-h_2),(k_1-k_2)\right],\left[(\beta-x);(h_1-h_2),(k_1-k_2)\right] \end{matrix}\right] dx$$

$$=\left[2\mathrm{Sin}\left(\frac{c}{2}\right)\right]^{(\alpha+\beta-2)} \mathrm{Cos}\left[\frac{c}{2}(\beta-\alpha)\right] I^{m,n:P}_{p+1,q:Q,r}\left[\begin{matrix} Z_1\left[2\mathrm{Sin}\left(\frac{c}{2}\right)\right]^{-2(h_1-h_2)} \\ Z_2\left[2\mathrm{Sin}\left(\frac{c}{2}\right)\right]^{-2(k_1-k_2)} \end{matrix}\middle|\begin{matrix} T_1 \\ T_2;\left[(\alpha+\beta+1);-2(h_1-h_2),-2(k_1-k_2)\right] \end{matrix}\right]$$

Provided that,

1 $(\alpha+\beta)<1,\alpha>0,\beta>0$

2 $0<c<\pi,h_1>0,h_2>0,k_1>0,k_2>0$3 $\Delta_1<\frac{1}{2}\pi A_i,\Delta_2<\frac{1}{2}\pi B_i,\forall i=1,2,---r$

Where A_i and B_i are given in (1.9) and (1.10) respectively (3.4)

PROOF-

The result (3.1) can be established by replacing the generalized I-Function of two variables on the left hand side as contour integral (1.1) we get,

$$\int_{-\infty}^{\infty} \mathrm{Sin}\,(cx)\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi)\,\phi_2(\eta)\,\psi(\xi,\eta)$$

$$\frac{Z_1^{\xi} Z_2^{\eta}}{\Gamma\left(\alpha+(h_1+h_2)\xi+(k_1+k_2)\eta+x\right)\Gamma\left(\beta+(h_1+h_2)\xi+(k_1+k_2)\eta-x\right)}\,dx.d\xi.d\eta$$

Interchanging the order of integral involved in the process, evaluating the inner integral with the help of (2.1) and applying (1.1) the definition of generalized I-Function of two variables, the value of integral is obtained

Note-

Proceeding on similar lines the integrals (3.2), (3.3) , (3.4) have been obtain by using (2.1) and (2.2) integrals respectively.

Applications-

1. *On choosing* c =π/2 in equation (3.1) we get following result which are useful in space science and used in the explanation of quantum gravitational

$$\int_{-\infty}^{\infty} \operatorname{Sin}\left(\frac{\pi x}{2}\right) I^{m_1,n_1:m_2,n_2;m_3,n_3}_{p,q+2:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}}\left[\begin{array}{c|c} Z_1 & T_1 \\ Z_2 & T_2;\left[(1-\alpha-x);(h_1+h_2),(k_1+k_2)\right],\left[(1-\beta+x);(h_1+h_2),(k_1+k_2)\right]\end{array}\right] dx$$

$$= \left[\sqrt{2}\right]^{(\alpha+\beta-2)} \operatorname{Sin}\left[\frac{\pi}{4}(\beta-\alpha)\right]$$

$$I^{m_1,n_1:m_2,n_2;m_3,n_3}_{p,q_1+1:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}}\left[\begin{array}{c|c} Z_1\left[\sqrt{2}\right]^{2(h_1+h_2)} & T_1 \\ Z_2\left[\sqrt{2}\right]^{2(k_1+k_2)} & T_2;\left[(-\alpha-\beta);2(h_1+h_2),2(k_1+k_2)\right]\end{array}\right]$$

The conditions of validity of the above result easily follow from integral (3.1)

2 On choosing $c = \pi/2$ in equation (3.3) we get following result which are useful in space science and used in the explanation of quantum gravitational

$$\int_{-\infty}^{\infty} \operatorname{Cos}\left(\frac{\pi x}{2}\right) I^{m_1,n_1:m_2,n_2;m_3,n_3}_{p,q+2:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}}\left[\begin{array}{c|c} Z_1 & T_1 \\ Z_2 & T_2;\left[(1-\alpha-x);(h_1+h_2),(k_1+k_2)\right],\left[(1-\beta+x);(h_1+h_2),(k_1+k_2)\right]\end{array}\right] dx$$

$$= \left[\sqrt{2}\right]^{(\alpha+\beta-2)} \operatorname{Cos}\left[\frac{\pi}{4}(\beta-\alpha)\right]$$

$$I^{m_1,n_1:m_2,n_2;m_3,n_3}_{p,q_1+1:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}}\left[\begin{array}{c|c} Z_1\left[\sqrt{2}\right]^{2(h_1+h_2)} & T_1 \\ Z_2\left[\sqrt{2}\right]^{2(k_1+k_2)} & T_2;\left[(-\alpha-\beta);2(h_1+h_2),2(k_1+k_2)\right]\end{array}\right]$$

The conditions of validity of the above result easily follow from integral (3.3)

3. If we put $h_2 = 0$ and $k_2 = 0$ in equation (3.1), we get the following result

$$\int_{-\infty}^{\infty} \operatorname{Sin}(cx)\, I^{m_1,n_1:m_2,n_2;m_3,n_3}_{p,q+2:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}}\left[\begin{array}{c|c} Z_1 & T_1 \\ Z_2 & T_2;\left[(1-\alpha-x);h_1,k_1\right],\left[(1-\beta+x);h_1,k_1\right]\end{array}\right] dx$$

$$= \left[2\operatorname{Cos}\left(\frac{c}{2}\right)\right]^{(\alpha+\beta-2)} \operatorname{Sin}\left[\frac{c}{2}(\beta-\alpha)\right]$$

$$I^{m_1,n_1:m_2,n_2;m_3,n_3}_{p,q_1+1:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}}\left[\begin{array}{c|c} Z_1\left[2\cos\left(\frac{c}{2}\right)\right]^{2h_1} & T_1 \\ Z_2\left[2\cos\left(\frac{c}{2}\right)\right]^{2k_1} & T_2;\left[(-\alpha-\beta);2h_1,2k_1\right]\end{array}\right]$$

The conditions of validity of the above result easily follow from integral (3.1)

4 If we put $h_2 = 0$ and $k_2 = 0$ in equation (3.3), we get the following

Result

$$\int_{-\infty}^{\infty} \operatorname{Cos}(cx)\, I^{m_1,n_1:m_2,n_2;m_3,n_3}_{p,q+2:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}}\left[\begin{matrix} Z_1 \\ Z_2 \end{matrix}\left|\begin{matrix} T_1 \\ T_2;\left[(1-\alpha-x);h_1,k_1\right],\left[(1-\beta+x);h_1,k_1\right]\end{matrix}\right.\right]dx$$

$$= \left[2\operatorname{Sin}\left(\frac{c}{2}\right)\right]^{(\alpha+\beta-2)} \operatorname{Cos}\left[\frac{c}{2}(\beta-\alpha)\right]$$

$$I^{m_1,n_1:m_2,n_2;m_3,n_3}_{p,q_1+1:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}}\left[\begin{matrix} Z_1\left[2\operatorname{Sin}\left(\frac{c}{2}\right)\right]^{2h_1} \\ Z_2\left[2\operatorname{Sin}\left(\frac{c}{2}\right)\right]^{2k_1} \end{matrix}\left|\begin{matrix} T_1 \\ T_2;\left[(-\alpha-\beta);2h_1,2k_1\right]\end{matrix}\right.\right]$$

The conditions of validity of the above result easily follow from integral (3.3)

5 If we set $m_1 = 0$, $r = 1$ in equation (3.1) the I-Function of two variables then we have ,

$$\int_{-\infty}^{\infty} \operatorname{Sin}(cx)\, H^{0,n_1:m_2,n_2;m_3,n_3}_{p,q+2:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}}\left[\begin{matrix} Z_1 \\ Z_2 \end{matrix}\left|\begin{matrix} T_1 \\ T_2;\left[(1-\alpha-x);(h_1+h_2),(k_1+k_2)\right],\left[(1-\beta+x);(h_1+h_2),(k_1+k_2)\right]\end{matrix}\right.\right]dx$$

$$= \left[2\operatorname{Cos}\left(\frac{c}{2}\right)\right]^{(\alpha+\beta-2)} \operatorname{Sin}\left[\frac{c}{2}(\beta-\alpha)\right]$$

$$H^{0,n_1:m_2,n_2;m_3,n_3}_{p,q_1+1:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}}\left[\begin{matrix} Z_1\left[2\cos\left(\frac{c}{2}\right)\right]^{2(h_1+h_2)} \\ Z_2\left[2\cos\left(\frac{c}{2}\right)\right]^{2(k_1+k_2)} \end{matrix}\left|\begin{matrix} T_1 \\ T_2;\left[(-\alpha-\beta);2(h_1+h_2),2(k_1+k_2)\right]\end{matrix}\right.\right]$$

The conditions of validity of the above result easily follow from integral (3.1)

6. If we set $m_1 = 0$, $r = 1$ in equation (3.3) the I-Function of two variables then we have ,

$$\int_{-\infty}^{\infty} \operatorname{Cos}(cx)\, H^{0,n_1:m_2,n_2;m_3,n_3}_{p,q+2:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}}\left[\begin{matrix} Z_1 \\ Z_2 \end{matrix}\left|\begin{matrix} T_1 \\ T_2;\left[(1-\alpha-x);(h_1+h_2),(k_1+k_2)\right],\left[(1-\beta+x);(h_1+h_2),(k_1+k_2)\right]\end{matrix}\right.\right]dx$$

$$= \left[2\operatorname{Sin}\left(\frac{c}{2}\right)\right]^{(\alpha+\beta-2)} \operatorname{Cos}\left[\frac{c}{2}(\beta-\alpha)\right]$$

$$H^{0,n_1:m_2,n_2;m_3,n_3}_{p,q_1+1:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}}\left[\begin{matrix} Z_1\left[2\operatorname{Sin}\left(\frac{c}{2}\right)\right]^{2(h_1+h_2)} \\ Z_2\left[2\operatorname{Sin}\left(\frac{c}{2}\right)\right]^{2(k_1+k_2)} \end{matrix}\left|\begin{matrix} T_1 \\ T_2;\left[(-\alpha-\beta);2(h_1+h_2),2(k_1+k_2)\right]\end{matrix}\right.\right]$$

The conditions of validity of the above result easily follow from integral (3.3)

7. If we put $m_1 = n_1 = p = q = 0$, in equation (3.1) we have the following result in terms of product of I-Function of One variables.

$$\int_{-\infty}^{\infty} \mathrm{Sin}\,(cx)\, I^{m_1,n_1:m_2,n_2;m_3,n_3}_{p,q+2:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}} \left[Z \,\middle|\, \begin{matrix} T_1 \\ T_2;\left[(1-\alpha-x);(h_1+h_2),(k_1+k_2)\right],\left[(1-\beta+x);(h_1+h_2),(k_1+k_2)\right] \end{matrix} \right] dx$$

$$= \left[2\,\mathrm{Cos}\left(\frac{c}{2}\right)\right]^{(\alpha+\beta-2)} \mathrm{Sin}\left[\frac{c}{2}(\beta-\alpha)\right]$$

$$I^{m_1,n_1:m_2,n_2;m_3,n_3}_{p,q_1+1:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}} \left[Z \begin{vmatrix} \left[2\cos\left(\frac{c}{2}\right)\right]^{2(h_1+h_2)} \\ \left[2\cos\left(\frac{c}{2}\right)\right]^{2(k_1+k_2)} \end{vmatrix} \begin{matrix} T_1 \\ T_2;\left[(-\alpha-\beta);2(h_1+h_2),2(k_1+k_2)\right] \end{matrix} \right]$$

The conditions of validity of the above result easily follow from integral (3.1)

8. If we put $m_1 = n_1 = p = q = 0$, in equation (3.3) we have the following result in terms of product of I-Function of One variables.

$$\int_{-\infty}^{\infty} \mathrm{Cos}\,(cx)\, I^{m_1,n_1:m_2,n_2;m_3,n_3}_{p,q+2:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}} \left[Z \,\middle|\, \begin{matrix} T_1 \\ T_2;\left[(1-\alpha-x);(h_1+h_2),(k_1+k_2)\right],\left[(1-\beta+x);(h_1+h_2),(k_1+k_2)\right] \end{matrix} \right] dx$$

$$= \left[2\,\mathrm{Sin}\left(\frac{c}{2}\right)\right]^{(\alpha+\beta-2)} \mathrm{Cos}\left[\frac{c}{2}(\beta-\alpha)\right]$$

$$I^{m_1,n_1:m_2,n_2;m_3,n_3}_{p,q_1+1:p_i^{(1)},q^{(1)};q_i^{(2)},q_i^{(2)}} \left[Z \begin{vmatrix} \left[2\,\mathrm{Sin}\left(\frac{c}{2}\right)\right]^{2(h_1+h_2)} \\ \left[2\,\mathrm{Sin}\left(\frac{c}{2}\right)\right]^{2(k_1+k_2)} \end{vmatrix} \begin{matrix} T_1 \\ T_2;\left[(-\alpha-\beta);2(h_1+h_2),2(k_1+k_2)\right] \end{matrix} \right]$$

The conditions of validity of the above result easily follow from integral (3.3)

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00007.0
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

INTEGRAL SOLUTIONS OF THE TRINITY CUBIC EQUATION

$$5(p^2+q^2)-6(pq)+4(p+q+1)=1600r^3$$

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ABSTRACT

The trinity cubic equation $5(p^2+q^2)-6(pq)+4(p+q+1)=1600r^3$ is analyzed. To the trinity cubic equation under consideration of sequence of non-zero integral solutions is obtained. A few fascinating properties of the solutions are seen, as well as few distinct polygonal numbers.

Keywords: Cubic equation, integrable solutions, polygonal number.

INTRODUCTION:

The Diophantine equation has a diverse range. Many people have shown interest in the integer solution of the non-homogeneous cubic equation with three unknowns. Due to the diversity in the way the theory of numbers was discussed in [1-3], it offers a limitless field of study. Numerous concerns gave been examined in depth in [4-9]. Several fascinating properties of the solutions towards the trinity cubic equation $5(p^2+q^2)-6(pq)+4(p+q+1)=1600r^3$ is examined in this communication, as well as some special numbers.

Notations:

$T_{m,n} = n\left[1+\frac{(n-1)(m-2)}{2}\right]$ (Polygonal number)

$P_n^m = \frac{1}{6}n(n+1)\left[(m-2)n+(5-m)\right]$ (Pyramidal number)

$O_n = \frac{1}{3}(2n^3+n)$ (Octrahedral number)

$CS_n = n^2+(n-1)^2$ (Centered square number)

$CC_n = (2n-1)(n^2-n+1)$ (Centered cube number)

$Gno_n = 2n-1$ (Gnomonic number)

$SO_n = n(2n^2-1)$ (Stella Octangula number)

$Star_n = 6n(n-1)+1$ (Star number)

$CH_n = 3n^2 - 3n + 1$ (Centered hexagonal number)

$RD_n = (2n-1)(2n^2 - 2n + 1)$ (Rhombic dodecagonal number)

METHOD OF ANALYSIS:

The corresponding cubic equation, which requires a solution with non-zero integral, is

$$5(p^2+q^2)-6(pq)+4(p+q+1)=1600r^3 \quad (1)$$

After modified the transformation,

$$p=l+m\,, q=l-m \quad (2)$$

In (1) leads to,

$$(l+1)^2+4m^2=400r^3 \quad (3)$$

Below, we demonstrate various distinct non-zero unique integrable solution patterns (1).

Pattern: 1

Assume $r=r(a,b)=a^2+4b^2$ (4)

Write $400=(16+6i\sqrt{4})(16-6i\sqrt{4})$ (5)

Applying the factorization method and substituting (4) and (5) in (3),

$$((l+1)+i\sqrt{4}m)((l+1)-i\sqrt{4}m)=(16+6i\sqrt{4})(16-6i\sqrt{4})((a+i\sqrt{4}b)^3(a-i\sqrt{4}b)^3) \quad (6)$$

Equating the actual and imagined portions, we obtain

$$l=l(a,b)=16a^3-192ab^2-72a^2b+96b^3-1$$

$$m=m(a,b)=6a^3-72ab^2+48a^2b-64b^3$$

The non-zero distinct integral solutions to equation (1) are determined by modifying the values of l & m in equation (2), we obtain

$$p=p(a,b)=22a^3-264ab^2-24a^2b+32b^3-1$$
$$q=q(a,b)=10a^3-120ab^2-120a^2b+160b^3-1$$
$$r=r(a,b)=a^2+4b^2$$

Properties:

1. $q(1,1)-p(1,1)+25r(1,1)$ is a Perfect square
2. $q(1,1)-p(1,1)-15r(1,1)$ is a Disarium number
3. $p(1,a)-q(1,a)+32ar(1,a)+48CH_a-8Gno_a \equiv 0 \pmod{52}$
4. $q(a,1)-p(a,1)+12ar(a,1)+32T_{8,a}-64Gno_a \equiv 0 \pmod{192}$

5. $p(1,b)-q(1,b)+36r(1,b)+64SO_b-16Gno_b \equiv 0(\mathrm{mod}\,64)$

6. $q(b,1)-p(b,1)+6O_b+48CS_b-27Gno_b \equiv 3(\mathrm{mod}\,230)$ 7. $q(1,1)-p(1,1)-28r(1,1)$ is a Nasty number

8. $q(1,1)-p(1,1)+113r(1,1)$ is a Cubical integer

Pattern:2

In place of (5), Write $400=(12+8i\sqrt{4})(12-8i\sqrt{4})$ (7)

If (7) and (4) are substituted in (3) and the factorization method, the corresponding integrable solutions of (1) for pattern 1 are represented by

$$p=p(a,b)=20a^3-240ab^2-60a^2b+80b^3-1$$
$$q=q(a,b)=4a^3-48ab^2-132a^2b+176b^3-1$$
$$r=r(a,b)=a^2+4b^2$$

Properties:

1. $q(1,1)-p(1,1)+5r(1,1)$ is a Perfect square

2. $p(2,1)-q(2,1)+19r(2,1)$ is a Palindrome number 3. $p(1,a)-q(1,a)+48r(1,a)+48SO_a-12Gno_a \equiv 0(\mathrm{mod}\,76)$

4. $q(a,1)-p(a,1)+16ar(a,1)+12T_{14,a}-98Gno_a \equiv 0(\mathrm{mod}\,194)$

5. $p(1,b)-q(1,b)+48CC_b+48CH_b+96T_{6,b}+12Gno_b \equiv 0(\mathrm{mod}\,4)$

6. $p(b,1)-q(b,1)+72r(b,1)+16P_b^3-88Gno_b \equiv 6(\mathrm{mod}\,188)$ 7. $q(1,1)-2p(1,1)+19r(1,1)$ is a Perfect number

8. $q(1,1)-p(1,1)+160r(1,1)$ is a Cubical integer

Pattern: 3

In place of (5), Write $400=\dfrac{(48+32i\sqrt{4})(48-32i\sqrt{4})}{16}$ (8)

Equating the actual and imagined portions, we obtain

$$l=\frac{1}{4}(48a^3-576ab^2-384a^2b+512b^3)-1$$
$$m=\frac{1}{4}(32a^3-384ab^2+144a^2b-192b^3)$$
$$r=a^2+4b^2$$

Since finding integrable solutions is the topic we are interested in, we have chosen a and b in such a way that *l*, *m,* and *r* are integers.

Let us take $a=4A$ and $b=4B$, we have

$l = l(A,B) = 768A^3 - 9216AB^2 - 6144A^2B + 8192B^3 - 1$

$m = m(A,B) = 512A^3 - 6144AB^2 + 2304A^2B - 3072B^3$

$r = r(A,B) = 16A^2 + 64B^2$

The integral solutions to (1) in terms of (2) are obtained by

$$p = p(A,B) = 1280A^3 - 15360AB^2 - 3840A^2B + 5120B^3 - 1$$
$$q = q(A,B) = 256A^3 - 3072AB^2 - 8448A^2B + 11264B^3 - 1$$
$$r = r(A,B) = 16A^2 + 64B^2$$

Properties:

1. $91q(1,1) - p(1,1) - 76r(1,1)$ is a Duck number
2. $48q(1,1) - p(1,1) - 37r(1,1)$ is a Perfect number
3. $p(1,A) - q(1,A) + 192r(1,A) + 3072SO_A - 768Gno_A \equiv 0 \pmod{4864}$
4. $q(A,1) - p(A,1) + 64Ar(A,1) + 1536CH_A - 5888Gno_A \equiv 0 \pmod{13568}$
5. $q(A,1) - p(A,1) + 1024PA^5 - 1792T_{6,A} + 5248Gno_A \equiv 2 \pmod{11312}$
6. $q(B,1) - p(B,1) + 288r(B,1) + 512CC_B + 512CH_B - 6144Gno_B \equiv 0 \pmod{30720}$
7. $21q(1,1) - p(1,1) - 102r(1,1)$ is a Nasty number 8. $275q(1,1) - p(1,1) - 100r(1,1)$ is a Silverback number

Pattern: 4

In place of (5), Write $400 = \dfrac{(60+40i\sqrt{4})(60-40i\sqrt{4})}{25}$ (9)

Equating the actual and imagined portions, we obtain

$l = l(a,b) = \frac{1}{5}(60a^3 - 720ab^2 - 480a^2b + 640b^3) - 1$

$m = m(a,b) = \frac{1}{5}(40a^3 - 480ab^2 + 180a^2b - 240b^3)$

$r = r(a,b) = a^2 + 4b^2$

Since finding integral solutions is the topic we are interested in, we have chosen a and b in such a way that *l, m*, and *r* are integers.

Let us take $a = 5A$ and $b = 5B$, we have

$l = l(A,B) = 1500A^3 - 18000AB^2 - 12000A^2B + 16000B^3 - 1$

$m = m(A,B) = 1000A^3 - 12000AB^2 + 4500A^2B - 6000B^3$

$r = r(A,B) = 25A^2 + 100B^2$

The integral solutions to (1) in terms of (2) are obtained by

$$p = p(A,B) = 2500A^3 - 30000AB^2 - 7500A^2B + 10000B^3 - 1$$
$$q = q(A,B) = 500A^3 - 6000AB^2 - 16500A^2B + 22000B^3 - 1$$
$$r = r(A,B) = 25A^2 + 100B^2$$

Properties:

1. $q(1,1) - p(1,1) + 16r(1,1)$ is a Cubical integer
2. $2q(1,1) - 2p(1,1) + 17r(1,1)$ is a Palindrome number
3. $p(1,A) - q(1,A) + 240r(1,A) + 6000SO_A - 1500Gno_A \equiv 0(\text{mod}\,9500)$
4. $q(A,1) - p(A,1) + 80Ar(A,1) + 1500Star_A - 11500Gno_A \equiv 0(\text{mod}\,25000)$
5. $q(A,1) - p(A,1) + 1000SO_A + 4500CS_A - 7000Gno_A \equiv 0(\text{mod}\,19000)$
6. $2q(A,1) - p(A,1) + 1020r(A,1) + 375RD_A + 375Star_A - 8625Gno_A \equiv 0(\text{mod}\,144624)$
7. $120q(1,1) - p(1,1) - 199r(1,1)$ is a Perfect number 8. $q(1,1) - 2p(1,1) - 242r(1,1)$ is a Emrip number

CONCLUSION:

For the non-homogeneous trinity cubic equation $5(p^2+q^2)-6(pq)+4(p+q+1)=1600r^3$, we have given numerous non-zero unique integrable solution patterns. To conclude, one can look for further options for solutions and their respective attributes among the various choices.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00008.2
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

FUZZY SCALE RELIABILITY ASSESSMENT FOR ANALYSIS OF MATHEMATICS LEARNING STYLES

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ABSTRACT

When using Likert scales, majority of statistical techniques cannot be used directly, & even if they could, doing so would significantly diminish results' interpretability & reliability. It is examined whether fuzzy scale is reliable for assessing survey respondents' opinions. So, validity of a conventional Likert scale-based (SB) questionnaire along with its fuzzy rating counterpart are compared. To do this, a set of students who used both student learning style scales (SLLS) provided answers to a few questions (Qs) from each scale. Since we know Likert scale version of this survey to be reliable, we can use corresponding Cronbach's alpha coefficients to gauge how well fuzzy version of survey performs.

Keywords: Fuzzy scale, Reliability, SLSS, Cronbach's $\tilde{\alpha}$ coefficient

1. INTRODUCTION

Likert scales -broadly employed to evaluate characteristics/viewpoints frequently linked to opinions, values, & other concepts. Data is generated from a collection of pre-fixed categories in a questionnaire using a Likert scale. These categories are frequently categorized using integer values from a scale that commonly ranges from 1 to 5, or from 1 to 7. These closed-format questions have become more common in practice since they are straightforward to administer & do not require a general explanation of responses **[1].**

Reliability is the degree to which an experiment, test, or other measuring method yields same results throughout a number of runs. The primary focus of this research will be on examining fuzzy scale's suitability in comparison to conventional Likert scale. To do this, a group of students' replies to a few questions from standard SLSS questionnaire will be compared in both Likert & fuzzy formats. Questions were only allowed in a mathematical framework if they matched dependent & independent learning types. To investigate trustworthiness of SLSS questionnaire within context of fuzzy logic, we shall explain an extension of Cronbach's alpha to fuzzy scenario.**[2].**

1.2. Preliminaries

Fuzzy set, also called fuzzy number, is a function that is convex in nature having range [0,1] to ℝ, where each x value of ℝ is related to membership function $U(x) \in [0,1]$. A fuzzy set (FS) U's 'cuts' or 'levels', stated by "U_α,"; intervals given by a value set substantiating $U(x) \geq \alpha$, per$\alpha \in [0,1]$. In this analysis, we focus on trapezoidal and triangular forms of FSs; latter is a particular case of former **[3].**

A trapezoidal FS that satisfies requirements that $[a,d]$ is 0 –level & $[b,c]$ is1-level is frequently referred to as Tra (a,b,c,d). The mathematical equation of fuzzy trapezoidal number having vertices in $\{a,b,c,d\}$ is

$$\text{Tra }(a,b,c,d) = \begin{cases} \dfrac{x-a}{b-a} & \text{if } x \in [a,b) \\ 1 & \text{if } x \in [b,c] \\ \dfrac{d-x}{d-c} & \text{if } x \in (c,d] \\ 0 & \text{otherwise} \end{cases} \qquad \dots (1.1)$$

A trapezoidal FS's description & its α-cuts is collected in Figure 1.1

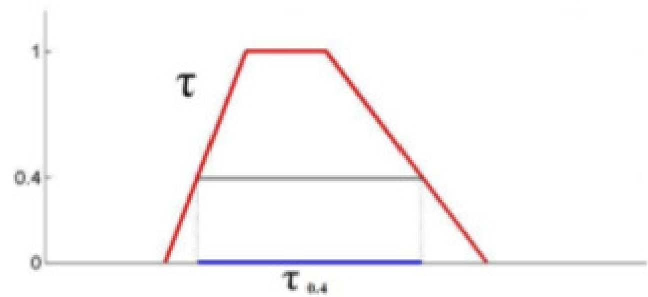

Figure 1.1: A trapezoidal FS's representation [4].

2. CRONBACH'S ALPHA PER RANDOM FUZZY SETS

Cronbach'sα**[2]** coefficient is used to calculate a test's internal consistency & is frequently used to determine how reliable test results are. By examining correlation between items, it takes values ≤ 1 & reflects how well a group of items assesses a single 1-D latent construct. Higher Cronbach's alpha values are preferred, so closer index is to 1,more reliable scale is **[5]**.

The Cronbach's alpha for related measuring scale is defined as follows in classical framework ; k items expressed via k real random variables $X_1, \dots, X_k$with sample –replies per $i \in \{1, \dots, k\}$:

$$\alpha = \left(\frac{k}{k-1}\right)\left(1 - \frac{\sum_{i=1}^{k} \hat{\sigma}_{X_i}^2}{\hat{\sigma}_T^2}\right) \qquad \dots . (2.1)$$

Where, $n = n_1 + \cdots + n_k$ and $\hat{\sigma}_T^2$ isvariance of all observed values, where n is the sample size of students. Since variance previously mentioned for random FSs is a marker of distribution of fuzzy values in relation with sample mean (SM), Cronbach's alpha may be applied to fuzzy framework in the same way it was in classical situation.

In light of k RFSs $\mathcal{X}_1, \ldots, \mathcal{X}_k \& n_i$ answers per$i \in \{1, \ldots, k\}$,
Cronbach's $\tilde{\alpha}$ per RFSs:

$$\tilde{\alpha} = \left(\frac{k}{k-1}\right)\left(1 - \frac{\sum_{i=1}^{k} \hat{\sigma}_{\mathcal{X}_i}^2}{\hat{\sigma}_{\mathcal{J}}^2}\right) \qquad \ldots . (2.2)$$

Where ,$\hat{\sigma}_{\mathcal{J}}^2$ is variance of all observed fuzzy values.

3. METHODOLOGY

Usually, idea of dependability provides a study of a construct's internal structure. The Cronbach's $\tilde{\alpha}$ index for RFSs is used to examine SLSS questionnaire's reliability in fuzzy context.

To evaluate a test's dependability, there are two key conditions **[2].** The test must, first & foremost, consist of a set of things that may be added together to provide a final score. The desired attribute must also be measured consistently across all goods. 20 questions from SLSS survey have been taken & used in this study. A dependent LS in mathematics was subject of 10 questions, while an independent LS was subject of 10 questions. These inquiries are compiled in appendix.

The degree program in elementary teaching at the asked a group of 110 students to respond to these 20 questions using both fuzzy and Likert scales.

In1st scenario, students select an answer ranging - 1 to 5, where 1 denotes utter disagreement, 2 moderate disagreement, 3 undecided, 4 moderate agreement, & 5 utter agreement.

In second instance, respondents used trapezoidal FSs on a scale from 0 to 10 (0 shows total disagreement&10 shows absolute agreement). Each response's 0-level represents set of values that student believes, to some extent, to be compatible with his or her view (i.e., student believes their opinion cannot exist outside this set).

Alternatively, the student views set of values in trapezoidal FS's first level to be wholly consistent with his or her view point. Finally, a trapezoidal can be created by linearly interpolating appropriate bounds of 0 – level & 1 - level.

The view points of 2 students A & B provided by trapezoidal FSs on 3 questionnaire Qs are shown in Figures 2 &3 below. It should be noted that when both students were asked to respond to identical questions utilizing a value on a Likert scale - 1 to 5, their responses - 4, 1 & 5, which demonstrates adaptability & greater diversity of fuzzy-type responses compared to Likert-type responses **[6].**

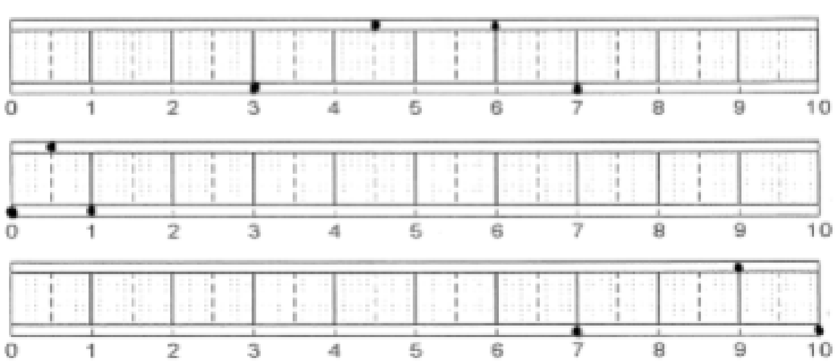

Figure 1.2: Student *A's* responses

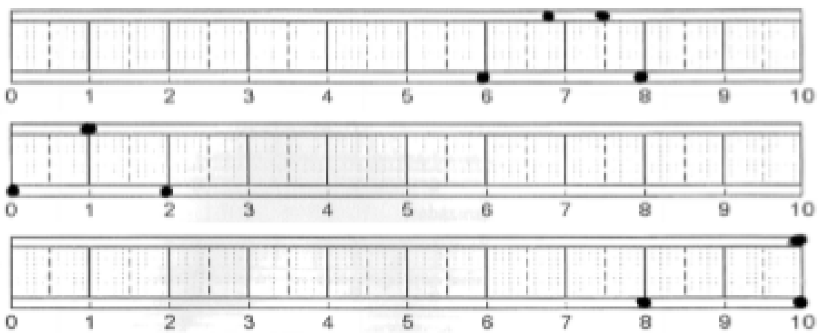

Figure 1.3: Student *B's* responses

4. EXPERIMENTAL RESULTS

To examine SLSS questionnaire's reliability when Likert & fuzzy replies are used. A descriptive analysis of responses is provided as a first step. Trapezoidal FSs'SMs that encode responses to SLSS questionnaire's questions corresponding to dependent & independent LSs are gathered in Figures 1.4 & 1.5. Additionally, Table 1.1 below shows SMs of Likert-type responses.

Table 2: Likert answers' sample means

Question	Dependent style	Independent style
1	4.194	3.12
2	4.037	2.806
3	4.454	2.407
4	3.676	2.954
5	2.852	3.398
6	3.657	3.620
7	3.602	3.611
8	3.843	2.861
9	4.593	3.324
10	2.583	3.639

Considerations about fuzzy-type & Likert-type sample means are noteworthy, particularly:

1. In regards to questions that correspond to dependent LS, if we pay close attention to supremum of 0-levels of fuzzy SMs, we observe those supremum's optimal values are attained per Qs D_3& D_9; they are also highest means attained for Likert answers. Additionally, questions D_5 & D_{10} obtain lowest values for maximum of 0-levels in fuzzy case as well as least values of category means.

2. However, in independent LS questions, questions I_6, I_7 & I_{10} have highest values of supremum of 0-levels of fuzzy means & categorical means, whereas questions I_2 & I_3 have lowest values in both circumstances.

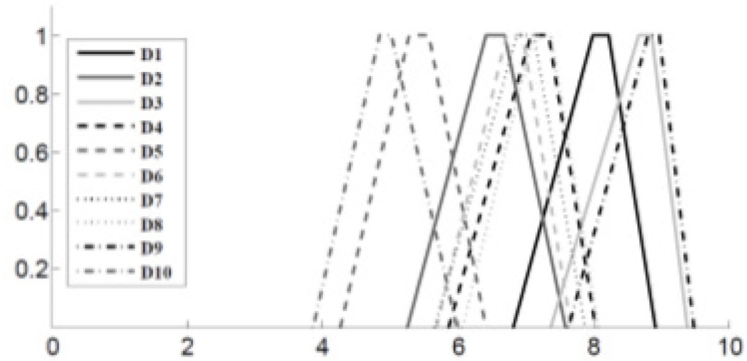

Figure 1.4: Examples of ways for responses to questions about dependent LSs

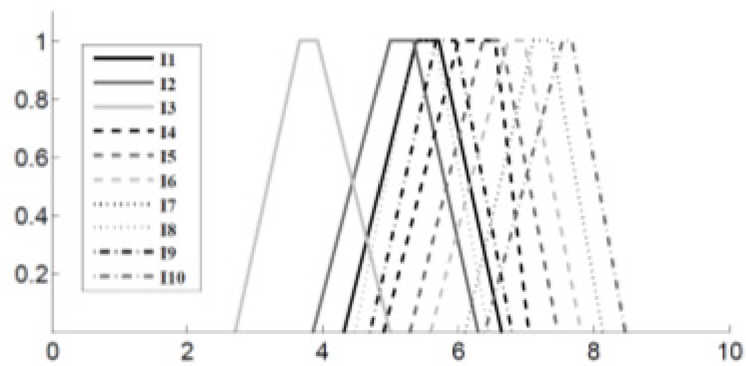

Figure 1.5: Examples of ways for responses to questions about independent LSs

Table 1.3: Sample variances in both cases

Question	Dependent style	Independent style
1	4.194	3.12
2	4.037	2.806
3	4.454	2.407
4	3.676	2.954
5	2.852	3.398
6	3.657	3.620
7	3.602	3.611
8	3.843	2.861
9	4.593	3.324
10	2.583	3.639

These findings confirm that fuzzy-based responses to SLSS questionnaire are coherent with respect to traditional categorical approach.

The overall sample mean of fuzzy answers for dependent & independent LSs is shown in Figure 1.6.

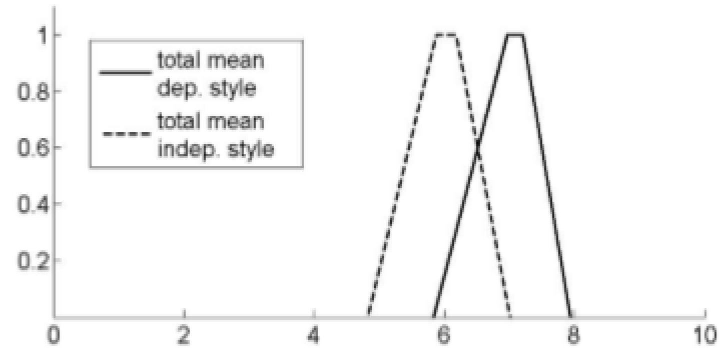

Figure 1.6: SMs of answers corresponding to independent LS questions

Additionally, overall sample averages for Likert responses are 3.174 for independent case & 3.749 for dependent case. In light of findings from both frameworks, we can therefore say that respondents generally had more reliant than independent LSs. Regarding replies' variability, Table 1.3 compiles matching variances of Likert & fuzzy responses for dependent & independent learning methods.

Table 1.3: Sample variances in both cases

	Fuzzy	Likert		Fuzzy	Likert
D_1	3.261	0.879	I_1	5.126	0.884
D_2	4.392	1.017	I_2	2.872	0.434
D_3	1.713	0.507	I_3	5.438	0.982
D_4	2.9	0.497	I_4	4.671	0.748
D_5	5.867	1.293	I_5	3.969	0.74
D_6	4.967	1.003	I_6	6.139	1.272
D_7	3.56	0.795	I_7	3.68	0.96
D_8	4.469	1.003	I_8	7.728	1.305
D_9	1.539	0.501	I_9	5.789	0.96
D_{10}	7.597	1.447	I_{10}	4.017	0.823

Changes in Likert-type responses are less than those of fuzzy-type replies given that answer variation scale spans from 1 to 5 in 2nd instance, yet from 0 to 10 in 1st. Nevertheless, it is simple to verify that for both response types, questions D_{10} and I_6 have highest variances while questions D_9 and I_2 have lowest variances. This fact supports previously observed consistency between fuzzy & Likert responses.

Both traditional Cronbach's alpha & RFS version of Cronbach's alpha account for both dependent & independent LSs when calculating reliability..

Cronbach'sα and $\tilde{\alpha}$ are calculated with variances of numerators shown in Table 1.3. In addition,values acquired by adding each person's responses tospecialized LS questions are used to calculate variances of total observed values utilised in denominators, $\hat{\sigma}_T^2$ and $\hat{\sigma}_{\tilde{T}}^2$. Table 1.4 displays calculated findings.

	Independent style	Dependent style
α	0.7356	0.6276
$\tilde{\alpha}$	0.7258	0.6049

Table 1.4: Mentioned Cronbach's $\tilde{\alpha}$ per mathematical dependent & independent LSs, as well as for categorical & fuzzy situations

The results shown in Table 1.4 indicate that Cronbach'sα and $\tilde{\alpha}$ on both answer styles are extremely close. However, a widely accepted criteria for expressing a test's internal consistency using Cronbach's alpha states that when index is between range [0.6,0.7) it is acceptable, however it is desirable when it exceeds value 0.7. As a result, α and $\tilde{\alpha}$ are acceptable for independent LS questions & acceptable for dependent LS questions using both types of answers.

The most noteworthy element is that, even if values found in Table 1.4 are not particularly high, reliability of SLSS questionnaire's questions about dependent & independent LSs is maintained in presence of ambiguous responses. This suggests that using a fuzzy scale to respond to opinion surveys is highly advised because it seems to maintain test's internal consistency & offers some benefits (such as those discussed throughout this work) that help capture inherent human reliability in forming accurate judgments better.

5. CONCLUSIONS

When categorical values & FSs are given as answers, internal consistency (or reliability) of some questions from SLSS questionnaire regarding a dependent or independent LS in mathematics was examined in this work. For this reason, a Cronbach's α has been created for variables with values in space of FSs. As a result, tests with fuzzy responses were shown to be just as reliable as tests with values selected from a Likert scale. The first scale is a particularly suitable instrument to capture & express imprecision inherent in human beliefs since fuzzy scale, in contrast to categorical one, has some advantages. Additionally, instructors might gain from usage of fuzzy scale since data from fuzzy questionnaire may help them develop their teaching techniques & broaden their perspectives on subjects they teach.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com

DOI No. : 10.5958/2394-9309.2023.00009.4
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

TRANSCENDENTAL EQUATION INVOLVING PALINDROME NUMBER

$$p^5 + \sqrt{p^7 + q^7 - (pq)^5} + \sqrt[3]{r^2 + s^2} = 20402\,h^3$$

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ABSTRACT

We make an effort to justify the integral solutions of transcendental equation $p^5 + \sqrt{p^7 + q^7 - (pq)^5} + \sqrt[3]{r^2 + s^2} = 20402h^3$ ***under frequent patterns providing specific numerical examples.***

Keywords : Transcendental equation, Palindrome number, Integral solutions.

I. INTRODUCTION

A mathematical equation is said to be transcendental if all of its constituent variables have transcendental functions. A function is transcendental when it cannot be represented by a polynomial solution and is analytical in nature. These equations are simple to solve because the variables can be roughly known. Numerous equations that appear to allow for only straightforward solutions are used to solve transcendental functions. The opposite of each of the aforementioned functions, as well as the logarithmic, exponential, trigonometric and hyperbolic ones, are instances of well-known transcendental functions. There are also some surprising transcendental function along with specific analytical functions like elliptic, zeta and gamma. In this paper for solving transcendental equation we have to use Palindrome number and sum of its squares of the same palindrome number.

[1-2] has been recommended for fundamental notions and principle in number theory. Fundamental concepts and ideas of number theory have been investigated in [3-7]. Transcendental equation related thoughts and problem were collected in [8-12].

II. METHOD OF ANALYSIS

The equation to be solved is, $p^5 + \sqrt{p^7 + q^7 - (pq)^5} + \sqrt[3]{r^2 + s^2} = 20402\,h^3$ (1)

The following linear transformation $p = (u - v)^3$, $q = (v - u)^3$, $r = u^3 + 3uv^2$, $s = 3u^2v - v^3$

leads to $p^5 + \sqrt{p^7 + q^7 - (pq)^5} + \sqrt[3]{r^2 + s^2} = u^2 + v^2$ hence, $p^5 + \sqrt{p^7 + q^7 - (pq)^5} + \sqrt[3]{r^2 + s^2} = u^2 + v^2$ reduces to,

$u^2 + v^2 = 20402\,h^3$ (2)

Now, we find various patterns of solutions of (1) using (2).

PATTERN 1:

Let $h = c^2 + d^2$, for $c, d \geq 0$

$$u^2 + v^2 = \left(101^2 + 101^2\right)\left(c^2 + d^2\right)^3 \qquad (3)$$

Using factorization and equating real and imaginary parts we get,

$$u = 101\left(c^3 - 3cd^2\right) - 101\left(3c^2 d - d^3\right)$$

$$v = 101\left(3c^2 d - d^3\right) + 101\left(c^3 - 3cd^2\right)$$

Therefore, $u = 101f(c,d) - 101g(c,d)$ and $v = 101f(c,d) + 101g(c,d)$

where, f(c, d) = $c^3 - 3cd^2$ and g(c, d) = $3c^2 d - d^3$

Hence, the non zero integral solutions are

$$p = (-202g(c,d))^3$$

$$q = (202g(c,d))^3$$

$$r = (101f(c,d) - 101g(c,d))\left((101f(c,d) - 101g(c,d))^2 - 3(101f(c,d) + 101g(c,d))^2\right)$$

$$s = (101f(c,d) + 101g(c,d))\left(3(101f(c,d) - 101g(c,d))^2 - (101f(c,d) + 101g(c,d))^2\right)$$

Numerical examples satisfying the solution are listed below.

TABLE 1

p	q	r	s	t	LHS=RHS
-65939264	65939264	-65939264	0	2	163216
65939264	-65939264	3950174034	-991149562	5	2550250
-33760903168	33760903168	-33760903168	0	8	10445824
48069723456	-48069723456	47344391552	79061177536	10	20402000

PATTERN 2:

consider (3) can be rewritten as,

$$u^2 + v^2 = 1 \cdot \left(101^2 + 101^2\right)\left(c^2 + d^2\right)^3$$

Taking , $1 = \left(\frac{4^2 + 3^2}{5^2}\right)$

$$u^2 + v^2 = \left(\frac{4^2 + 3^2}{5^2}\right)\left(101^2 + 101^2\right)\left(c^2 + d^2\right)^3$$

Using factorization and equating real and imaginary parts we get,

$$u = \frac{1}{5}\left(101\left(4c^3 - 9c^2 d - 12cd^2 + 3d^3\right) - 101\left(3c^3 + 12c^2 d - 9cd^2 - 4d^3\right)\right)$$

$$v = \frac{1}{5}\left(101\left(3c^3 + 12c^2 d - 9cd^2 - 4d^3\right) + 101\left(4c^3 - 9c^2 d - 12cd^2 + 3d^3\right)\right)$$

Take c = 5X and d = 5Y

$$u = 101\left(100X^3 - 225X^2Y - 300XY^2 + 75Y^3\right) - 101\left(75X^3 + 300X^2Y - 225XY^2 - 100Y^3\right)$$

$$v = 101\left(75X^3 + 300X^2Y - 225XY^2 - 100Y^3\right) + 101\left(100X^3 - 225X^2Y - 300XY^2 + 75Y^3\right)$$

Therefore, $u = 101f(X,Y) - 101g(X,Y)$ and $v = 101g(X,Y) + 101f(X,Y)$

where, $f(X,Y) = 100X^3 - 225X^2Y - 300XY^2 + 75Y^3$ and $g(X,Y) = 75X^3 + 300X^2Y - 225XY^2 - 100Y^3$

Hence, the non zero integral solutions are

$$p = (-202g(X,Y))^3$$

$$q = (202g(X,Y))^3$$

$$r = (101f(X,Y) - 101g(X,Y))\left((101f(X,Y) - 101g(X,Y))^2 - 3(101f(X,Y) + 101g(X,Y))^2\right)$$

$$s = (101f(X,Y) + 101g(X,Y))\left(3(101f(X,Y) - 101g(X,Y))^2 - (101f(X,Y) + 101g(X,Y))^2\right)$$

Numerical examples satisfying the solutions are listed below.

TABLE 2

p	q	r	s	t	LHS=RHS
-1030301000000	1030301000000	45333244000000	-120545217000000	50	2550250000
8876171902625000	-8876171902625000	-903799355343750	7902827229781250	125	39847656250
-527514112000000	527514112000000	23210620928000000	-61719151104000000	200	163216000000
434658234375000000	-434658234375000000	-177082984375000000	32196906250000000	250	318781250000

PATTERN 3:

Taking (3) as in pattern 2 and replacing 1 by $\left(\frac{6^2 + 8^2}{10^2}\right)$

$$u^2 + v^2 = \left(\frac{6^2 + 8^2}{10^2}\right)\left(101^2 + 101^2\right)\left(c^2 + d^2\right)^3$$

Using factorization and equating real and imaginary parts we get,

$$u = \frac{1}{10}\left(101\left(6c^3 - 24c^2d - 18cd^2 + 8d^3\right) - 101\left(8c^3 + 18c^2d - 24cd^2 - 6d^3\right)\right)$$

$$v = \frac{1}{10}\left(101\left(8c^3 + 18c^2d - 24cd^2 - 6d^3\right) + 101\left(6c^3 - 24c^2d - 18cd^2 + 8d^3\right)\right)$$

Take $c = 10X$ and $d = 10Y$

$$u = 101\left(600X^3 - 2400X^2Y - 1800XY^2 + 800Y^3\right) - 101\left(800X^3 + 1800X^2Y - 2400XY^2 - 600Y^3\right)$$

$$v = 101\left(800X^3 + 1800X^2Y - 2400XY^2 - 600Y^3\right) + 101\left(600X^3 - 2400X^2Y - 1800XY^2 + 800Y^3\right)$$

Therefore, $u = 101f(X,Y) - 101g(X,Y)$ and $v = 101g(X,Y) + 101f(X,Y)$

where, $f(X,Y) = 600X^3 - 2400X^2Y - 1800XY^2 + 800Y^3$ and $g(X,Y) = 800X^3 + 1800X^2Y - 2400XY^2 - 600Y^3$

Hence, the non-zero integral solutions are

$$p = (-202g(X,Y))^3$$

$$q = (202g(X,Y))^3$$

$$r = (101f(X,Y) - 101g(X,Y))\left((101f(X,Y) - 101g(X,Y))^2 - 3(101f(X,Y) + 101g(X,Y))^2\right)$$

$$s = (101f(X,Y) + 101g(X,Y))\left(3(101f(X,Y) - 101g(X,Y))^2 - (101f(X,Y) + 101g(X,Y))^2\right)$$

Numerical examples satisfying the solution are listed below.

TABLE 3

p	q	r	s	t	LHS=RHS
527514112000000	-527514112000000	61719151104000000	-23210620928000000	200	163216000000
-4220112896000000	4220112896000000	1203391568000000	-2654055376000000	100	20402000000
270087225344000000	-270087225344000000	31600205365248000000	-11883837915136000000	800	10445824000000
8242408000000000000	-8242408000000000000	-3348478250000000000	2318177250000000000	500	2550250000000

PATTERN 4:

Taking (3) as in pattern 2 and replacing 1 by $\left(\frac{5^2+12^2}{13^2}\right)$

$$u^2 + v^2 = \left(\frac{5^2+12^2}{13^2}\right)\left(101^2+101^2\right)\left(c^2+d^2\right)^3$$

Using factorization and equating real and imaginary parts we get,

$$u = \frac{1}{13}\left(101\left(5c^3 - 36c^2d - 115cd^2 + 12d^3\right) - 101\left(12c^3 + 15c^2d - 36cd^2 - 5d^3\right)\right)$$

$$v = \frac{1}{13}\left(101\left(12c^3 + 15c^2d - 36cd^2 - 5d^3\right) + 101\left(5c^3 - 36c^2d - 115cd^2 + 12d^3\right)\right)$$

Take c = 13X and d = 13Y

$$u = 101\left(845X^3 - 6084X^2Y - 2535XY^2 + 2028Y^3\right) - 101\left(2028X^3 + 2523X^2Y - 6084XY^2 - 845Y^3\right)$$

$$v = 101\left(2028X^3 + 2523X^2Y - 6084XY^2 - 845Y^3\right) + 101\left(845A^3 - 6084A^2B - 2535AB^2 + 2028B^3\right)$$

Therefore, $u = 101f(X,Y) - 101g(X,Y)$ and $v = 101g(X,Y) + 101f(X,Y)$

where, f(X,Y) = $845X^3 - 6084X^2Y - 2535XY^2 + 2028Y^3$ and

g(X,Y) = $2028X^3 + 2523X^2Y - 6084XY^2 - 845Y^3$

Hence, the non-zero integral solutions are

$$p=(-202g(X,Y))^3$$

$$q=(202g(X,Y))^3$$

$$r=(101f(X,Y)-101g(X,Y))\left((101f(X,Y)-101g(X,Y))^2-3(101f(X,Y)+101g(X,Y))^2\right)$$

$$s=(101f(X,Y)+101g(X,Y))\left(3(101f(X,Y)-101g(X,Y))^2-(101f(X,Y)+101g(X,Y))^2\right)$$

Numerical examples satisfying the solution are listed below.

TABLE 4

p	q	r	s	t	LHS=RHS
109168747894502000	-109168747894502000	647692134009652000	263532720864861000	338	787812457744
-68747666312572400	68747666312572400	28475776714828500	-12004981660774700	169	98476557218
2546209863428610000	-2546209863428610000	6146550610316660000	14579597677992200000	676	6302499661952
176324432071979000	-176324432071979000	452183104828628000	375963257238011000	845	701911734130

CONCLUSION

To demonstrate the key solutions to the transcendental equation under complex pattern, we have used a variety of numerical examples. Additionally, one could look for in-depth answers to these associated issues.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00010.0
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

MODELING AND ANALYTICAL STUDY OF A COMPLEX BI-SERIAL QUEUE NETWORK CONNECTED TO A COMMON SERVER WITH FEEDBACK AND BATCH ARRIVAL

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ABSTRACT

A queue network model comprised of two bi-serial systems has been studied in the present work. Both the systems are heterogeneous in nature and are centrally connected to a common server. Arrival of customers follow Poisson process and are in fixed size batches. Method of generating function technique, law of calculus and classical formula are used to determine all queue characteristics. Particular cases are taken to check the validity of the model. The model is demonstrated with numerical illustration and graphical analysis where comparative graphical analysis is done in three cases, first case is when feedback is allowed and batch arrival of customers at all servers is considered, second case is when there is no facility of feedback, customers arrive in batches and the last case is only one time visit of customer is allowed also the arrival is single arrival.

Keywords: queuing, heterogeneous servers; bi-serial servers; feedback; batch arrival; comparative analysis

INTRODUCTION

Maggu[1] introduced the concept of bi-series in Mathematical theory of Queuing. It was Hafiz Noor Mohammad who studied bi-serial queues with bulk arrival. After that many researchers extended this work to models with more servers with single and bulk arrival. T.P.Singh & Kusum[4] discussed feedback queue model under different parameters and augmentations. T.P.Singh, Reeta etal.[7] combined both concepts bi-tandem and feedback together in one paper. The feedback arises due to unsuccessful services to make a successful one from the initial stage. Further T.P.Singh, Meenu & Deepak Gupta[8] analysed queue model with threshold effect on a queue model where arrivals are in batches. Gupta et.al [5],[6] discussed queue network models with parallel and bi-serial servers with single arrival.. Aggarwal S & Singh B.K consider tri-cum bi-serial queuing model (2018) with single arrival and derive various queue performance measures. After single arrival, Aggarwal Sachin et. al [10] analyze a tri-cum bi-serial queue model ,where arrival of customers is taken as batch arrival. This study was further extended by Gupta D, Gupta R[11]. They described the behaviour of two bi-serial subsystems which are connected to a common server and arrival is taken as bulk arrival. The proposed work is further an extension of work done in [11]. In order to maintain the satisfaction level of customers the concept of feedback is introduced in the present paper. Two systems comprised of bi-serial subsystems are considered which are centrally connected to common servers. Customers can visit any of the servers at most two times.

NOTATIONS USED

Table 1: Notations used throughout the work done

Servers		C_{11}	C_{12}	C_3	C_{21}	C_{22}
Arrival Rate		λ_1	λ_2			
Mean Service Rate		μ_1	μ_2	μ_3	μ_4	μ_5
No. of batches in queues		m_1	m_2	m_3	m_4	m_5
Probability of moving the customers in batches from one server to another	1st visit	$C_{11} \to C_{12}$ p_{12} $C_{11} \to C_3$ p_{13}	$C_{12} \to C_{11}$ p_{21} $C_{12} \to C_3$ p_{23}	$C_3 \to C_{21}$ p_{34} $C_3 \to C_{22}$ p_{35} $C_3 \to C_{11}$ p_{31} $C_3 \to C_{12}$ p_{32}	$C_{21} \to C_{22}$ p_{45} $C_{21} \to C_3$ p_{43} $C_{21} \to exit$ p_4	$C_{22} \to C_{21}$ p_{54} $C_{22} \to C_3$ p_{53} $C_{22} \to exit$ p_5
	2nd visit	$C_{11} \to C_{12}$ q_{12} $C_{11} \to C_3$ q_{13}	$C_{12} \to C_{11}$ q_{21} $C_{12} \to C_3$ q_{23}	$C_3 \to C_{21}$ q_{34} $C_3 \to C_{22}$ q_{35}	$C_{21} \to C_{22}$ q_{45} $C_{21} \to exit$ q_4	$C_{22} \to C_{21}$ q_{54} $C_{22} \to exit$ q_5
Probability of leaving the server	1st visit	a	b	c	d	e
	2nd visit	a_1	b_1	c_1	d_1	e_1

DESCRIPTION OF THE MODEL

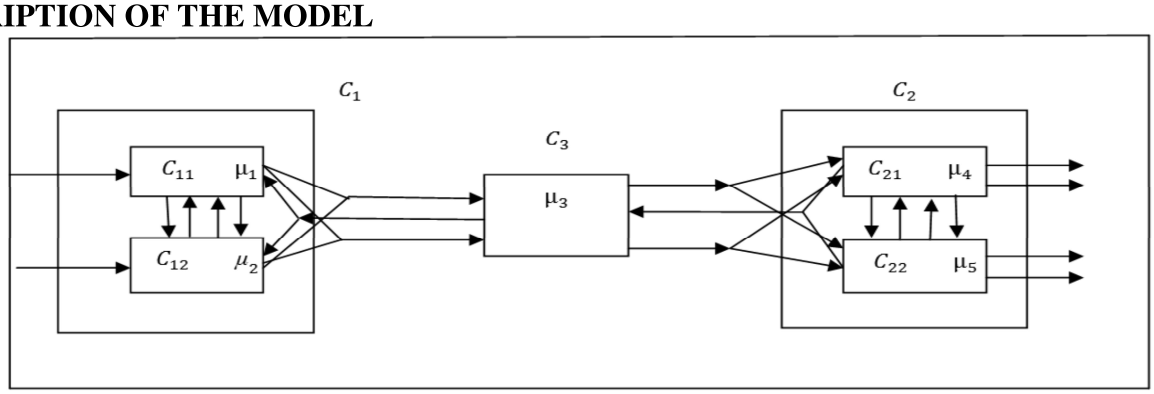

Figure 1: Feedback queue model with bi-serial servers and centrally connected to a common server

The present model as shown in figure1 is comprised of three servers namely C_1, C_2, C_3 where C_1 & C_2 are containing bi-serial servers C_{11}, C_{12} & C_{21}, C_{22} respectively. C_3 is a central server to which both the servers C_1 & C_2 are connected. In order to avail the services firstly the customers in batches will come at the server C_1 in front of the servers C_{11} & C_{12}. After availing service from server C_{11} customers may either move to server C_{12} or to the server C_3. Similar opportunity is there after taking service from C_{12} server . If the customers are satisfied then they will either move to the server C_{21} or to the server C_{22} for service. If they are not satisfied then they can revisit the servers C_{11} & C_{12} . After availing service from servers C_{21} & C_{22} they can either leave the system or visit the server in front of the server from which they taken service. Also they can revisit any of the servers at most once as per their need or satisfaction.

PRACTICAL SITUATION

The feedback queue model with batch arrival that is considered in this paper is applicable to various fields like as gaming zones, computer networking, administrative offices, production management and many more. In order to understand the model clearly we can take a particular example of a mall or shopping complex having three different sections that may be on three different floors. Assuming first section on ground floor for clothing, where there are two bi-serial servers for men clothe and women clothes. First floor is centrally linked to ground floor and second floor, which is for food section. Second floor again has two bi-serial servers, where one is gaming zone for children and other is leisure space like spa, massage centre.

After entering in the mall customers can visit any of the clothing section at the ground floor then they will move to food section for refreshment or food. After taking service from food section they will move to second floor, where either they can visit to gaming zone or leisure space according to their need. After taking service either they will exit the mall or revisit any of the servers at most once for feedback.

MATHEMATICAL MODELLING

Let p_{m_1,m_2,m_3,m_4,m_5} denotes the probability of m_1, m_2, m_3, m_4, m_5 customers waiting for service at the queues in batches in front of the servers $C_{11}, C_{12}, C_3, C_{21}, C_{22}$ respectively where $m_1, m_2, m_3, m_4, m_5 \geq 0$ .The differential difference equations in steady state is as follows:

For $m_1 > b_1, m_2 > b_2, m_3, m_4, m_5 > 0$

$$(\lambda_1+\lambda_2+\mu_{11}+\mu_{12}+\mu_3+\mu_{21}+\mu_{22})p_{m_1,m_2,m_3,m_4,m_5}=\lambda_1 p_{m_1-b_1,m_2,m_3,m_4,m_5}+\lambda_2 p_{m_1,m_2-b_2,m_3,m_4,m_5}$$
$$+\mu_{11}AP_{m_1+1,m_2-1,m_3,m_4,m_5}+\mu_{11}BP_{m_1+1,m_2,m_3-1,,m_4,m_5}+\mu_{12}CP_{m_1-1,m_2+1,m_3,m_4,m_5}+\mu_{12}DP_{m_1,m_2+1,m_3-1,m_4,m_5}$$
$$+\mu_3 EP_{m_1,m_2,m_3+1,m_4-1,m_5}+\mu_3 FP_{m_1,m_2,m_3+1,m_4,m_5-1}+\mu_3 HP_{m_1-1,m_2,m_3+1,m_4,m_5}+\mu_3 IP_{m_1,m_2-1,m_3+1,m_4,m_5}$$
$$+\mu_{21}JP_{m_1,m_2,m_3,m_4+1,m_5-1}+\mu_{21}KP_{m_1,m_2,m_3,m_4+1,m_5}+\mu_{21}LP_{m_1,m_2,m_3-1,m_4+1,m_5}$$
$$+\mu_{22}MP_{m_1,m_2,m_3,m_4-1,m_5+1}+\mu_{22}NP_{m_1,m_2,m_3,m_4,m_5+1}+\mu_{22}OP_{m_1,m_2,m_3-1,m_4,m_5+1}\text{.................}(1)$$

Here,

$$A=(ap_{12}+a_1q_{12}),B=(ap_{13}+a_1q_{13}),C=(bp_{21}+b_1q_{21}),D=(bp_{23}+b_1q_{23}),E=(cp_{34}+c_1q_{34})$$
$$F=(cp_{35}+c_1q_{35}),H=(cp_{31}),I=(cp_{32}),J=(dp_{45}+d_1q_{45}),K=(dp_4+d_1q_4),L=(dp_{43}),$$
$$M=(ep_{54}+e_1q_{54}),N=(ep_5+e_1q_5),O=(ep_{53})$$

71 more equations like equation (1) will form when we consider all possible cases for m_1, m_2, m_3, m_4, m_5

We solve these equations with the help of probability generating function technique. For this purpose we consider probability generating function and partial generating functions as follows,

$$G(X,Y,Z,,R,S)=\sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}\sum_{m_3=0}^{\infty}\sum_{m_4=0}^{\infty}\sum_{m_5=0}^{\infty}p_{m_1,m_2,m_3,m_4,m_5}(X)^{m_1}(Y)^{m_2}(Z)^{m_3}(R)^{m_4}(S)^{m_5}\text{.........}(2)$$

$$|X|=1,|Y|=1,|z|=1,|R|=1,|S|=1$$

$$G_{m_2,m_3,m_4,m_5}(X)=\sum_{m_1=0}^{\infty}p_{m_1,m_2,m_3,m_4,m_5}X^{m_1}\text{..}(3)$$

$$G_{m_3,m_4,m_5}(X,Y)=\sum_{m_2=0}^{\infty}G_{m_2,m_3,m_4,m_5}(X)Y^{m_2}\text{.....................}(4)$$

$$G_{m_4,m_5}(X,Y,Z)=\sum_{m_3=0}^{\infty}G_{m_3,m_4,m_5}(X,Y)Z^{m_3}\text{.....................}(5)$$

$$G_{m_5}(X,Y)=\sum_{m_4=0}^{\infty}G_{m_4,m_5}(X,Y,Z)R^{m_4}\text{..}(6)$$

$$G(X,Y,Z,R,S)=\sum_{m_5=0}^{\infty}G_{m_5}(X,Y,Z,R)\,S^{m_5}\text{.......................................}(7)$$

By solving all equation with the help of above defined probability generating function we get,

$$G(X,Y,Z,R,S)=\frac{\begin{gathered}\mu_{11}\left[1-\frac{AY}{X}-\frac{BZ}{X}\right]G_0(Y,Z,R,S)+\mu_{12}\left[1-\frac{CX}{Y}-\frac{DZ}{Y}\right]G_0(X,Z,R,S)\\+\mu_3\left[1-\frac{HX}{Z}-\frac{IY}{Z}-\frac{ER}{Z}-\frac{FS}{Z}\right]G_0(X,Y,R,S)+\mu_{21}\left[1-\frac{LZ}{R}-\frac{JS}{R}-\frac{K}{R}\right]G_0(X,Y,Z,S)\\+\mu_{22}\left[1-\frac{OZ}{S}-\frac{MR}{S}-\frac{N}{S}\right]G_0(X,Y,Z,R)\end{gathered}}{\begin{gathered}\lambda_1(1-X^{b_1})+\lambda_2(1-Y^{b_2})+\mu_{11}\left[1-\frac{AY}{X}-\frac{BZ}{X}\right]+\mu_{12}\left[1-\frac{CX}{Y}-\frac{DZ}{Y}\right]\\+\mu_3\left[1-\frac{HX}{Z}-\frac{IY}{Z}-\frac{ER}{Z}-\frac{FS}{Z}\right]+\mu_{21}\left[1-\frac{LZ}{R}-\frac{JS}{R}-\frac{K}{R}\right]\\+\mu_{22}\left[1-\frac{OZ}{S}-\frac{MR}{S}-\frac{N}{S}\right]\end{gathered}}\text{.....(8)}$$

$$G(X,Y,Z,R,S)=\frac{\begin{gathered}\mu_{11}\left[1-\frac{AY}{X}-\frac{BZ}{X}\right]G_1+\mu_{12}\left[1-\frac{CX}{Y}-\frac{DZ}{Y}\right]G_2+\mu_3\left[1-\frac{HX}{Z}-\frac{IY}{Z}-\frac{ER}{Z}-\frac{FS}{Z}\right]G_3\\+\mu_{21}\left[1-\frac{LZ}{R}-\frac{JS}{R}-\frac{K}{R}\right]G_4+\mu_{22}\left[1-\frac{OZ}{S}-\frac{MR}{S}-\frac{N}{S}\right]G_5\end{gathered}}{\begin{gathered}\lambda_1(1-X^{b_1})+\lambda_2(1-Y^{b_2})+\mu_{11}\left[1-\frac{AY}{X}-\frac{BZ}{X}\right]+\mu_{12}\left[1-\frac{CX}{Y}-\frac{DZ}{Y}\right]+\mu_3\left[1-\frac{HX}{Z}-\frac{IY}{Z}-\frac{ER}{Z}-\frac{FS}{Z}\right]\\+\mu_{21}\left[1-\frac{LZ}{R}-\frac{JS}{R}-\frac{K}{R}\right]+\mu_{22}\left[1-\frac{OZ}{S}-\frac{MR}{S}-\frac{N}{S}\right]\end{gathered}}\text{....(9)}$$

Where,

$G_0(Y,Z,R,S)=G_1, G_0(X,Z,R,S)=G_2, G_0(X,Y,R,S)=G_3, G_0(X,Y,Z,S)=G_4$

$G_0(X,Y,Z,R)=G_5$

By taking X=Y=Z=R=S=1 and substituting these values in equation (9), we obtain 0/0 form. In order to find the values of G_1, G_2, G_3, G_4, G_5 we will take limits and use L. Hospital's Rule.

Now differentiating equation (9) w.r.t X and using L. Hospital's Rule, taking Y=Z=R=S=1 and X→1

$$-\lambda_1 b_1+\mu_{11}(A+B)+\mu_{12}(-C)+\mu_3(-H)=\mu_{11}(A+B)G_1+\mu_{12}(-C)G_2+\mu_3(-H)G_3\text{.................(10)}$$

Differentiating equation (9) w.r.t Y and using L. Hospital's Rule,taking X=Z=R=S=1 and Y→1

$$-\lambda_2 b_2+\mu_{11}(-A)+\mu_{12}(C+D)+\mu_3(-I)=\mu_{11}(-A)G_1+\mu_{21}(C+D)G_2+\mu_3(-I)G_3\text{.................(11)}$$

Differentiating equation (9) w.r.t Z and using L. Hospital's Rule, taking X=Y=R=S=1 and Y→1

$$\begin{gathered}\mu_{11}(-B)+\mu_{12}(-D)+\mu_3(E+F+H+I)+\mu_{21}(-L)+\mu_{22}(-O)\\=\mu_{11}(-B)G_1+\mu_{12}(-D)G_2+\mu_3(E+F+H+I)G_3+\mu_{21}(-L)G_4+\mu_{22}(-O)G_5\text{.................(12)}\end{gathered}$$

Differentiating equation (9) w.r.t R and using L. Hospital's Rule, taking X=Y=Z=S=1 and R→1

$$\mu_3(-E)+\mu_{21}(J+K+L)+\mu_{22}(-M)=\mu_3(-E)G_3+\mu_{21}(J+K+L)G_4++\mu_{22}(-M)G_5\text{........(13)}$$

Differentiating equation (9) w.r.t S and using L. Hospital's Rule, taking X=Z=R=S=1 and Y→1

$$\mu_3(-F)+\mu_{21}(-J)+\mu_{22}(M+N+O)=\mu_3(-F)G_3+\mu_{21}(-J)G_4++\mu_{22}(M+N+O)G_5 \quad \text{........}(14)$$

Solving equations from (10) to (14) we obtain,

$$G_1 = 1-\left[\frac{(\lambda_1 b_1 + C\lambda_2 b_2)}{\mu_{11}(1-AC)} + \frac{(H+IC)}{\mu_{11}(1-AC)}\left(\frac{\lambda_1 b_1(1-MJ)(AD+B)+\lambda_2 b_2(1-MJ)(D+BC)}{A_1}\right)\right] \quad \text{..........} (15)$$

$$G_2 = 1-\left[\frac{(\lambda_1 b_1 A + \lambda_2 b_2)}{\mu_{12}(1-AC)} + \frac{(I+HA)}{\mu_{12}(1-AC)}\left(\frac{\lambda_1 b_1(1-MJ)(AD+B)+\lambda_2 b_2(1-MJ)(D+BC)}{A_1}\right)\right] \quad \text{..........} (16)$$

$$G_3 = 1-\left[\frac{\lambda_1 b_1(1-MJ)(AD+B)+\lambda_2 b_2(1-MJ)(D+BC)}{A_1}\right] \quad \text{..........} (17)$$

$$G_4 = 1-\frac{(E+MF)}{\mu_{21}}\left[\frac{\lambda_1 b_1(AD+B)+\lambda_2 b_2(D+BC)}{A_1}\right] \quad \text{..........} (18)$$

$$G_5 = 1-\frac{(EJ+F)}{\mu_{22}}\left[\frac{\lambda_1 b_1(AD+B)+\lambda_2 b_2(D+BC)}{A_1}\right] \quad \text{..........} (19)$$

Where,

$$A_1 = (1-MJ)[-D(HA+I)-B(H+IC)+(1-AC)]+(1-AC)[-L(E+MF)-O(EJ+F)]$$

Also, from equations (15) to (19),

$$\rho_1 = \left[\frac{(\lambda_1 b_1 + C\lambda_2 b_2)}{\mu_{11}(1-AC)} + \frac{(H+IC)}{\mu_{11}(1-AC)}\left(\frac{\lambda_1 b_1(1-MJ)(AD+B)+\lambda_2 b_2(1-MJ)(D+BC)}{A_1}\right)\right] \quad (20)$$

$$\rho_2 = \left[\frac{(\lambda_1 b_1 A + \lambda_2 b_2)}{\mu_{12}(1-AC)} + \frac{(I+HA)}{\mu_{12}(1-AC)}\left(\frac{\lambda_1 b_1(1-MJ)(AD+B)+\lambda_2 b_2(1-MJ)(D+BC)}{A_1}\right)\right] \quad (21)$$

$$\rho_3 = \left[\frac{\lambda_1 b_1(1-MJ)(AD+B)+\lambda_2 b_2(1-MJ)(D+BC)}{A_1}\right] \quad (22)$$

$$\rho_4 = \frac{(E+MF)}{\mu_{21}}\left[\frac{\lambda_1 b_1(AD+B)+\lambda_2 b_2(D+BC)}{A_1}\right] \quad (23)$$

$$\rho_5 = \frac{(EJ+F)}{\mu_{22}}\left[\frac{\lambda_1 b_1(AD+B)+\lambda_2 b_2(D+BC)}{A_1}\right] \quad (24)$$

Particular Cases:

(i) If feedback is not there, then

$A = p_{12}, B = p_{13}, C = p_{21}, D = p_{23}, E = p_{34} F = p_{35}, H = 0, I = 0, J = p_{45}, K = p_4, L = 0,$
$M = p_{54}, N = p_5, O = 0$

And from equations (20) to (24), utilization factors for various servers become,

$$\rho_1 = \left[\frac{(\lambda_1 b_1 + p_{21}\lambda_2 b_2)}{\mu_{11}(1-p_{12}p_{21})}\right] \quad \text{..........} (25) \qquad \rho_2 = \left[\frac{(\lambda_1 b_1 p_{12} + \lambda_2 b_2)}{\mu_{12}(1-p_{12}p_{21})}\right] \quad \text{..........} (26)$$

$$\rho_3 = \left[\frac{\lambda_1 b_1(1-p_{45}p_{54})(p_{12}p_{23}+p_{13})+\lambda_2 b_2(1-p_{45}p_{54})(p_{13}p_{21}+p_{23})}{(1-p_{45}p_{54})(1-p_{12}p_{21})}\right] \quad \text{..........} (27)$$

$$\rho_4 = \frac{(p_{34}+p_{54}p_{35})}{\mu_{21}}\left[\frac{\lambda_1 b_1(p_{12}p_{23}+p_{13})+\lambda_2 b_2(p_{23}+p_{13}p_{21})}{(1-p_{45}p_{54})(1-p_{12}p_{21})}\right] \quad \text{..........} (28)$$

$$\rho_5 = \frac{(p_{34}p_{54} + p_{35})}{\mu_{22}}\left[\frac{\lambda_1 b_1(p_{12}p_{23} + p_{13}) + \lambda_2 b_2(p_{23} + p_{13}p_{21})}{(1-p_{45}p_{54})(1-p_{12}p_{21})}\right] \text{..........} (29)$$

This result tally with the result given by [8]

(ii) If arrival is taken as single arrival and feedback is not considered, then

$A = p_{12}, B = p_{13}, C = p_{21}, D = p_{23}, E = p_{34} F = p_{35}, H = 0, I = 0, J = p_{45}, K = p_4, L = 0,$
$M = p_{54}, N = p_5, O = 0, b_1 = 1, b_2 = 1$

$$\rho_1 = \left[\frac{(\lambda_1 + p_{21}\lambda_2)}{\mu_{11}(1-p_{12}p_{21})}\right] \text{..........} (30) \qquad \rho_2 = \left[\frac{(\lambda_1 p_{12} + \lambda_2)}{\mu_{12}(1-p_{12}p_{21})}\right] \text{..........} (31)$$

$$\rho_3 = \left[\frac{\lambda_1(1-p_{45}p_{54})(p_{12}p_{23} + p_{13}) + \lambda_2(1-p_{45}p_{54})(p_{13}p_{21} + p_{23})}{(1-p_{45}p_{54})(1-p_{12}p_{21})}\right] \text{..........} (32)$$

$$\rho_4 = \frac{(p_{34} + p_{54}p_{35})}{\mu_{21}}\left[\frac{\lambda_1(p_{12}p_{23} + p_{13}) + \lambda_2(p_{23} + p_{13}p_{21})}{(1-p_{45}p_{54})(1-p_{12}p_{21})}\right] \text{..........} (33)$$

$$\rho_5 = \frac{(p_{34}p_{54} + p_{35})}{\mu_{22}}\left[\frac{\lambda_1(p_{12}p_{23} + p_{13}) + \lambda_2(p_{23} + p_{13}p_{21})}{(1-p_{45}p_{54})(1-p_{12}p_{21})}\right] \text{..........} (34)$$

The result obtained in this case tally with [5]

COMPARATIVE ANALYSIS

In this section we analysis comparatively mean queue lengths and total queue length of servers in case of

Case I: Feedback and batch arrival of customers at all servers

Case II: Without feedback and batch arrival of customers

Case III: Without feedback and single arrival of customers

For this purpose, taking numerical values as,

$p_{12} = 0.6, p_{13} = 0.4, p_{21} = 0.5, p_{23} = 0.5, p_{31} = 0.3, p_{32} = 0.3, p_{34} = 0.2, p_{35} = 0.2, p_{43} = 0.3,$
$p_4 = 0.4, p_{45} = 0.3, p_{53} = 0.3, p_5 = 0.4, p_{54} = 0.3,$
$q_{12} = 0.3, q_{13} = 0.7, q_{21} = 0.4, q_{23} = 0.6, q_{34} = 0.5, q_{35} = 0.5, q_{45} = 0.2, q_4 = 0.8, q_{54} = 0.3, q_5 = 0.7,$
$b_1 = 3, b_2 = 4, \lambda_1 = 2, \lambda_2 = 3, \mu_{11} = 30, \mu_{12} = 30, \mu_3 = 40, \mu_{21} = 20, \mu_{22} = 20$

In all cases partial queue lengths and mean queue length (L) are derived by using the formula,

$$L_i = \frac{\rho_i}{1-\rho_i} \quad \text{where i=1 to 5}$$

$$L = \sum L_i$$

Table 2: partial queue lengths and mean queue length in all the three cases

Queue Lengths	L_1	L_2	L_3	L_4	L_5	L
Case I	3.680333	13.43001	7.635579	7.038585	5.958942	37.74345
Case II	1.333178	2.888025	0.818182	0.346076	0.346076	5.731537
Case III	0.199904	0.25	0.142857	0.07689	0.07689	0.746541

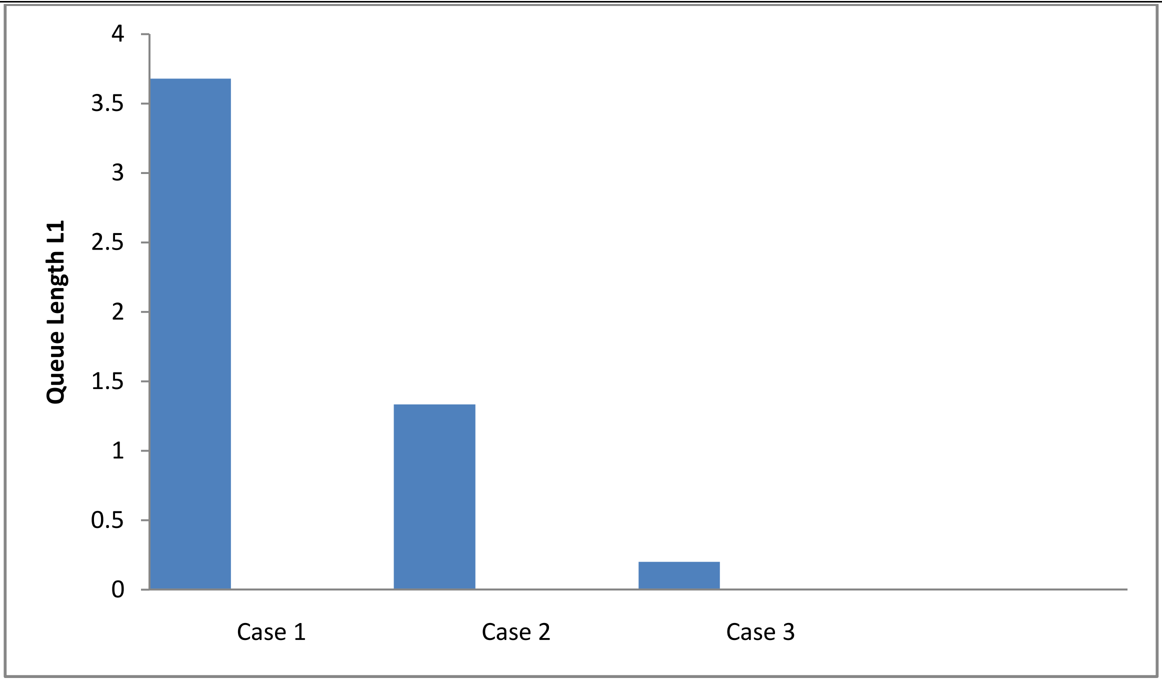

Figure 2: Partial queue lengths L_1 in all cases

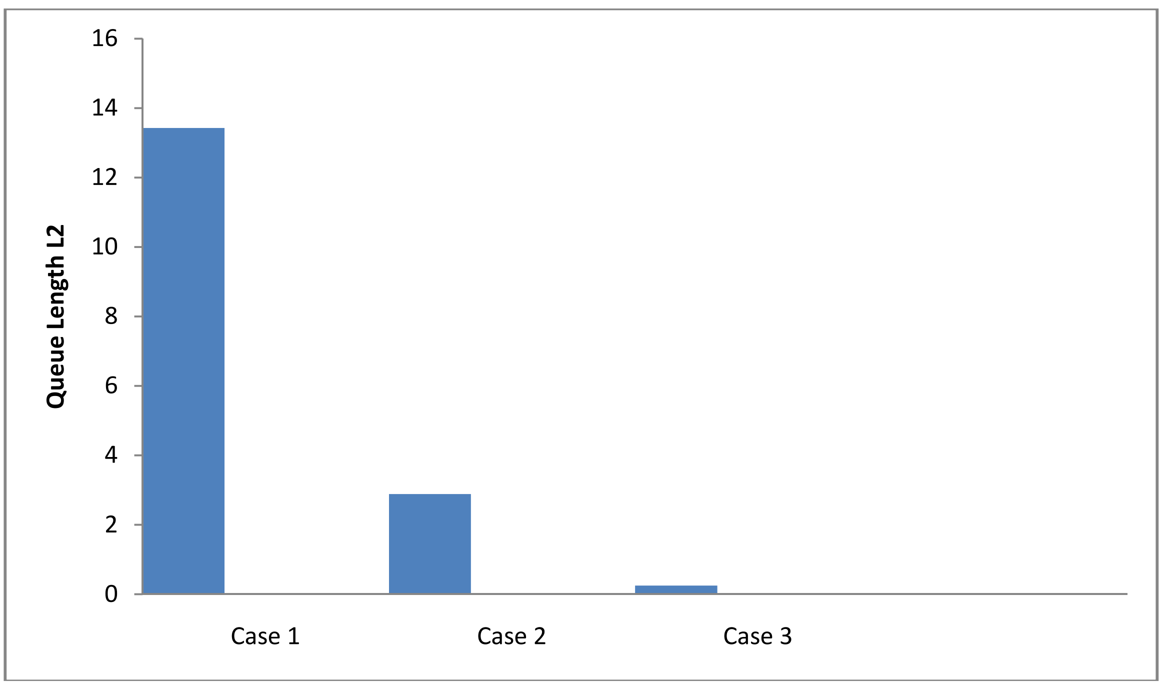

Figure 3: Partial queue lengths L_2 in all cases

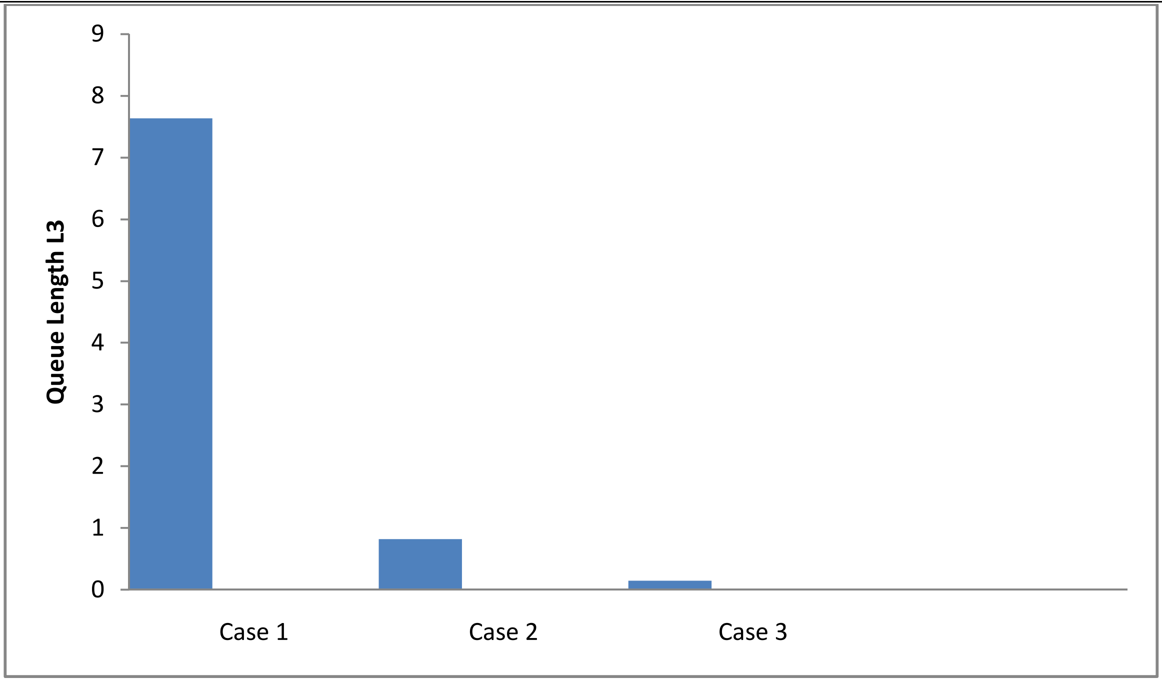

Figure 4: Partial queue lengths L_3 in all cases

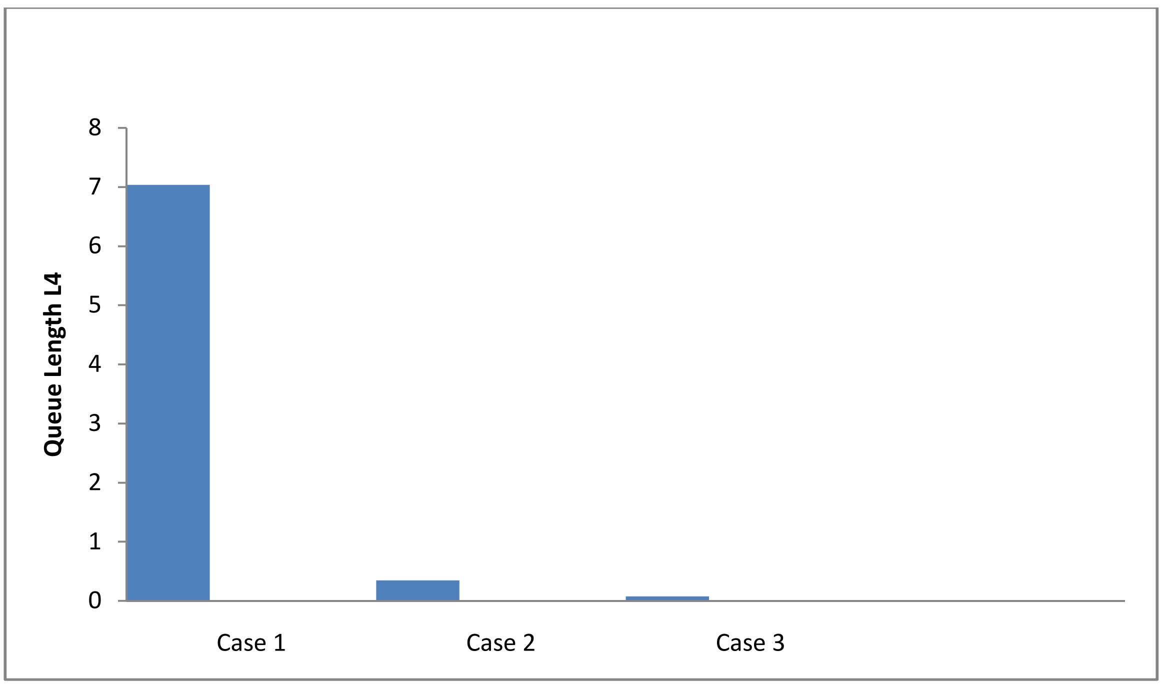

Figure 5: Partial queue lengths L_4 in all cases

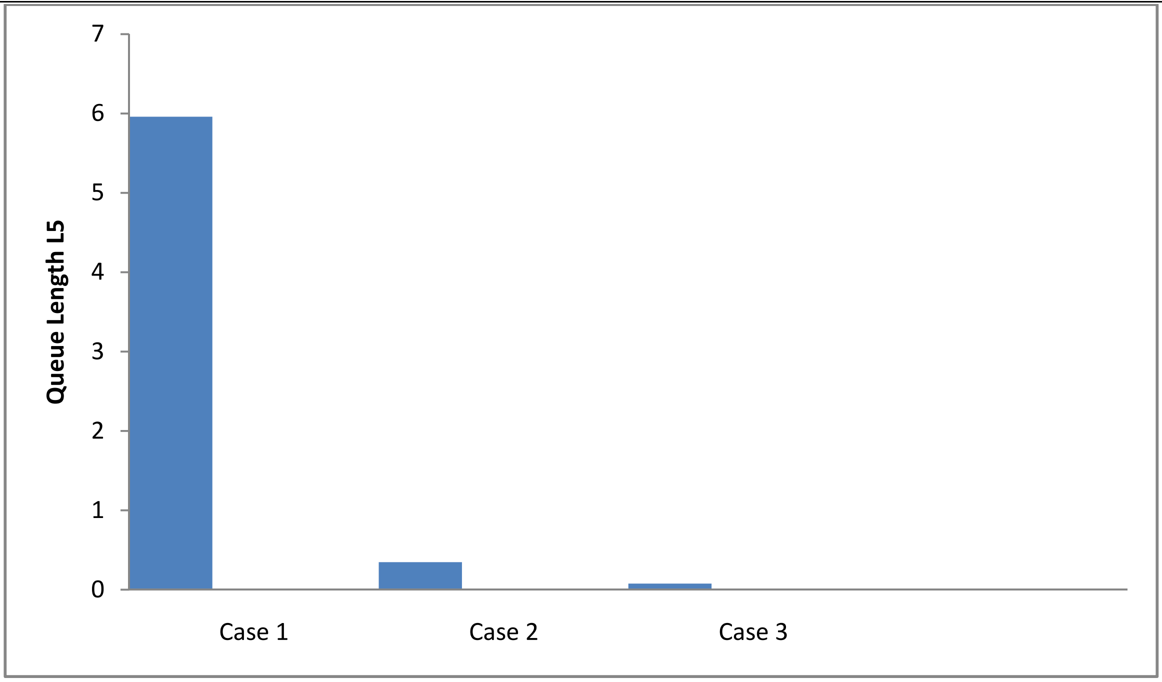

Figure 6: Partial queue lengths L_5 in all cases

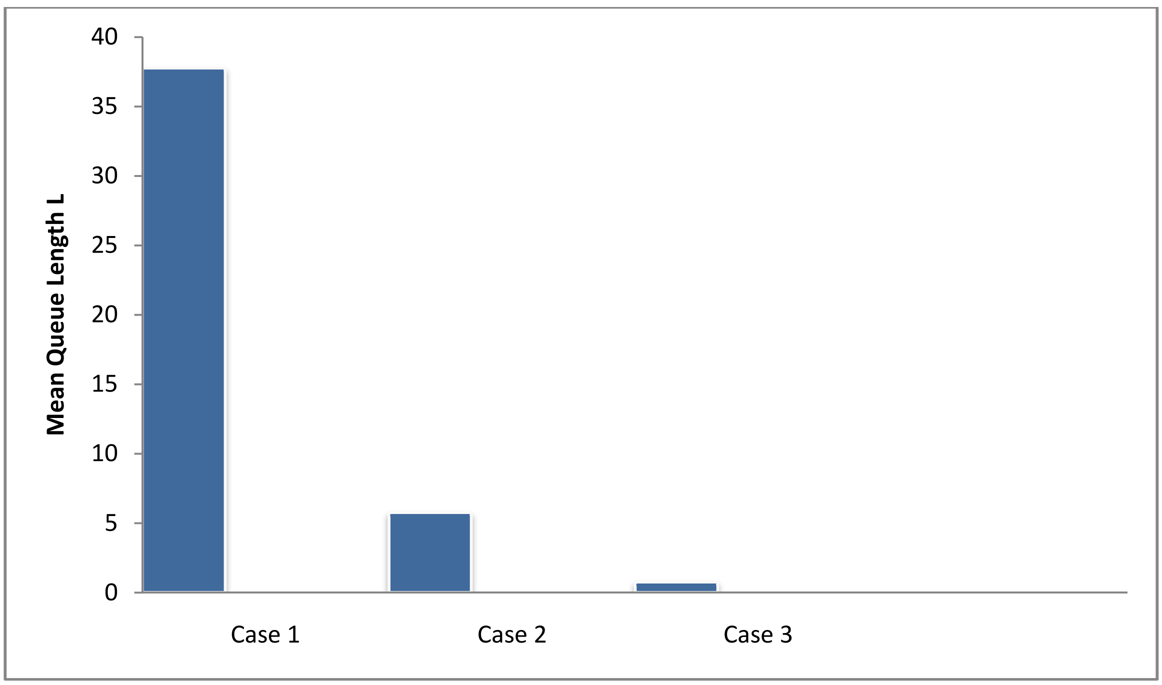

Figure 7: Mean queue lengths L in all cases

RESULT AND DISCUSSION

It is clear from table 2 and from figures 2 to 7 that partial queue lengths and total queue length of the system are increasing very fast when arrival is in batches and feedback is allowed to customers that is in 1st case. It clearly indicates the increase in sale; also if possible, service rate can be increased according to capacity. In 2nd case queue lengths are increasing at a constant speed and also more than the queue lengths as compared to 3rd case. In 3rd case that is, if feedback is not there then queue lengths are very small which indicates the loss of time and money on behalf of service provider or seller.

CONCLUSION:

In the present work we analyse a queue model with bi-serial servers in three cases; when arrival of customers is in batches and feedback is allowed to customers at all servers, 2nd is when arrival is in batches and one time visit is allowed to customers and 3rd is when arrival is single arrival and one time visit is allowed. After analysing the system we observe that in 1st case, if we give customers the facility of revisit then queue lengths are increasing at very high speed. It will increase the satisfaction level of customers also the sale of products. Also if it is not possible for a service provider to increase the service rate then the customers can be engaged in other activities by giving them some extra facilities so that they can wait for service. It can be concluded that feedback is important and arrival can be according to type of service whether it is single or batch arrival.

ACKNOWLEDGEMENT: The authors are grateful to Dr. T.P.Singh for his all time help and guidance.

Conflict of Interest: The Authors have no conflict of interest to declare.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00011.2
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

APPLICATION OF RELIABILITY MODELLING IN PERFORMANCE ANALYSIS OF DENTISTRY SYSTEM

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ABSTRACT

Our mouth is a mirror of our organs and a tool to look for any symptom or illness that may be present but not visible. For example, mouth sores or other oral problems are the initial symptoms of many systemic disorders, such as AIDS or diabetes. In reality, according to the Academy of General Dentistry, more than 90% of all systemic disorders manifest as mouth symptoms. This paper deals with a dentistry system wherein problems are classified as major and minor based on the effort and expense required for repair. A stochastic model has been created in this paper and by using the semi-Markov process and regeneration point approach, this model's performance is examined. Various conclusions are given in light of the graphical study.

Keywords : Dentistry, Diseases, Regenerative Point, Semi-Markov, Stochastic.

INTRODUCTION

A person's entire health, especially their heart health, and oral health are closely related. Neglecting oral health can have major health consequences or make pre-existing issues worse. A healthy mouth may help lower the risk of several diseases and conditions, such as diabetes, heart disease, and stroke. For our body to function correctly, our dental system needs to be kept in good condition. Many researchers including Dalal et al. [1], Raza [5] and Nguyen [9] have contributed in the area of healthcare. Several researchers have examined theoretical aspects and data analysis in relation to the operation of the dental system. Yet, there isn't much information about analysis of the dental system's survivability in the literature on reliability modeling. We are creating a model to address dental issues in order to close this gap.

ASSUMPTIONS:

1. All problems are self- announcing.
2. The single Dentist is available for treatment/diagnose.

3. The patient reaches the clinic in negligible time.
4. The treatment in the clinic starts with all medical perfections and accuracy.
5. After each treatment (repair)/ operation(replacement) the dental system is as good as new.
6. The time to failure, inspection, repair, operation distributions are exponential whereas other time distributions are general.
7. The treatment is perfect and instantaneous.

NOTATIONS:

- λ_3 Rate of occurrence of micro minor tooth problems.
- λ_4 Rate of occurrence of micro minor gum related problems.
- λ_{1T} Rate of occurrence of minor tooth problems.
- λ_{1G} Rate of occurrence of minor gum related problems.λ_{1T}
- λ_{2T} Rate of occurrence of major tooth problems.
- λ_{2G} Rate of occurrence of major gum problems.
- O Working tooth/Operating Stage.
- $G_{1T}(t)$, $g_{1T}(t)$ c.d.f./ p.d.f. of treatment time of minor tooth problem.
- $G_{1G}(t)$, $g_{1G}(t)$ c.d.f./ p.d.f. of treatment time of minor gum related problem.
- $K_3(t)$ /$k_3(t)$ c.d.f./p.d.f. of time to maintenance of tooth related micro minorproblems.
- $K_4(t)$ /$k_4(t)$ c.d.f./p.d.f. of time to maintenance of gum related micro minor problems.
- $H_{1T}(t)$ /$h_{1T}(t)$ c.d.f./p.d.f. of time to operate ofmajor tooth problems.
- $H_{1G}(t)$ /$h_{1G}(t)$ c.d.f./p.d.f. of time to operate of major gum problems.
- © /Ⓢ Laplace Convolution/ Laplace Stieltjes Convolution.
- */** Laplace transformation /Laplace Stieltjes transformation
- $q_{ij}(t)/Q_{ij}(t)$ p.d. f/c.d.f for the transformation of the system from one regenerative stage S_i to another stage S_j or to a failed stage S_j

MODEL DESCRIPTION: Different stages of the Dental system models according to Semi Markov process and Regenerative Point Technique are as follows:

Stage 0: Initial working stage.

Stage 1: Tooth working but have some (tooth related) micro problems.

Stage 2: Tooth working but have some (gum related) micro problems.

Stage 3: Tooth is working with pain due to some minor tooth related problem.

Stage 4: Tooth is working with pain due to some minor gum related problem.

Stage 5: Tooth is not working (severe pain) due to some major tooth problems after stage 3 and tooth undergoes for operation. After operation the dental system works efficiently.

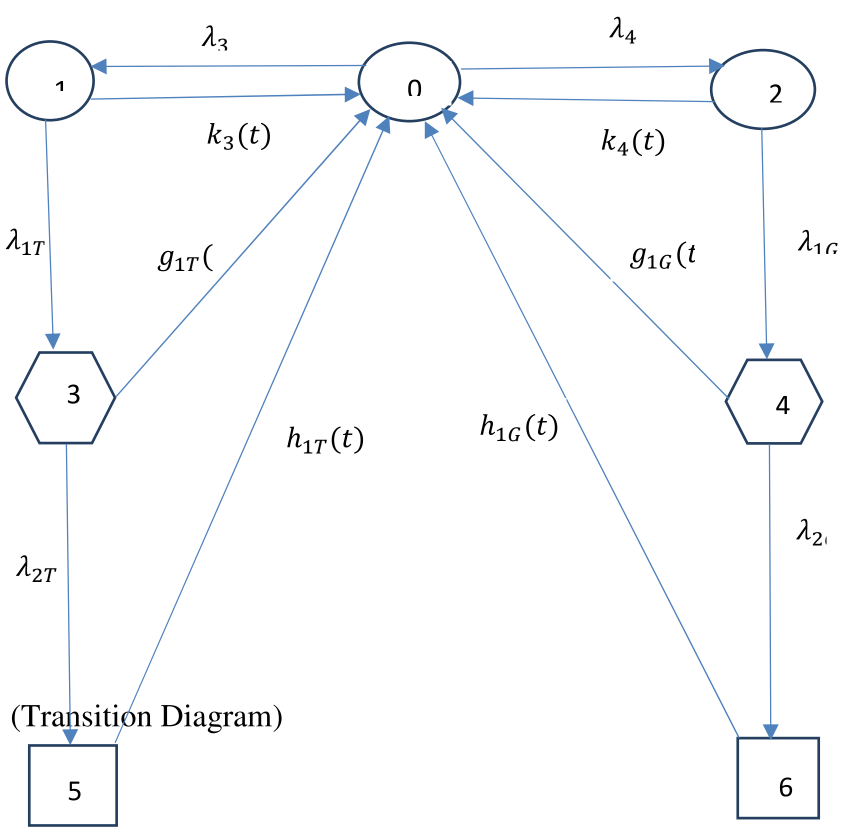
Stage6: Tooth is not working (severe pain) due to some major problems after stage 4 and tooth undergoes for operation. After operation the dental system works efficiently.

Here, Stages 0,1,2 are working properly whereas stages 3 and 4 are partially working (having some problem), 5 and 6 are failed stages.

MEASURES OF SYSTEM EFFECTIVENESS:

Using Semi Markov Process and Regenerative Point Technique, following measures of dental system effectiveness are obtained:

- Transition Probabilities.
- Mean Sojourn Time.
- Mean time to system failure.
- Expected survival time with full capacity.
- Expected survival time with reduced capacity.
- Busy period of a Dentist (treatment time only).
- Busy period of a Dentist (Operation time only).



(Transition Diagram)

TRANSITION PROBABILITIES ARE :

$dQ_{01}=\lambda_3 e^{-(\lambda_3+\lambda_4)}dt$ $dQ_{02}=\lambda_4 e^{-(\lambda_3+\lambda_4)}dt$ $dQ_{10}= e^{-\lambda_{1T}t}k_1$(t)dt

$dQ_{13}=\lambda_{1T}\, e^{-(\lambda_{1T})t}\overline{K_1(t)}dt$ $dQ_{20}= e^{-\lambda_{1G}t}k_2$(t) dt $dQ_{30=e^{-(\lambda_{2T})t}}g_{1T}$

(t) dt $dQ_{35}=\lambda_{2T}\, e^{-(\lambda_{2T})t}\overline{G_{1T}(t)}dt$ $dQ_{50=}h_{1T}$(t) dt$dQ_{24}=\lambda_{1G}\, e^{-(\lambda_{1G})t}\overline{K_2(t)}dt$

$dQ_{40=e^{-(\lambda_{2G})t}}\, g_{1G}$(t) dt $dQ_{46}=\lambda_{2G}\, e^{-(\lambda_{2G})t}\overline{G_{1G}(t)}dt$ $dQ_{60=}h_{1G}$(t) dt

Taking L. S. T$Q_{ij}^{**}(s)$ and p_{ij}=$\lim_{n\to\infty} Q_{ij}{}^{**}$(s), the non- zero $p_{ij}$are obtained as under

p_{01}=$\frac{\lambda_3}{\lambda_3+\lambda_4}$ p_{02}=$\frac{\lambda_4}{\lambda_3+\lambda_4}$ p_{10}=$k_1^*(\lambda_{1T})$

p_{20}=$k_2^*(\lambda_{1G})$$p_{13}$=1-$k_1^*(\lambda_{1T})$ p_{30}=$g_{1T}^*(\lambda_{2T})$

p_{35}=$1-g_{1T}^*(\lambda_{2T})$ p_{50}=$h_{1T}^*(0)$ p_{24}=$1-k_2^*(\lambda_{1G})$

p_{40}=$g_{1G}^*(\lambda_{2G})$ p_{46}=$1-g_{1G}^*(\lambda_{2G})$ p_{60}=$h_{1G}^*(0)$

Also, $p_{01}+p_{02}$=1 $p_{13}+p_{10}$=1 p_{20}+p_{24}=1 $p_{30}+p_{35}$=1

$p_{40}+p_{46}$=1 p_{50}=p_{60} =1

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into the state i, is mathematically stated as

m_{ij}=$\int_0^\infty tdQ_{ij}$(t) = $-q_{ij}*'(0)$

$m_{01}=\frac{\lambda_3}{(\lambda_3+\lambda_4)^2}$ $m_{02}=\frac{\lambda_4}{(\lambda_3+\lambda_4)^2}$ $m_{10}=-k_1{}^{*'}(\lambda_{1T})$

$m_{13}=\frac{1}{\lambda_{1T}}+k_1{}^{*'}(\lambda_{1T})-\frac{k_1{}^*(\lambda_{1T})}{\lambda_{1T}}$ $m_{20}=-k_2{}^{*'}(\lambda_{1G})$

$m_{24}=\frac{1}{\lambda_{1G}}+k_2{}^{*'}(\lambda_{1G})-\frac{k_2{}^*(\lambda_{1G})}{\lambda_{1G}}$ $m_{35}=\frac{1}{\lambda_{2T}}+g_{1T}{}^{*'}(\lambda_{1T})-\frac{g_{1T}{}^*(\lambda_{2T})}{\lambda_{2T}}$

$m_{30}=-g_{1T}{}^{*'}(\lambda_{2T})$ m_{40}=$-g_{1G}*'(\lambda_{2G})$ m_{50}= $-h_{1T}{}^{*'}(0)$

$m_{46}=\frac{1}{\lambda_{2G}}+g_{1G}{}^{*'}(\lambda_{2G})-\frac{g_{1G}{}^*(\lambda_{2G})}{\lambda_{2G}}$ m_{60}= $-h_{1G}{}^{*'}(0)$

The mean sojourn time in the regenerative state (μ_i)is defined as the time of stay in that state before transition to any other state, then we have

$\mu_0=\frac{1}{\lambda_3+\lambda_4}$ μ_1= $\frac{1-k_1{}^*(\lambda_{1T})}{\lambda_{1T}}$ μ_2= $\frac{1-k_2{}^*(\lambda_{1G})}{\lambda_{1G}}$

μ_3= $\frac{1-g_{1T}{}^*(\lambda_{2T})}{\lambda_{2T}}$ μ_4= $\frac{1-g_{1G}{}^*(\lambda_{2G})}{\lambda_{2G}}$

μ_5=$-h_{1T}*'(0)$ μ_6=$-h_{1G}*'(0)$

Thus, we see that

μ_0= m_{01}+ m_{02} μ_1= m_{10}+ m_{13} μ_2= m_{20}+ m_{24}

μ_3= m_{30}+ m_{35} μ_4=m_{40}+ m_{46} μ_5= m_{50} μ_6= m_{60}

OTHERS MEASURES OF SYSTEM EFFECTIVENESS:

Using probabilistic argument for regenerative processes, various recursive relations are obtained and solved by Matrix method to find different measures of system effectiveness. Which are as follows.

Mean Time to System Failure$(T_0) = \frac{N(S)}{D(S)}$

Where N(S) = $\mu_0+p_{01}\mu_1+p_{02}\mu_2+p_{01}p_{13}\mu_3+p_{02}p_{24}\mu_4$

And D(S) = $1-p_{01}p_{10}-p_{02}p_{20}-p_{01}p_{13}p_{30}-p_{02}p_{24}p_{40}$

Expected Survival Time with Full Capacity UT_0 (t)=$\frac{N_1(s)}{D_1(s)}$

Expected survivaltime with reduced capacity (DT_0 (t)) = $\frac{N_2(s)}{D_1(s)}$

Busy period of a Dentist (Repair time only) (BR_0(t)) = $\frac{N_3(s)}{D_1(s)}$

Busy period of a Dentist (Operate time only) (BO_0(t)) = $\frac{N_4(s)}{D_1(s)}$

Where $N_1 = \mu_0+p_{01}\mu_3+p_{02}\mu_4$

$N_2 = p_{01}p_{13}\mu_3+p_{02}p_{24}\mu_4$

$N_3 = p_{01}p_{13}\mu_3+p_{02}p_{24}\mu_4$

$N_4 = (p_{01}p_{13}p_{35}\mu_5+p_{24}p_{46}\mu_6)$

And $D_1= \mu_0+p_{01}\mu_1+p_{02}\mu_2+p_{01}p_{13}\mu_3+p_{02}p_{24}\mu_4+p_{01}p_{13}p_{35}\mu_5+p_{02}p_{24}p_{46}\mu_6$

NUMERICAL STUDY AND GRAPHICAL ANALYSIS:

Giving some particular values to the parameters and considering

$g_1(t)=\alpha_1 e^{-\alpha_1 t}$, $g_2(t)=\alpha_2 e^{-\alpha_2 t}$,$h_1(t) = \gamma e^{-\gamma t}$, λ_1=0.141, λ_2= 0.095, $\alpha_1 = 0.968$,$\alpha_2 = 0.591$,γ = 0.063

Mean time to System Failure=10.52632

Expected survival time with full capacity =0.64069

Expected survival time with reduced capacity=0.084989

Busy period of a dentist (Inspection time only) =0.069633

Busy period of a dentist (repair time only) =0.192372

Busy period of a dentist (operate time only) =0.0984907

Using above numerical values, various graphs has been developed. The interpretation and conclusion from the graphs are as under:

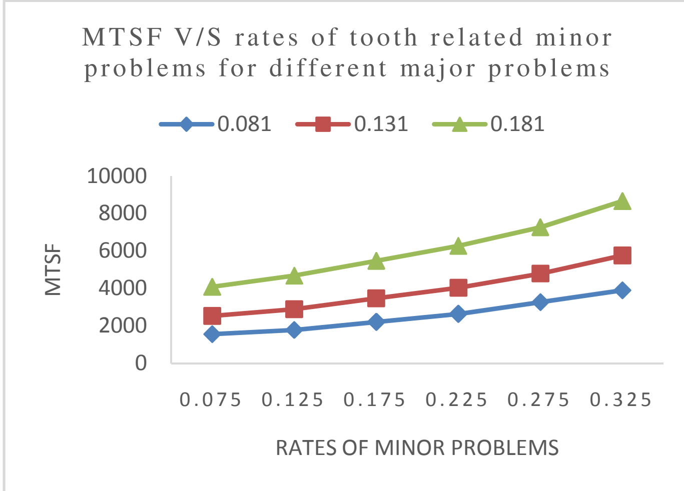


Fig. 1

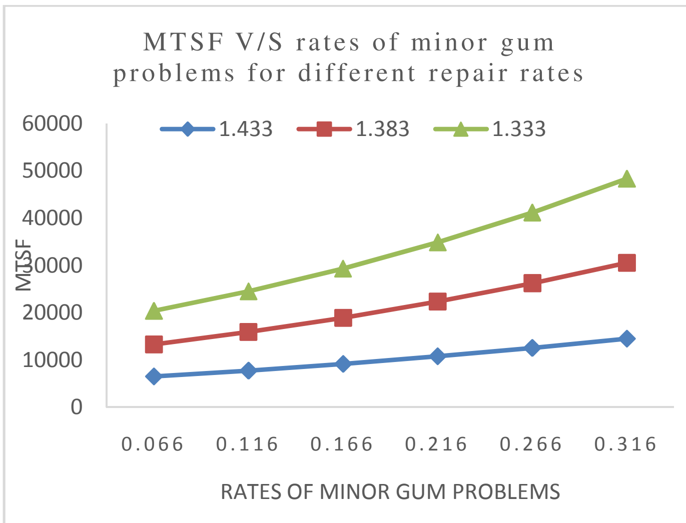


Fig. 2

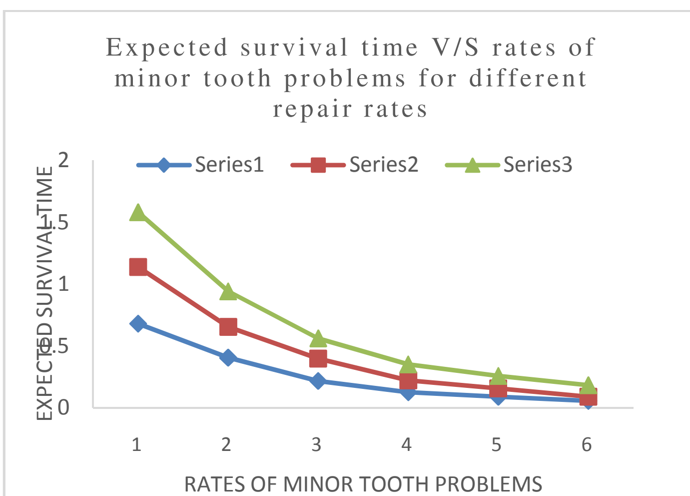


Fig. 3

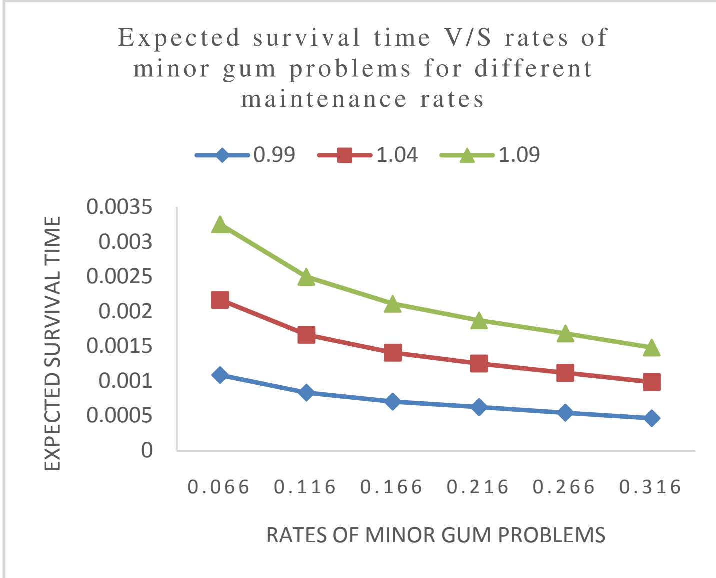


Fig. 4

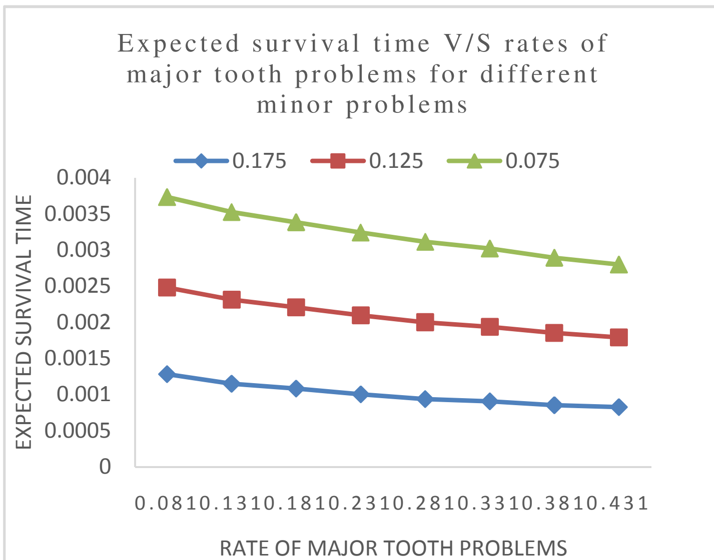


Fig. 5

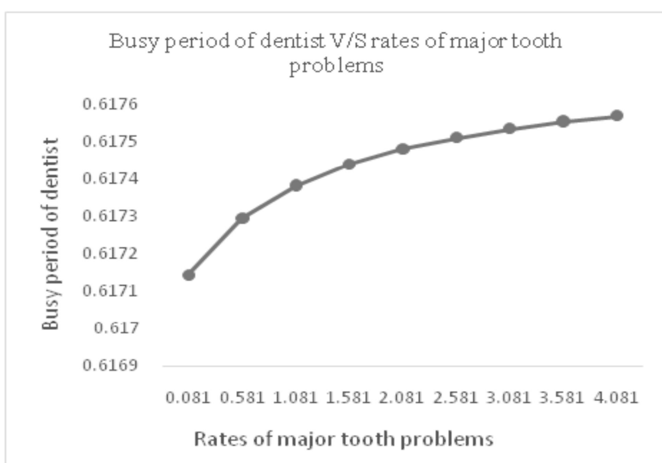


Fig. 6

Following interpretations and conclusion are drawn from fig.1 to fig. 6: -

1. Figure 1 shows that 'mean time to system failure' increases with the increase in minor problems. It also increases as we increase rates of major problems.
2. Figure 2 represents that 'mean time to system failure' increases with the increase in minor gum problems. It decreases as we increase repair rates.
3. In Figure 3 'Expected survival time' decreases with the increase in minor tooth problems. Also, expected survival time increases as we increase repair rates.
4. Figure 4 declared that 'Expected survival time' decreases with the increase in minor gum problems. Also, it increases as we increase maintenance rates.
5. Figure 5 reveals that 'Expected survival time' declines with the increase in major tooth problems. Also, it decreases as we increase in rates of minor problems.
6. Figure 6 indicates that busy period of dentist increases with the increase in major tooth problems.

CONCLUSION:

From the analysis of above graphs, expected survival time decreases with the increase in major problems also, busy period of a dentist increases with the increases in problems. Hence, as a result, while undergoing routine cleanings and checkups might seem like an unnecessary investment, missing them can result in more expensive operations. For instance, if a small, affordable cavity is not treated, it may spread and require an expensive root canal or cap. So, we should care at the earlier stage of the problem and go for regular check-ups. Poor oral health issues like cavities, gingivitis, bad breath, tooth decay, or periodontal disease can all be avoided with proper dental care.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00012.4
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

SPECIAL DIOPHANTINE TRIPLES INVOLVING OCTAGONAL PYRAMIDAL NUMBERS

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ABSTRACT

In this paper, we strive for three particular polynomials with integer coefficients to such an extent that the product of any two numbers added by a non-zero number (or polynomials with integer coefficients) is a perfect square.

KEYWORDS: Triples, Diophantine triples, octagonal pyramidal number, triples, perfect square, pyramidal number.

NOTATION:

$p_n^8 = \frac{n(n+1)}{6}[6n-3] =$ octagonal pyramidal number of rank n

1. INTRODUCTION:

In number theory, a Diophantine equation is a polynomial equation with two or more unknowns that solely considers or searches for integer solutions [1-4]. The term "Diophantine" relates to the Greek mathematician Diophantus of Alexandria who, in the third century, pioneered the introduction of symbolism to variable-based mathematics and studied related problems. Numerous mathematicians have investigated the issue of the occurrence of Dio-triples and quadruples with the property D(n) for any integer n as well as for any linear polynomial in n [5-8]. In this case, [9-16] provides a full study of the many challenges on Diophantine triples. Triple sequences with a half buddy were examined in [17].

Our search for Diophantine triples utilizing octagonal pyramidal numbers was prompted by these findings. This study tries to build Dio-Triples that satisfy the necessary property when the product of any two of the triple's members plus a non-zero integer or a polynomial with integer coefficients is added. The Diophantine triples from octagonal pyramidal numbers of various ranks are also presented in each of the three parts along with the relevant features.

2. BASIC DEFINITION:

A set of three distinct polynomials with integer coefficients (a_1, a_2, a_3) is said to be Special Diophantine triple with property $D(n)$ if $a_i * a_j + a_i + a_j + n$ is a perfect square for all$1 \le i < j \le 3$, where n may be non zero or polynomial with integer coefficients.

3. METHOD OF ANALYSIS:

Construction of the Special Diophantine triples involving octagonal pyramidal numbers:

Section – A:

Let $a = 6P_n^8$ and $b = 6P_{n-1}^8$

$a = 6n^3 + 3n^2 - 3n$ and $b = 6n^3 - 15n^2 + 9n$

ab = $(6n^3 + 3n^2 - 3n)(6n^3 - 15n^2 + 9n)$

$ab = 36n^6 - 90n^5 + 54n^4 + 18n^5 - 45n^4 + 27n^3 - 18n^4 - 45n^3 - 27n^2.$

$ab + (a + b) = 36n^6 - 72n^5 - 12n^4 + 84n^3 - 20n^2 - 24n + 9$

$ab + (a + b) + (-3n^4 + 19n^2 - 30n + 9) = (6n^3 - 6n^2 - 4n + 3)^2$

$$ab + (a + b) + (-3n^4 + 19n^2 - 30n + 9) = \alpha^2 \qquad (3.1)$$

Equation (3.1) is a perfect square.

$$bc+(b+c)+ (-3n^4 + 19n^2 - 30n + 9) = \beta^2 \qquad (3.2)$$

$$ac+(a+c)+ (-3n^4 + 19n^2 - 30n + 9) = \gamma^2 \qquad (3.3)$$

Solving (3.2) and (3.3)

$$c(a-b)+(a-b) (-3n^4 + 19n^2 - 30n + 9) = a\beta^2 - b\gamma^2 \qquad (3.4)$$

The equation (3.3) and (3.2)

$$c(a-b)+(a-b) = \gamma^2-\beta^2 \qquad (3.5)$$

setting $\beta = x + (b + 1)y$ and $\gamma = x + (a + 1)y$

$$x^2 = (ab + a + b + 1)y^2 - 3n^4 + 19n^2 - 30n + 9$$

If $y_0 = 1$,

$x^2 = (ab + a + b + 1) - 3n^4 + 19n^2 - 30n + 9$

$x^2 = (6n^3 - 6n^2 - 4n + 3)^2$

$x = (6n^3 - 6n^2 - 4n + 3)$

$\beta = x + (b + 1)y = 12n^3 - 21n^2 + 5n + 4$

$bc+(b+c)+ (-3n^4 + 19n^2 - 30n + 9) = \beta^2$

c = $24n^3 - 24n^2 - 2n + 7$

c = $4(6P_n^8) - 36n^2 + 10n + 7$

Therefore, the triples

(a, b, c) = $\{4(6P_n^8) - 36n^2 + 10n + 7\}$ is a Diophantine triple with the property $D(-3n^4 + 19n^2 - 30n + 9)$

Some numerical examples are given below in the following table.

TABLE 3.1

n	Diophantine Triples	$D(-3n^4 + 19n^2 - 30n + 9)$
0	(0,0,7)	9
1	(6,0,5)	-5
2	(54,6,99)	-23
3	(180,54,433)	-153
4	(420,180,1151)	-575

SECTION - B:

Let a = $6P_n^8$ and b = $6P_{n-2}^8$ be octagonal pyramidal numbers of rank n and n-2 respectively.

a = $6P_n^8$ and b = $6P_{n-2}^8$

a = $6n^3 + 3n^2 - 3n$ b = $6n^3 - 33n^2 + 57n - 30$

ab = $36n^6 - 180n^5 + 225n^4 + 90n^3 - 261n^2 + 90n$

$$ab + (a + b) + (6n^3 + 21n^2 - 144n + 111) = (6n^3 - 15n^2 + 9)^2$$

$$ab + (a + b) + (6n^3 + 21n^2 - 144n + 111) = \alpha^2 \quad (3.6)$$

Equation (3.6) is perfect square.

$$bc + (b + c) + (6n^3 + 21n^2 - 144n + 111) = \beta^2 \quad (3.7)$$

$$ac + (a + c) + (6n^3 + 21n^2 - 144n + 111) = \gamma^2 \quad (3.8)$$

Solving (3.7) and (3.8)

$$c(a-b)+(a-b)\,(6n^3 + 21n^2 - 144n + 111) = a\beta^2 - b\gamma^2 \quad (3.9)$$

The equation (3.8) and (3.7)

$$c(a-b)+(a-b) = \gamma^2-\beta^2 \quad (3.10)$$

setting $\beta = x + (b + 1)y$ and $\gamma = x + (a + 1)y$

$$x^2 = (ab + a + b + 1)y^2 - 6n^3 + 21n^2 - 144n + 111$$

If $y_0 = 1$,

$$x^2 = (ab + a + b + 1) - 6n^3 + 21n^2 - 144n + 111$$

$$x^2 = (6n^3 - 15n^2 + 9)^2$$

$$x = (6n^3 - 15n^2 + 9)$$

$\beta = x + (b + 1)y = 12n^3 - 48n^2 + 57n - 20$

bc+(b+c)+ $(6n^3 + 21n^2 - 144n + 111) = \beta^2$

c = $24n^3 - 60n^2 + 54n - 11$

c = $4(6P_n^8) - 72n^2 + 66n - 11$

Therefore, the triples

(a, b, c) = $\{4(6P_n^8) - 72n^2 + 66n - 11\}$ is a Diophantine triple with the property

D$(6n^3 + 21n^2 - 144n + 111)$.

Some numerical examples are given below in the following table.

TABLE 3.2

n	Diophantine Triples	D$(6n^3 + 21n^2 - 144n + 111)$
0	(0,-30,-11)	111
1	(6,0,7)	-6
2	(54,0,49)	-45
3	(180,6,259)	30
4	(420,54,781)	255

SECTION –C:

Let a = $6P_n^8$ and b = $6P_{n-3}^8$ be octagonal pyramidal numbers of rank n and n-3 respectively.

a = $6P_n^8$ and b = $6P_{n-3}^8$

a = $6n^3 + 3n^2 - 3n$ b = $6n^3 - 51n^2 + 141n - 126$

ab = $36n^6 - 288n^5 + 675n^4 - 180n^3 - 801n^2 + 378n$

$$ab + (a + b) + (9n^4 - 126n^2 - 120n + 610)$$

$$= (6n^3 - 24n^2 + 9n + 22)^2$$

$$ab + (a + b) + (9n^4 - 126n^2 - 120n + 610) = \alpha^2 \qquad (3.6C)$$

Equation (3.6C) is perfect square.

$$bc + (b + c) + (9n^4 - 126n^2 - 120n + 610) = \beta^2 \qquad (3.7C)$$

$$ac + (a + c) + (9n^4 - 126n^2 - 120n + 610) = \gamma^2 \qquad (3.8C)$$

Solving (3.7C) and (3.8C)

$$c(a-b)+(a-b)\,(9n^4 - 126n^2 - 120n + 610) = a\beta^2 - b\gamma^2 \qquad (3.9C)$$

The equation (3.8C) and (3.7C)

$$c(a-b)+(a-b) = \gamma^2 - \beta^2 \qquad (3.10C)$$

setting β = $x + (b + 1)y$ and γ = $x + (a + 1)y$

$$x^2 = (ab + a + b + 1)y^2 - (9n^4 - 126n^2 - 120n + 610)$$

If $y_0 = 1$,

$$x^2 = (ab + a + b + 1) - (9n^4 - 126n^2 - 120n + 610)$$

$$x^2 = (6n^3 - 24n^2 + 9n + 22)^2$$

$$x = (6n^3 - 24n^2 + 9n + 22)$$

$\beta = x + (b + 1)y$

$= 6n^3 - 24n^2 + 9n + 22 + 6n^3 - 51n^2 + 141n - 126 + 1$

bc+(b+c)+ $(9n^4 - 126n^2 - 120n + 610) = \beta^2$

c = $24n^3 - 96n^2 + 156n - 81$

c = $4(6P_n^8) - 108n^2 + 168n - 81$

Therefore, the triples

(a, b, c) = $\{4(6P_n^8) - 108n^2 + 168n - 81\}$ is a Diophantine triple with the property D$(6n^3 - 24n^2 + 9n + 22)$

Some numerical examples are given below in the following table.

TABLE 3.3

n	Diophantine Triples	D$(6n^3 - 24n^2 + 9n + 22)$
0	(0,-126,-84)	22
1	(6,-30,0)	13
2	(54,0,36)	-8
3	(180,0,168)	-5
4	(420,6,540)	226

4. CONCLUSION:

We find special Diophantine triples for octagonal pyramidal numbers. To conclude one may look for triples or quadruples for different numbers with their relating properties.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00013.6
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

AN INTRODUCTION TO PLUS WEIGHTED DENDROLANGUAGE AND ITS PROPERTIES

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ABSTRACT

The most important work of this paper is to introduce plus weighted Dendrolanguage generating system and to discuss normal form of plus weighted context-free Dendrolanguage producing system.

Keywords : Trees and Pseudoterms, Plus Weighted Generating Systems, Normal of P-CFDS.

1 INTRODUCTION

Weighted context free grammars and weighted finite automata were initially introduced in significant articles by Marcel-Paul Schutzenberger (1961) and Noam Chomsky (1963), respectively. Weighted finite automata are standard nondeterministic finite automata in which the transitions have weights. We consider the following scenarios to demonstrate the variation of weighted finite automata. we may determine the wide range of a word by counting the number of trees that can be used to represent it as follows; Let each transition have a weight of 1, and for a tree that is taken again, the sum of the weights of its successful tree. The wide range of a word equals the sum of its successful trees. Applications for weighted automata are numerous. weighted automata and their accompanying algorithm are developed by contemporary spoken-dialog or handheld speech recognition systems to express their concepts and promote successful combination and search [1,7].

If there is connection between context-free grammars and grammars of natural languages, it is undoubtedly, as Chomsky proposes, through some stronger concept like that of transformational grammar. In this framework, it is not the context-free language itself that is of interest, but, rather, the set of derivation trees, i.e., the structural descriptions of markers. From the viewpoint of the syntax directed description of fuzzy meanings, sets of trees rather than the sets of strings are of prime importance.

A plus weighted automata [8,9,10,11, 12,13] is an automata that deals with plus weights up to infinity. Many algebraic structures of plus weighted automata has been discussed in [8,9,10]. Thus we are motivated to study systems to manipulate plus weighted dendrolanguage generating system which is the generalization of fuzzy Context Free Dendrolanguage generating System. Plus weighted Dendrolanguage generating System can also be extended to max weighted automata cited as [2,3,4,5,6]. This work can be further extended in the field of graph theory [13,14,15,16,17].

This paper comprises of 6 sections including this section phase 2 offers some fundamental ideas which are needed for the succeeding section. Section 3 use the records about trees and Pseudo terms. Section 4 offers with Plus Weighted Dendrolanguage Generating System. Section 5 gives the Normal Form of P-CFDS

2. PRELIMINARIES

In this section we review some basic notations and definitions about grammar and its types.

Definition 2.1

A phrase-structure grammar or grammar is a four tuple G = <V_N, V_N, S, P> Where,

V_N is a set of non-terminal symbols, V_T is a set of terminal symbols called alphabets, S is a special element of V_N and is called the starting symbol, P is the production. Relation on $(V_T \cup V_N)^*$, the set of strings of elements of terminal and non-terminal.

Types of grammar

1. **Type 0 or unrestricted grammar**:

A grammar in which there are no restrictions on its productions.

2. **Type 1 or context sensitive grammar:**

Grammar that contains only productions of the form $\alpha \to \beta$ where $|\alpha| = |\beta|$ and $\alpha \in V_N$.

Example 2.2

Find context sensitive language. G=<{S,B,C},{a,b,c},S,P> be a grammar where

P consists S→aSBc, S→aBc, cB→BC, aB→ab, B→bb, bc→bc, cC→cC

S $\Rightarrow$ aSB $\Rightarrow$ aaBcBc $\Rightarrow$ aaBBCc $\Rightarrow$ aabBCc $\Rightarrow$ aabbbCc $\Rightarrow$ aabbbcC $\Rightarrow$ aabbbcc

3. Type 2 or context free grammar.

Grammar that contains only productions of the form $\alpha \to \beta$ where $|\alpha| = |\beta|$ and $\alpha \in V_N$.

Example 2.3

< V_N, V_T, S, P> where V_N={S}, V_T = {a, b} and P consists of productions S→a, S→Sa, S→b, S→bS.

Then show that L (G) = {b^m, a^n /m, n≥0}.

S $\Rightarrow$ Sa $\Rightarrow$ bSa $\Rightarrow$ bSaa $\Rightarrow$ bbSaa $\Rightarrow$ bbaaa $\Rightarrow$ b^2 a^3

Therefore, L(G)={b^m,a^n /m,n≥0}.

4. Type 3 or regular grammar.

Grammar that contains productions of the form $\alpha \to \beta$ where $|\alpha| \leq \beta, \alpha \in \mathcal{V}_N$ and β is of the form a or aB where $a \in \mathcal{V}_T$ and $\beta \in \mathcal{V}_N$.

Example 2.4

G=<{S, B, C},{a,b,c},S,P> Where P consists of the productions S→aB, A→aB, B→bA, B→b. Show that L (G) = {$(ab)^n$ /n≥1}.

S $\Rightarrow$ aB $\Rightarrow$ abA $\Rightarrow$ abaB $\Rightarrow$ abab $\Rightarrow a^2b^2 \Rightarrow$ $(ab)^2$

Therefore, L (G) = {$(ab)^n$ /n≥1}.

3 TREES AND PSEUDOTERMS

As before, Let ℕ be the set of natural numbers and $\mathbb{N}^*$ be the set of all strings on ℕ including the null string (∧). A finite closed subset U of $\mathbb{N}^*$ is called a finite tree domain if the following conditions hold:

(1) q ∈U and q=u v implies u∈U, where u, v, q∈ $\mathbb{N}^*$.

(2) q $k \in U$ and j≥ k implies qj∈ U, where q∈ $\mathbb{N}^*$, jk∈ℕ.

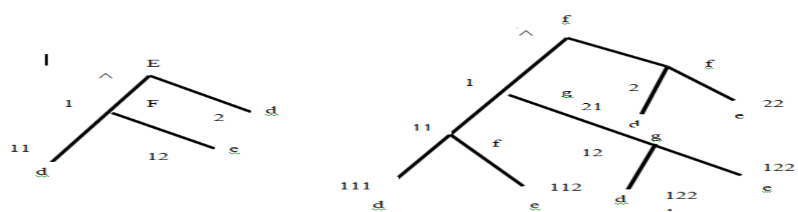
Let U be a finite tree domain. Then the subset $\overline{U} = \{q\in U | q.1 \notin U\}$ is called the leaf node set. A pair (N: T) of finite alphabets N and T, where $N \cap T = \emptyset$ is called a partially ranked alphabet. A tree t on a partially ranked alphabet (N: T) is a function from a finite tree domain U into $N \cup T$, written t: U→ $(N: T)$, such that

t (q)∈N for q∈ $U/\overline{U}$

t (q)∈ T for q∈ $\overline{U}$

Of course, a finite tree t: $U \to (N: T)$ can be represented by a finite set of pairs (q,t(q)), ie) $\{(q,t(q)) \mid q \in U\}$.

Trees on (N: T) can be represented graphically by constructing a rooted tree (where the successors of each node are ordered), representing the domain of the mapping, and labeling the nodes with elements of $N \cup T$, representing the values of the function. Thus in the following figures there are two examples; as a mapping, the left-hand tree has the domain $U = \{\wedge,1,2,11,12\}$ and the value at 11 is a note also that $\overline{U}$ $= \{2,11,12\}$.



The definition of a tree and the corresponding pictorial representation provide a good basis for intuition for considering tree manipulating systems.

However, the development of the theory is simpler if the familiar linear representation of such trees is considered. Thus, we define the set $D^p_{(N:T)}$ of Pseudo terms on $N \cup T$ as the smallest subset $[N \cup T \cup \{(,)\}]^*$ satisfying the following conditions:

1) $T \subset D^p_{(N;T)}$

2) If $n > 0$ and $B \in N$ and $b_1, b_2 \ldots b_n \in D^p_{(N;T)}$ then $B(b_1, b_2, \ldots b_n \in D^p_{(N;T)}$

(We note that parentheses are not symbols of $N \cup T$)

We consider trees and Pseudo terms to be equivalent formalizations. The translation between the two is the usual one. By way of examples; the trees of the above figure correspond to the following Pseudo terms; $E(F(de)d), f(g(f(de))f(de)$.

This correspondence call be more precise in the following manner:

1) If a Pseudo terms $b^p \in D^p_{(N:T)}$ is atomic (i.e) $b^p = a \in T$, then the corresponding tree b has domain $\{\wedge\}$ and $b\{\wedge\} = a$.

2) If $b^p = B(b_1^p, \ldots\ldots, b_m^p)$, then b has domain $U_i \le j\{iq \mid q \in \text{domain} \quad (b_i)\} \cup \{\wedge\}, b(n) = B,$ and for q=iq' in the domain of b, $b(q) = b_i(q')$.

A plus weighted set T of trees is defined by a membership function $\gamma_T : D_{(N:T)} \to [0,\infty]$ the set of all plus set of trees is denoted by $F[D_{(N:T)}]$.

4. PLUS WEIGHTED DENDROLANGUAGE GENERATING SYSTEM

We present a plus weighted system then generate plus sets of trees, as an extension of the Dendrolanguage generating system

Definition 4.1

A Plus Weighted Context Free Dendrolanguage Generating System (P-CFDS) is 6-tuple S= (N_0, N, T, W, P, λ_0) such that the following conditions hold:

1) N_0 is a finite set of symbols whose elements are called non terminal node symbols.
2) N is a finite set of symbols whose elements are called node symbols.
3) T is a finite set of symbols whose elements are called leaf symbols.
4) W weighting space (ie) W=($[0,\infty)$,+,·)
5) P is a finite set of plus rewriting rules of the form.
   $\mu(\lambda, t) = c$

Which are also represented by $t \in D_{(N:N_0 \cup T)} : c \in [0,\infty)$ or equivalently by $\lambda \xrightarrow{c} p(t)$

P (t) is a Pseudo term corresponding to a tree t.

6) $\lambda_0 \in N_0$ Is an initial non terminal node symbol. Define the plus $\overset{c}{\Rightarrow}$ on the set $D_{(N:N_0 \cup T)}$ of trees as follows: for all $\gamma \overset{c}{\Rightarrow} \delta$ if and only if
   (i) $P(\gamma) = x\lambda y,$
   (ii) $P(\delta) = xP(t)y,$ and
   (iii) $\lambda \overset{c}{\to} t$ Is in P, where $x, y \in [N \cup N_0 \cup T \cup \{(,)\}]^*$, $\lambda \in N_0$ and $t \in D_{(N:N_0 \cup T)}$.

   Moreover, define the transitive closure $\overset{*c}{\Rightarrow}$ of plus weighted relation $\overset{c}{\Rightarrow}$ as follows.

   (1) $\gamma \overset{*1}{\Rightarrow} \gamma$ for all $\gamma \in D_{(N:N_0 \cup T)}$,

   (2) $\gamma \overset{*c}{\Rightarrow} \delta$ if and only if $c = \sum_{c \in D(N:N_0 \cup T)} \{c'.c'' \mid \gamma \overset{*c'}{\Rightarrow} \mu, \mu \overset{c''}{\Rightarrow} \delta\}$.

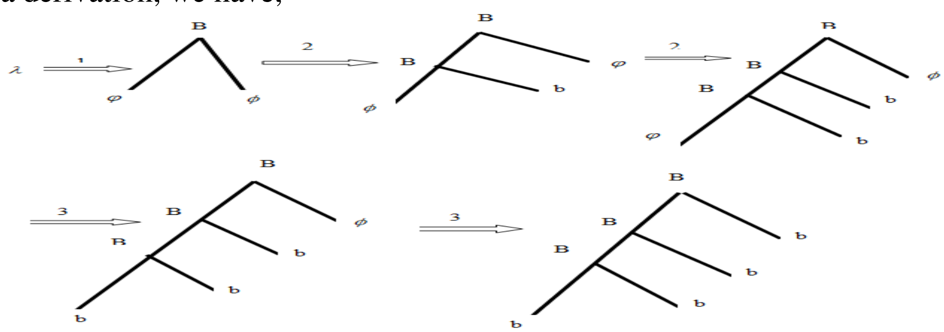
Definition 4.2

The plus weighted $D(S) = \{(t;c) \mid \lambda_0 \overset{*c}{\Rightarrow} t \in D_{(N:T)\}}$ is called a plus weighted context free Dendrolanguage (P-CFDL) generated by the P-CFDS, S.

Example 4.3

P-CFDS, S=(N_0,N,T,W,P,λ)generates P-CFDS, D(S)={(t;36)|P(t)=B(b,….B(bB(bb))…..),n≥0} ∪ {(t:)|P(t)=B(…..B(B(bb)b)…..),n≥1.

For example, as a derivation, we have,



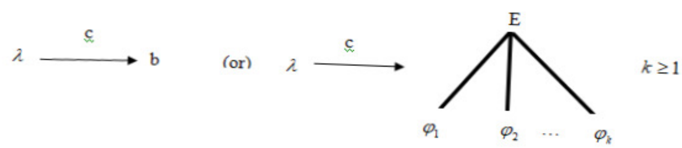
Or equivalently,

$$\lambda \overset{1}{\Rightarrow} B(\varphi\phi) \overset{2}{\Rightarrow} B(B(\varphi b)\phi) \overset{2}{\Rightarrow} B(B(B(\varphi b)b)\phi) \overset{3}{\Rightarrow} B(B(B(bb)b)\phi) \overset{3}{\Rightarrow} B(B(B(bb)b)b).$$

5 Normal Form of P-CFDS

The depth of tree t with domain U_t is defined by $d(t) = \sum\{|q| \| q \in U_t\}$, where |q| is the length of q. the order of P-CFDS is defined to be the maximum value of the depths of trees appearing in the right-hand side of the rules.

Two P-CFDS's are said to be equivalent if they generate the same Plus Weighted Dendrolanguage. In this section we show that for any P-CFDS'S there is an equivalent P-CFDS of order 1, one whose rules are of the form.



Where $\lambda, \varphi_i \in \mathcal{N}_0$ (i = 1,2,.....k), E $\in \mathcal{N}$ and c $\in \mathfrak{T}$

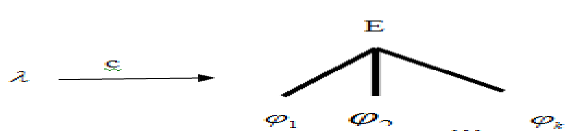
5. LEMMA 5.1

Let S= (N_0, N, T, W, P, λ_0) be a P-CFDS, of order $n(n \geq 2)$. Then there exists an equivalent P-CFDS of order n-1.

Proof:

We determine a new P-CFDS, S=(N'$_0$,N,T,W,P', λ_0) from a given P-CFDS, S-P' is defined as follows: for each rule. $\lambda \overset{c}{\rightarrow} t$

in P,(1) if d(t)<n, then $\lambda \overset{c}{\rightarrow} t$ is in P'; (2) if d(t) = n and P(t_1)…..P(t_k) then



And $\varphi_i \overset{1}{\rightarrow} t_i$ for all i such that P(t_i)$\notin N_0$ is in P', where φ_i is a new distinct nonterminal node symbol if P(t_i)$\notin N_0$ and $\varphi_i = P(t_i)$ if P(t_i)$\in N_0$.

Clearly, that N_0' is the union of N_0 and the set of all new nonterminal node symbols introduced by applying the above rule (2)

Suppose $\gamma \overset{c}{\Rightarrow} \delta$ under S. Then $P(\gamma) = x\lambda y, P(\delta) = xP(t)y,$ and $\lambda \overset{c}{\rightarrow} t$ is in P. If d (t) <n the above construction shows that $\gamma \overset{c}{\Rightarrow} \delta$ under S' since $\lambda \overset{c}{\rightarrow} t$ is also contained in P'. If d(t)=n, we have by the above construction that,

$$P(\gamma) = x\lambda y \overset{c}{\Rightarrow} x\mathrm{X}(\varphi_1 ....\varphi_k)y \overset{*1}{\Rightarrow} x\mathrm{X}(P(t_1)......P(t_k))y,$$

Conversely, if $x\lambda y \overset{c}{\Rightarrow} x\mathrm{X}(\varphi_1 ....\varphi_k)y$ under S', then $\varphi_i \overset{1}{\rightarrow} t_i$: i=1,2,……k should be applied since nonterminal symbols φ_i' s can be rewritten only by them

Hence $x\lambda y \overset{c}{\Rightarrow} x\mathrm{X}(\varphi_1 ....\varphi_k)y \overset{*1}{\Rightarrow} x\mathrm{X}(P(t_1)......P(t_k))y,$ for this derivation is $x\lambda y \overset{c}{\Rightarrow} xP(t)y$ Under S.

Thus D(S)=D(S') from the construction procedure of S'. it is clear that S' is of order n-1.

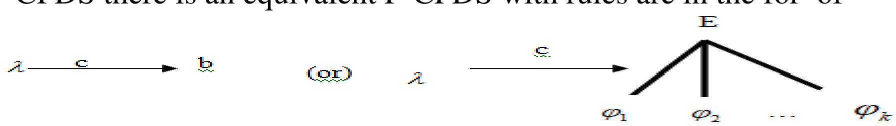
By repeated application of lemma we obtain the following lemma.

Lemma 5.2

For any P-CFDS, S, there exists an equivalent P-CFDS, S' of order 1.

Theorem 5.3

For any P-CFDS there is an equivalent P-CFDS with rules are in the for of

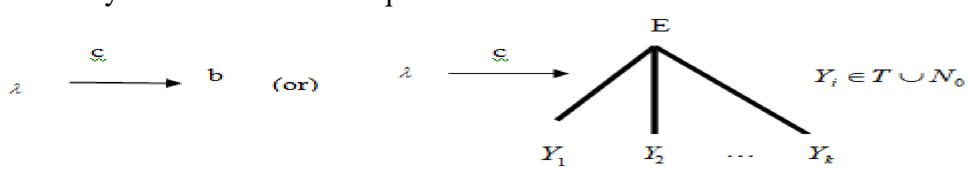


$k \geq 1$

Where λ, φ_i, i=1,2,……k are non terminal node symbols a is a leaf symbol, and E is a terminal node symbol.

Proof:

Let S be a P-CFDS. By lemma there is an equivalent P-CFDS whose rules are in the following form:



Here, if we replace the rule of type (2) by a rule.

Where $\varphi_i = Y_i$ if $Y_i \in N_0$ if φ_i is a new symbol if $Y_i \in T$, and rules

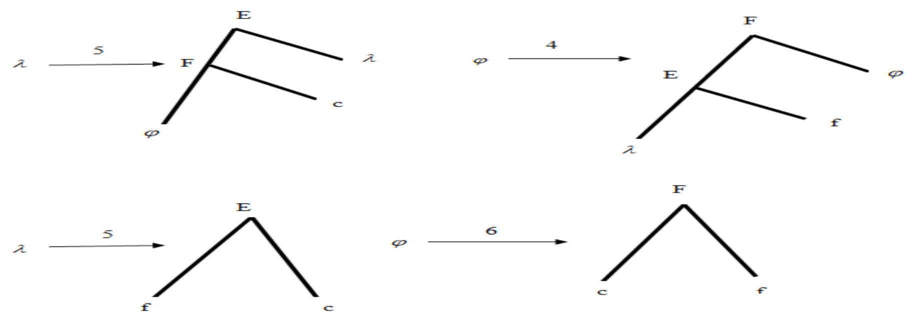
$$\varphi_i \overset{1}{\rightarrow} Y_i$$

for all $Y_i \in T$, then we obtain the desired P-CFDS.

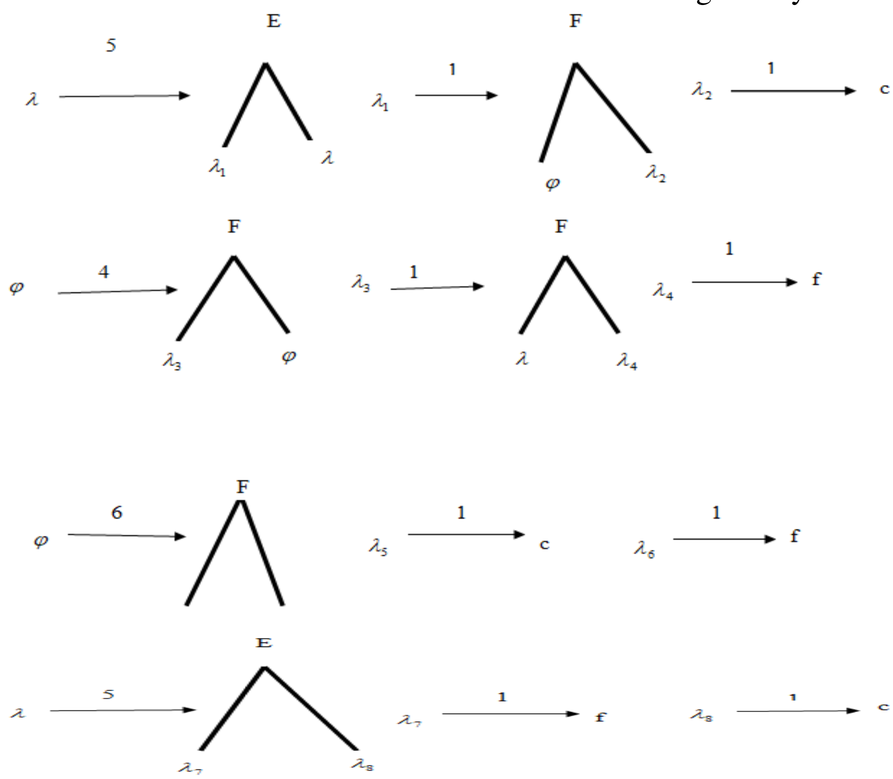
An P-CFDS whose rules are in the form of (1) or (2) of theorem is said to be normal.

Example 5.4

consider the following set of rules:



This gives an P-CFDS of order 2. The normal form for this P-CFDS is given by the following rules:



6. CONCLUSION :

In this paper Plus Weighted Context-Free Dendrolanguage is introduced and if there exist a P-CFDS of order $n(n \geq 2)$ then there is an equivalent P-CFDS of order (n-1) is proved further this topic can be extended to plus weighted derivation trees which will yield fruitful result in the field of tree Automata.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com

DOI No. : 10.5958/2394-9309.2023.00014.8
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

A NOTE ON KERNELS OF GROUP AND RING HOMOMORPHISMS

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ABSTRACT

Among various other results we exhibit that each group is a kernel of a group homomorphism and each additive group as well as each ring can be seen as a kernel of a ring homomorphism. In addition we discuss the algebraic implications of the idea presented in this paper.

Keywords : group, ring, ideal, quotient group, quotient ring.

MSC 2010: 97H40, 13A99.

1. INTRODUCTION

The idea of quotient groups and quotient rings is of fundamental importance in group and ring theory respectively. It may be noted that in mathematical literature generally one does not find a result expressing each group as a quotient group and each ring as a quotient ring. Here we provide such a result for groups as well as rings. Thus this article deals with visualizing each group as a quotient group and each ring as a quotient ring and we discuss the algebraic implications of this idea.

In addition it is well known that each additive group is a ring with trivial multiplication [1-4] and in this note we exhibit that each additive group can be visualized as a kernel of a ring homomorphism. Before this we exhibit that each group can be seen as a kernel of a group homomorphism.

In [6] we have noted that if G is a finite cyclic group of a given order and H is a finite cyclic group of the same order associated with a field, then H does have additional structure however G need not have additional structure. Similarly in this note we exhibit that if S_1 and S_2 are isomorphic subrings of a ring R, then one of them can have additional structure.

In the last section we discuss the algebraic implications of the idea presented in this note.

2. GROUPS AND RINGS AS KERNELS OF HOMOMORPHISMS

Proposition 1: Every group A is a quotient of a group G.

Proof. Case I. Let A is an additive group. Let us define $G = \{a + eb : a, b \in A\}$, then G is an additive group. It may be noted that here e is any symbol.

Clearly A is a proper normal subgroup of G. Let us define a mapping $f: G \to A$ such that $f(a+eb) = a-b$. Then f is a homomorphism of G onto A. The kernel of f is given by $K = \{a+ea : a \in G\}$. Clearly from the first isomorphism theorem [1-5] we have $\frac{G}{K} \cong A$.

Thus each additive group A is a quotient of a group G.

Case II. Let A is a multiplicative abelian group. Let us define $G = \{(a,b) : a, b \in A\}$, then G is an abelian group under coordinate wise multiplication.

Let $H = \{(e,a) : a \in A\}$. Here e is the identity of A. Clearly H is a normal subgroup of G and G is isomorphic to A. Let us define a mapping $f: G \to A$ such that $f(a,b) = a$. Then f is a homomorphism of G onto A. The kernel of f is given by $K = \{(e,a) : a \in A\}$ and the first isomorphism theorem [1-5] gives that $\frac{G}{K} \cong A$.

Case III. Let A is a multiplicative non-abelian group. Then $G = \{(a,b) : a, b \in A\}$, is a non-abelian group under coordinate wise multiplication.

It may be noted that the proof of case II will hold good in this case also provided $H = \{(e,a) : a \in A\}$ is a normal subgroup of G. Let (c,d) is any element of A.

Then $H(c,d) = \{(c,ad) : a \in A\}$ and $(c,d)H = \{(c,da) : a \in A\}$. Since $\{ad : a \in A\} = \{da : a \in G\}$. Therefore it follows that H is a normal subgroup of G. Hence the required result follows (rest part is similar to the proof given in case II).

Corollary 1. Every group A is a proper normal subgroup of a group G.

Corollary 2. Every group A is a kernel of a group homomorphism.

Proposition 2: Every additive group is a kernel of a ring homomorphism.

Proof. Let G be any additive group. Let us define $R = \{a+eb : a, b \in G\}$, then R is a ring with trivial multiplication. Here e is any symbol.

It is easy to note that G is a proper subgroup of the additive group of R. One may easily note that G is an ideal of R. Hence G is a kernel of a ring homomorphism.

Corollary 3. Every ring is a kernel of a ring homomorphism.

Proposition 4: Every additive group is a proper normal subgroup of the additive group of a ring.

Proof. Let G be any additive group. Let us define $R = \{a+eb : a, b \in G\}$, then R is a ring with trivial multiplication. Here e is any symbol.

Clearly G is a proper normal subgroup of the additive group of R.

Proposition 5: Every additive group G can be seen as a proper ideal of a ring R such that $\frac{R}{I} \cong G$. Here I is an ideal of R.

Proof. Let G be any additive group. Let us define $R=\{a+eb:a,b\in G\}$, then R is a ring with trivial multiplication. Here e is any symbol.

One may note that G is a proper ideal of R. Let us define a ring homomorphism $f:R\to G$ from R onto G by $f(a+eb)=a-b$. It may be noted that the kernel of f is given by $I=\{a+ea:a\in G\}$.

Using the first isomorphism theorem [1-3] we obtain $\frac{R}{I}\cong G$.

Proposition 6: Every ring R is a quotient of a ring F.

Proof. Let R is a ring. Let us define $F=\{a+eb:a,b\in R\}$, then R is a ring with trivial multiplication. Here e is any symbol. Now it is easy to note that R is an ideal of F. Now proceeding in the same way as we have done in the proof of proposition 5, we obtain $\frac{F}{I}\cong R$.

Note 1. Let R is a ring. Let us define $F=\{a+eb:a,b\in R\}$, then R is a ring under addition and multiplication with $e^2=1$. Now we can obtain $\frac{F}{I}\cong R$. But in this case the result does not hold for a ring of characteristic two.

Proposition 7: If S_1 and S_2 are isomorphic subrings of a ring F, then one of them can have additional structure.

Proof. Let $F=\{a+eb:a,b\in Q\}$. Here Q is the ring of rational numbers and F is a ring as described in the note 1.

Let $S_1=\{a:a\in Q\}$ and $S_2=\{a+ea:a\in Q\}$. Now one can easily verify that S_1 and S_2 are sub rings of F. Also S_1 and S_2 are isomorphic to each other. However S_2 is an ideal of F but S_1 is not an ideal of F. Thus S_2 has additional structure.

3. IMPORTANCE AND DISCUSSION:

For groups one may note that in the cases I, II and III, K is isomorphic to A and therefore $\frac{G}{K}\cong K$. In other words we have $\frac{G}{A}\cong A$. Thus this idea leads to find a group G having subgroup A such that $\frac{G}{A}\cong A$. It may be worth mentioning that there are several groups G whose every proper subgroup H satisfies $\frac{G}{H}\cong H$.

Further it may be noted that there does not exist any non-Abelian group G whose every proper subgroup H satisfies $\frac{G}{H}\cong H$. One reason for this is that each Hamiltonian group G has a normal subgroup of order two which does not satisfy the above property.

For rings, in proposition 5 we have seen that $\frac{R}{I} \cong G$. Let us define a mapping $\phi : I \to G$ as $f(a+ea) = a$. Then one can easily verify that I and G are isomorphic. In the light of this result we obtain $\frac{R}{I} \cong I$. One can verify that every proper ideal I of R satisfies $\frac{R}{I} \cong I$. Thus like groups this idea leads to find a ring R having an ideal I such that $\frac{R}{I} \cong I$. It is interesting to note that there are several rings R whose every proper ideal I satisfies $\frac{R}{I} \cong I$.

Declaration of Conflict of Interest: The author declares that there is no conflict of interest.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00015.X
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

LEHMER-4 MEAN LABELING FOR SOME PATH RELATED GRAPHS

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ABSTRACT

A graph G=(V,E) with p vertices and q edges is called Lehmer-4 mean graph, if it is possible to label vertices $x \in V$ with distinct label g(x) from 2,4,6,8,……….. 2p in such a way that when each edge e = uv is labeled with $g(e=uv)=\left\lfloor \frac{g(u)^4+g(v)^4}{g(u)^3+g(v)^3} \right\rfloor$ (or) $\left\lceil \frac{g(u)^4+g(v)^4}{g(u)^3+g(v)^3} \right\rceil$, then the edge labels are distinct. In this case, g is called Lehmer-4 mean labeling of G. In this paper, Lehmer-4 mean labeling for some path related graphs have been introduced.

Keywords: Star, Tree, Bistar,Graph.

1. INTRODUCTION

Graph labeling is an assignment of integer to its vertices or edges under certain condition. All Graphs in this paper are finite and undirected. The symbols V(G) and E(G) denote the vertex set and edge set of a graph G. The cardinality of the vertex set is called the order of G denoted by p. The cardinality of the edge set is called the size of G denoted by q edges is called a (p,q) graph. A graph labeling is an assignment of integers to the vertices or edges. Bloom and Hsu [2] extended the notion of graceful labeling to directed graphs. Further this work can be extended in the field of automata theory cited as Saridha , Rajaretnam [13,14,15,16,17] which has a wide range of application in automata theory. Saridha and Haridha Banu [18] discussed a new direction towards plus weighted grammar. Shalini, Paul Dhayabaran introduced different types of labelings[19,20]. Different types of labelings have been proved under connected and disconnected graphs[21,22,23,24,25,26,27,28,29,30]. Shalini.P, S.A. Meena [31] introduced “Lehmer -4 mean labelling of graph ”.Shalini.P S.Tamizharasi[32,33] studied “Power -3 Heronian Odd Mean Labeling Graphs”. Graph Labeling can be further extended to weiner index polynomial is cited as Palani Kumar, Rameshkumar [8,9,10,11,12]

2. BASIC DEFINITIONS

DEFINITION2.1

A graph G is said to be **Lehmer-4 mean graph** if it admits lehmer-4 mean labeling.

DEFINITION2.2

A Star S_n is the complete bipartite graph $K_{1,n}$

DEFINITION2.3

T_n be the tree of order n

DEFINITION2.4

A Bistar graph is the graph obtained by joining the centre(apex) vertices of two copies of $K_{1,n}$ by an edge and it is denoted by BS_n

DEFINITION2.5

Y_n is connected graph without any circuits

3. MAIN RESULTS

Theorem3.1

The Star $K_{1,n}$ is a Lehmer-4 mean graph for $n \geq 2$

Proof:

Let G be a graph of Star $K_{1,n}$

Let $K_{1,n}$ be a star with vertices as $\{v_1, u_1, u_2, u_3, \ldots, u_n\}$

Define $f:V(G) \rightarrow \{2,4,6,8,\ldots\ldots,2n+2\}$ by

$f(v_1) = 2$

$f(u_i) = 2i+2 \; ; \; 1 \leq i \leq n$

Then the edge labels as $f(e_i) = 2i + 1 \; ; \; 1 \leq i \leq n$

Therefore, the edges of the star graph receive distinct numbers

Hence, the Star $K_{1,n}$ is a Lehmer-4 mean graph.

Example3.1

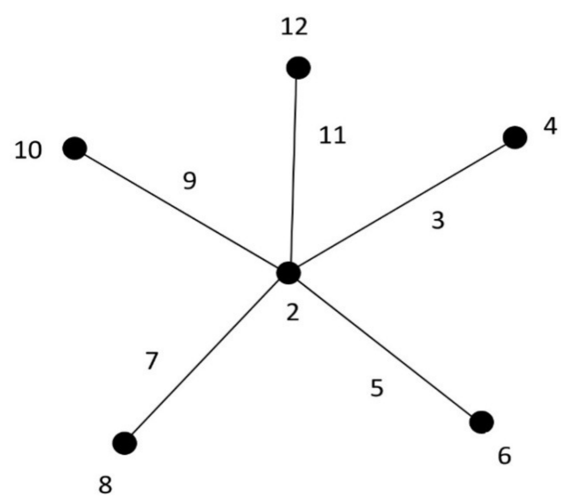

Figure 3.1: Star $K_{1,5}$

Theorem3.2

Y_n is a Lehmer-4 mean graph for $n \geq 2$

Proof:

Let G be a graph of Y_n

Let Y_n be a graph with vertices as $\{u_1, v_1, w_1, w_2, \ldots, w_n\}$

Define $f:V(G) \rightarrow \{2,4,6,8,\ldots\ldots,2n+4\}$ by ,

$f(u_1) = 2n+2$

$f(v_1)= 2n+4$

$f(w_i)=2i$; $1 \leq i \leq n$

Therefore, the edges of Y_n graph receive distinct numbers

Hence, $Y_n$is aLehmer-4 mean graph.

Example 3.2

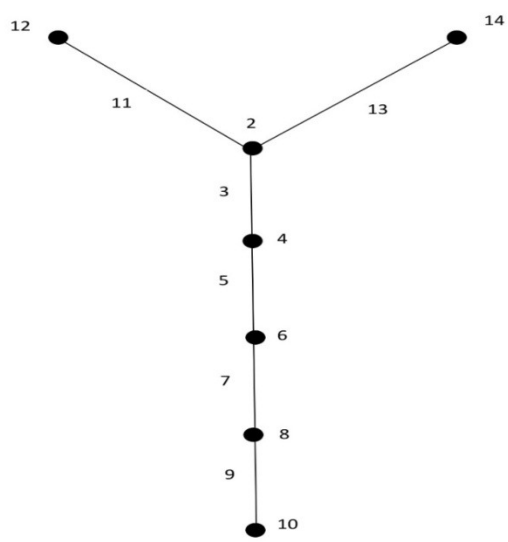

Figure 3.2 : Y_5

Theorem3.3

The Tree T_n is a Lehmer-4 mean graph for $n \geq 2$

Proof:

Let G be a graph of tree T_n

Let T_n be a tree with vertices as $\{u_1,v_1, v_2, \ldots,v_{n+1}\}$

Define $f:V(G) \rightarrow \{2,4,6,8,\ldots\ldots,2n+4\}$ by ,

$f(u_1) = 2$

$f(v_i)=2i+2$; $1 \leq i \leq n+1$

Then the edge labels as,

$f(e_i) = 2i+1$; $1 \leq i \leq n+1$

Therefore, the edges of the tree graph receive distinct numbers

Hence, the tree T_n is a Lehmer-4 mean graph.

Example 3.3

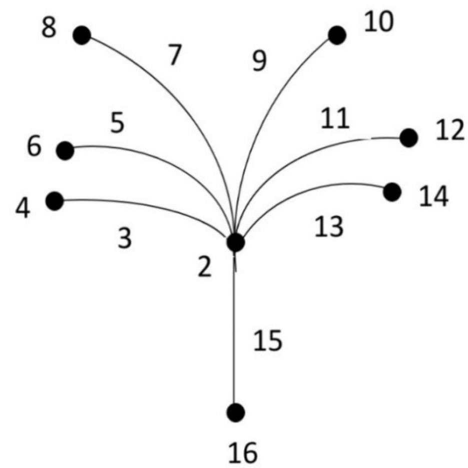

Figure3.3 : T_6

Theorem 3.4

The Bistar BS_n is a Lehmer-4 meangraph for $n \geq 2$

Proof:

Let G be a graph of Bistar BS_n

Let BS_n be a bistar with vertices as $\{u_1, v_1, v_2, \ldots, v_n, w_{n+1}, w_{n+2}, \ldots, w_{2n-1}\}$

Define $f:V(G) \rightarrow \{2,4,6,8,\ldots\ldots,4n+2\}$ by ,

$f(u_1)=2$

$f(v_i)=2i+2$,$1 \leq i \leq n$

$f(w_i)=2i+4$, $n \leq i \leq 2n-1$

Therefore, the edges of the Bistar graph BS_5 receive distinct numbers

Hence, the Bistar graph BS_5 is a Lehmer-4 mean graph

Example 3.4

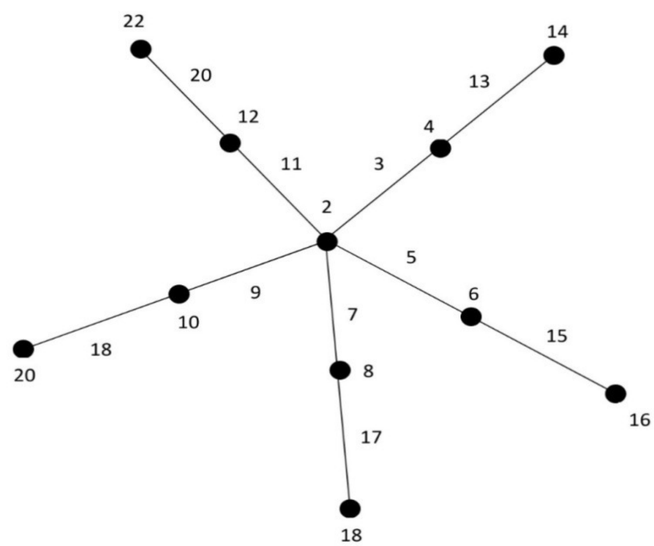

Figure 3.4 : BS_5

CONCLUSION

Finally, we conclude that Star, tree, bistar, Y_n is a Lehmer-4 mean graph.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com

DOI No. : 10.5958/2394-9309.2023.00016.1
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

ANALYSIS OF FUZZY PRIORITY QUEUING SYSTEM WITH HETEROGENEOUS SERVERS

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ABSTRACT

Fuzzy environment investigation of a priority queue network is presented in this research. For this, we considered a queuing model with two heterogeneous servers linked to a common server. Here arrival and service rates are taken fuzzy because in real these are not certain in nature. To convert fuzzy value into crisp value Yager's formula used. TFN, α - cut method and fuzzy computations are used to obtained fuzzy queuing performance measures of the suggested model. A numerical illustration is used to check the efficiency of the study. The model finds its application in hospitals, banks, offices, educations, communication network system, public transportation and many other fields.

Keywords : Heterogeneous, fuzzy, Priority, α - cut, triangular fuzzy number, membership function

1. INTRODUCTION:

Priority queue system is an essential part of queuing theory in which customers of different classes are assigned different type of importance based on priority discipline. Customers with high priority may sometimes benefit more than consumers with low priority. Such type of queuing model arises in many real-world situations. Practically, arrival, priority and service pattern are uncertain in nature. To remove this uncertainty fuzzy set theory has been used. Zadeh [1] mathematically framed theory of fuzzy sets. After that many researchers worked on fuzzy tandem queues Li and Lee [2], Buckley [3], Nagoorgani and Retha [4] and used α – cut method to evaluate fuzzy queues. Seema, Gupta, D.& Sharma, S. [7], M. Mittal, T. P. Singh, D. Gupta [8], Sameer Sharma [10], Noor [12] analyzed tandem and bi-tandem muti-server queueing system in fuzzy environment. J. Devaraj, D. Jayalakshmi [6], M. Mittal, T. P. Singh, D. Gupta [9], W. Ritha & S. Josephine Vinnarasi [11][13], K. Selvakumaria, S. Revathi [15], studied priority queuing model with equal and unequal service rate by using L-R Method and $\alpha - cut$ approach to solve triangular and trapezoidal fuzzy numbers. V. Saini, D. Gupta & A.K. Tripathi [14] [16-17] discussed heterogeneous queue network in both stochastic and fuzzy situations with feedback.

In the existing work we evaluated a priority queueing model with heterogeneous server instead of identical server because in reality it is not always true that servers are of same nature. Such type of situation is more realistic. System queue characteristics are obtained by using TFN and fuzzy arithmetic logics. A numerical analysis of proposed fuzzy model has been introduced.

2. FUZZY SET:

Sets with degree-of-membership in their elements are referred to as fuzzy sets. Zadeh and Klaua D. [1965] separately developed fuzzy sets as an enhancement to the standard idea of set. An element is either a member of a set or it is not, according to classical set theory. However, theory of fuzzy sets uses a membership function valued in the real unit range [0,1] to gradually evaluate an element's membership in a set. When there is ambiguous or insufficient information, the fuzzy set theory is frequently applied.

Definition – A fuzzy set $\tilde{F}: X \rightarrow \mu_{\tilde{F}}(\mathrm{X})$ is defined as $\tilde{F} = \{(x, \mu_{\tilde{F}}(\mathrm{x})): \mathrm{x} \in \mathrm{X},\ \mu_{\tilde{F}}(\mathrm{x}) \in [0,1]\}$ where X is the universal set and $\mu_{\tilde{F}}(\mathrm{x})$ represent the function of membership degree and the membership function's value, $\mu_{\tilde{F}}(\mathrm{x}) = \begin{cases} 0, if\ x \notin X \\ 1, if\ \ x\ \in X \end{cases}$

2.1 Crisp set

For any crisp set characteristic function $\mu_{\tilde{F}}$ takes either 0 or 1 value in classical set.

2.2 α – cut approach

A fuzzy set $\tilde{F}: X \rightarrow [0,1]$, for any $\alpha \in [0,1]$, the $\alpha - cut\ \ \alpha_F = \{x \in X, \mu_{\tilde{F}}(\mathrm{X}) \geq \alpha\}$ is a crisp set.

Strong $\alpha - cuts$: $\alpha +_F = \{x \in X, \mu_{\tilde{F}}(\mathrm{X}) > \alpha\};\ \alpha \in [0,1]$

Weak $\alpha - cuts$: $\alpha_F = \{x \in X, \mu_{\tilde{F}}(\mathrm{X}) \geq \alpha\};\ \alpha \in [0,1]$

As all members of a fuzzy set must be greater than or equal to α, it is important to view fuzzy sets as crisp sets.

2.3 Triangular Fuzzy Number

If the following criteria are met, a number F = (f, g, h) is said to represent a triangular fuzzy number with membership function $\mu_{\tilde{F}}(\mathrm{X})$.

$$\mu_{\tilde{F}}(\mathrm{X}) = \begin{cases} \dfrac{x-f}{g-f}, & f \leq x \leq g \\ \dfrac{h-x}{h-g}, & g \leq x \leq h \\ 0, otherwise & \end{cases}$$

2.4 Fuzzy Arithmetic Operations

The fundamental arithmetic operations on two triangular fuzzy numbers, F = (f_1, g_1, h_1) and G = (f_2, g_2, h_2), are the following:

i. Addition of two fuzzy numbers = $\tilde{F} + \tilde{G}$ = (f_1+ f_2, g_1+ g_2, h_1+ h_2)

ii. Difference of two fuzzy numbers = $\tilde{F} - \tilde{G}$ = (f_1- f_2, g_1- g_2, h_1-h_2) if DP($\tilde{F}$)≥ DP($\tilde{G}$), where DP($\tilde{F}$)=$\frac{h_1-f_1}{2}$ & DP($\tilde{G}$) = $\frac{h_2-f_2}{2}$, DP denotes different point of triangular fuzzy numbers, otherwise $\tilde{F} - \tilde{G}$ = (f_1- h_2, g_1- g_2, h_1- f_2)

iii. Multiplication = $\tilde{F} \times \tilde{G}$= ($f_1g_2$+ g_1f_2 - g_1g_2, g_1g_2, h_1g_2+g_1h_2 - g_1g_2)

iv. Division = $\tilde{A} / \tilde{B}$ = $(\frac{2f_1}{f_2+h_2}, \frac{g_1}{g_2}, \frac{2h1}{f_2+h_2})$

2.5 Defuzzification

A triangular fuzzy number $\tilde{F}$ = (f_1, g_1, h_1) fuzzified into crisp number F= $\frac{f1+2\,g1+h1}{4}$ by using Yager's [1981] formula.

3. DESCRIPTION OF MODEL:

The purposed model consists three server C_1, C_2 & C_3. The server C_1 is common server linked with two heterogeneous server C_2 & C_3. Two types of customers with pre-emptive priority, one is high priority and second is low priority with Poisson arrival rate λ_{1H} & λ_{1L}respectively enter into the system from outside. After getting service at C_1, customers join either heterogeneous server C_2 or C_3 for service. After receiving successful service, customer leave the system with possible conditions $\alpha_{12} + \alpha_{13} = 1, \alpha'_{12} + \alpha'_{13} = 1, \alpha_2 = 1, \alpha_3 = 1$. The service time of server C_1, C_2 & C_3 are exponentially distributed with service rate $\mu_{1H}, \mu_{1L}, \mu_2, \mu_3$ respectively.

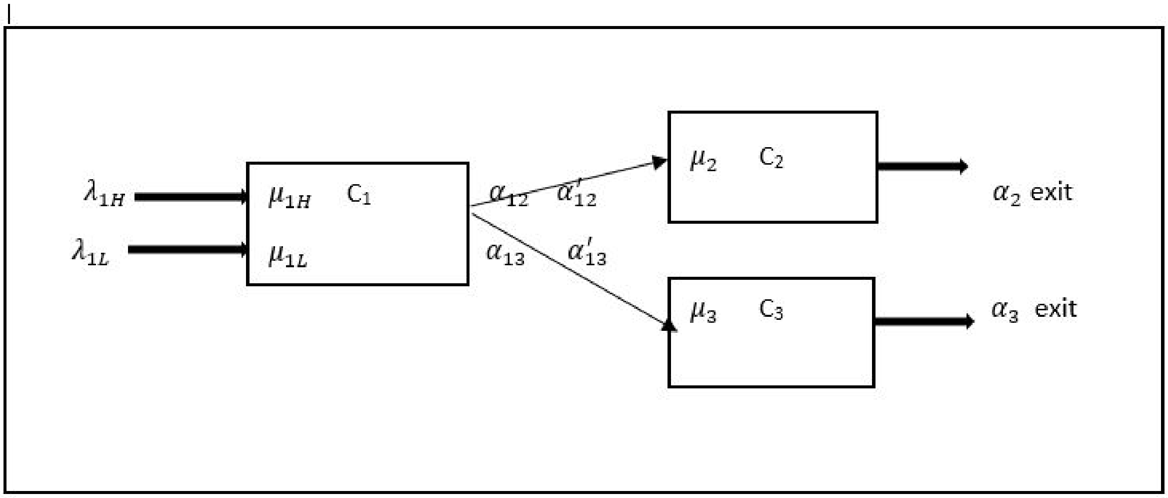

Figure1: Priority Queue Model

NOTATIONS:

The following notations are used in the paper

$m_{1L}, m_{1H,} m_2, m_3 = number\ of$ incoming customers

$\widetilde{\lambda_{1L}}, \widetilde{\lambda_{1H}}$ = fuzzy arrivals of low priority & high priority items

$\widetilde{\mu_{ij}}$ = fuzzy service rates where i=1,2 & j= L, H

$\widetilde{\alpha_{ij}}$ = fuzzy probabilities of high priority customers from i'th server to j'th server

$\widetilde{\alpha'_{ij}}$ = fuzzy probability of low priority customers from i'th server to j'th server

$\widetilde{\alpha_j}$ = fuzzy leaving probability where i=2,3

$\tilde{L}$ = System fuzzy average queue length

$\widetilde{L_i}$ = marginal Uncertain queue length of the servers

Mathematical Modeling in Stochastic Environment

Now, for mathematical modeling of proposed model in stochastic environment we define probability $P_{m_{1L},m_{1H},m_2,m_3}(t)$ of m_{1L}, m_{1H}, m_2, m_3 customers waiting in queues at any time t and time -independent differential equations of the model depicted as

$(\lambda_{1H}+\lambda_{1L}+\mu_{1H}+\mu_2+\mu_3)P_{m_{1L},m_{1H},m_2,m_3}=\lambda_{1L}P_{m_{1L-1},m_{1H},m_2,m_3}+\lambda_{1H}P_{m_{1L},m_{1H-1},m_2,m_3}+$
$\mu_{1H}\alpha_{12}P_{m_{1L},m_{1H}+1,m_{2-1},m_3}+\mu_{1H}\alpha_{13}P_{m_{1L},m_{1H}+1,m_2,m_{3-1}}+\mu_2\alpha_2P_{m_{1L},m_{1H},m_{2+1},m_3}+\mu_3\alpha_3P_{m_{1L},m_{1H},m_2,m_3+1}$
for $m_{1L},m_{1H},m_2,m_3>0$ (1)

For $m_{1L}=0,m_{1H},m_2,m_3>0$
$(\lambda_{1H}+\lambda_{1L}+\mu_{1H}+\mu_2+\mu_3)P_{0,m_{1H},m_2,m_3}=\lambda_{1H}P_{0,m_{1H-1},m_2,m_3}+\mu_{1H}\alpha_{12}P_{0,m_{1H}+1,m_{2-1},m_3}+$
$\mu_{1H}\alpha_{13}P_{0,m_{1H}+1,m_2,m_{3-1}}+\mu_2\alpha_2P_{0,m_{1H},m_{2+1},m_3}+\mu_3\alpha_3P_{0,m_{1H},m_2,m_3+1}$ (2)

For $m_{1H}=0,m_{1L},m_2,m_3>0$
$(\lambda_{1H}+\lambda_{1L}+\mu_{1L}+\mu_2+\mu_3)P_{m_{1L},0,m_2,m_3}=$
$\lambda_{1L}P_{m_{1L-1},0,m_2,m_3}+\mu_{1H}\alpha_{12}P_{m_{1L},1,m_{2-1},m_3}+\mu_{1H}\alpha_{13}P_{m_{1L},1,m_2,m_{3-1}}+\mu_{1L}\alpha'_{12}P_{m_{1L+1},0,m_{2-1},m_3}+$
$\mu_{1L}\alpha'_{13}P_{m_{1L+1},0,m_2,m_{3-1}}+\mu_2\alpha_2P_{m_{1L},0,m_{2+1},m_3}+\mu_3\alpha_3P_{m_{1L},0,m_2,m_3+1}$ (3)

For $m_2=0,m_{1L},m_{1H},m_3>0$
$(\lambda_{1H}+\lambda_{1L}+\mu_{1H}+\mu_3)P_{m_{1L},m_{1H},0,m_3}=$
$\lambda_{1L}P_{m_{1L-1},m_{1H},0,m_3}+\lambda_{1H}P_{m_{1L},m_{1H-1},0,m_3}+\mu_{1H}\alpha_{13}P_{m_{1L},m_{1H}+1,0,m_{3-1}}+\mu_2\alpha_2P_{m_{1L},m_{1H},1,m_3}+$
$\mu_3\alpha_3P_{m_{1L},m_{1H},0,m_3+1}$ (4)

For $m_3=0,m_{1L},m_{1H},m_2>0$
$(\lambda_{1H}+\lambda_{1L}+\mu_{1H}+\mu_2)P_{m_{1L},m_{1H},m_2,0}=$
$\lambda_{1L}P_{m_{1L-1},m_{1H},m_2,0}+\lambda_{1H}P_{m_{1L},m_{1H-1},m_2,0}+\mu_{1H}\alpha_{12}P_{m_{1L},m_{1H}+1,m_{2-1},0}+\mu_2\alpha_2P_{m_{1L},m_{1H},m_{2+1},0}+$
$\mu_3\alpha_3P_{m_{1L},m_{1H},m_2,1}$ (5)

For $m_{1L},m_{1H}=0,m_2,m_3>0$
$(\lambda_{1H}+\lambda_{1L}+\mu_2+\mu_3)P_{0,0,m_2,m_3}=\mu_{1H}\alpha_{12}P_{0,1,m_{2-1},m_3}+\mu_{1H}\alpha_{13}P_{0,1,m_2,m_{3-1}}+\mu_{1L}\alpha'_{12}P_{1,0,m_{2-1},m_3}+$
$\mu_{1L}\alpha'_{13}P_{1,0,m_2,m_{3-1}}+\mu_2\alpha_2P_{0,0,m_{2+1},m_3}+\mu_3\alpha_3P_{0,0,m_2,m_3+1}$ (6)

For $m_{1L},m_2=0,m_{1H},m_3>0$
$(\lambda_{1H}+\lambda_{1L}+\mu_{1H}+\mu_3)P_{0,m_{1H},0,m_3}=\lambda_{1H}P_{0,m_{1H-1},0,m_3}+\mu_{1H}\alpha_{13}P_{0,m_{1H}+1,0,m_{3-1}}+\mu_2\alpha_2P_{0,m_{1H},1,m_3}+$
$\mu_3\alpha_3P_{0,m_{1H},0,m_3+1}$ (7)

For $m_{1L},m_3=0,m_{1H},m_2>0$
$(\lambda_{1H}+\lambda_{1L}+\mu_{1H}+\mu_2)P_{0,m_{1H},m_2,0}=\lambda_{1H}P_{0,m_{1H-1},m_2,0}+\mu_{1H}\alpha_{12}P_{0,m_{1H}+1,m_{2-1},0}+\mu_2\alpha_2P_{0,m_{1H},m_{2+1},0}+$
$\mu_3\alpha_3P_{0,m_{1H},m_2,1}$ (8)

For $m_{1H},m_2=0,m_{1L},m_3>0$
$(\lambda_{1H}+\lambda_{1L}+\mu_{1L}+\mu_3)P_{m_{1L},0,0,m_3}=\lambda_{1L}P_{m_{1L-1},0,0,m_3}+\mu_{1H}\alpha_{13}P_{m_{1L},1,0,m_{3-1}}+\mu_{1L}\alpha'_{13}P_{m_{1L+1},0,0,m_{3-1}}+$
$\mu_2\alpha_2P_{m_{1L},0,1,m_3}+\mu_3\alpha_3P_{m_{1L},0,0,m_3+1}$ (9)

$$m_{1H},m_3=0,m_{1L},m_2>0$$

$(\lambda_{1H} + \lambda_{1L} + \mu_{1L} + \mu_2)P_{m_{1L},0,m_2,0} = \lambda_{1L}P_{m_{1L-1},0,m_2,0} + \mu_{1H}\alpha_{12}P_{m_{1L},1,m_{2-1},0} + \mu_{1L}\alpha'_{12}P_{m_{1L+1},0,m_{2-1},0} + \mu_2\alpha_2 P_{m_{1L},0,m_{2+1},0} + \mu_3\alpha_3 P_{m_{1L},0,m_2,1}$ (10)

$m_2, m_3 = 0, m_{1L}, m_{1H} > 0$

$(\lambda_{1H} + \lambda_{1L} + \mu_{1H})P_{m_{1L},m_{1H},0,0} = \lambda_{1L}P_{m_{1L-1},m_{1H},0,0} + \lambda_{1H}P_{m_{1L},m_{1H-1},0,0} + \mu_2\alpha_2 P_{m_{1L},m_{1H},1,0} + \mu_3\alpha_3 P_{m_{1L},m_{1H},0,1}$ (11)

For $m_{1L}, m_{1H}, m_2 = 0, m_3 > 0$

$(\lambda_{1H} + \lambda_{1L} + \mu_3)P_{0,0,0,m_3} = \mu_{1H}\alpha_{13}P_{0,1,0,m_{3-1}} + \mu_{1L}\alpha'_{13}P_{1,0,0,m_{3-1}} + \mu_2\alpha_2 P_{0,0,1,m_3} + \mu_3\alpha_3 P_{0,0,0,m_3+1}$

(12)

For $m_{1L}, m_{1H}, m_3 = 0, m_2 > 0$

$(\lambda_{1H} + \lambda_{1L} + \mu_2)P_{0,0,m_2,0} = \mu_{1H}\alpha_{12}P_{0,1,m_{2-1},0} + \mu_{1L}\alpha'_{12}P_{1,0,m_{2-1},0} + \mu_2\alpha_2 P_{0,0,m_{2+1},0} + \mu_3\alpha_3 P_{0,0,m_2,1}$

(13)

For $m_{1L}, m_2, m_3 = 0, m_{1H} > 0$

$(\lambda_{1H} + \lambda_{1L} + \mu_{1H})P_{0,m_{1H},0,0} = \lambda_{1H}P_{0,m_{1H-1},0,0} + \mu_2\alpha_2 P_{0,m_{1H},1,0} + \mu_3\alpha_3 P_{0,m_{1H},0,1}$ (14)

For $m_{1H}, m_2, m_3 = 0, m_{1L} > 0$

$(\lambda_{1H} + \lambda_{1L} + \mu_{1L})P_{m_{1L},0,0,0} = \lambda_{1L}P_{m_{1L-1},0,0,0} + \alpha_2 P_{m_{1L},0,1,0} + \mu_3\alpha_3 P_{m_{1L},0,0,1}$ (15)

For $m_{1L}, m_{1H}, m_2, m_3 = 0$

$(\lambda_{1H} + \lambda_{1L})P_{0,0,0,0} = \alpha_2 P_{0,0,1,0} + \mu_3\alpha_3 P_{0,0,0,1}$ (16)

To solve these equations, we are using generating function & partial generating functions as,

$G(Y_1,Y_2,Y_3,Y_4) = \sum_{m_{1L}=0}^{\infty}\sum_{m_{1H}=0}^{\infty}\sum_{m_2=0}^{\infty}\sum_{m_3=0}^{\infty} P_{m_{1L},m_{1H},m_2,m_3}\, Y_1^{m_{1L}}, Y_2^{m_{1H}}, Y_3^{m_2}, Y_4^{m_3}$

$G_{m_{1H},m_2,m_3}(Y_1) = \sum_{m_{1L}=0}^{\infty} P_{m_{1L},m_{1H},m_2,m_3} Y_1^{m_{1L}}$ (17)

$G_{m_2,m_3}(Y_1, Y_2) = \sum_{m_{1H}=0}^{\infty} G_{m_{1H},m_2,m_3}(Y_1) Y_2^{m_{1H}}$ (18)

$G_{m_3}(Y_1, Y_2, Y_3) = \sum_{m_2=0}^{\infty} G_{m_2,m_3}(Y_1, Y_2) Y_3^{m_2}$ (19)

$G(Y_1, Y_2, Y_3, Y_4) = \sum_{m_3=0}^{\infty} G_{m_3}(Y_1, Y_2, Y_3) Y_4^{m_3}$ (20)

And we get solution as,

$$G(Y_1,Y_2,Y_3,Y_4) = \frac{G_1\left[\mu_{1H}\left(1-\frac{\alpha_{12}Y_3}{Y_2}-\frac{\alpha_{13}Y_4}{Y_2}\right)-\mu_{1L}\left(1-\frac{\alpha_{12}Y_3}{Y_1}-\frac{\alpha_{13}Y_4}{Y_1}\right)\right]+\mu_2\left(1-\frac{\alpha_2}{Y_3}\right)G_2+\mu_3\left(1-\frac{\alpha_3}{Y_4}\right)G_3 +\mu_{1L}\left(1-\frac{\alpha_{12}Y_3}{Y_1}-\frac{\alpha_{13}Y_4}{Y_1}\right)G_4}{\lambda_{1L}(1-Y_1)+\lambda_{1H}(1-Y_2)+\mu_{1H}\left(1-\frac{\alpha_{12}Y_3}{Y_2}-\frac{\alpha_{13}Y_4}{Y_2}\right)+\mu_2\left(1-\frac{\alpha_2}{Y_3}\right)+\mu_3\left(1-\frac{\alpha_3}{Y_4}\right)} \quad (21)$$

Here for convenience, we denote

$G_1=G_0(Y_1, Y_3, Y_4)$, $G_2=G_0(Y_1, Y_2, Y_4)$, $G_3=G_0(Y_1, Y_2, Y_3)$, $G_4=G_{0,0}(Y_3, Y_4)$

At $|Y_1|=|Y_2|=|Y_3|=|Y_4|=1$ and $G(Y_1, Y_2, Y_3, Y_4) = 1$, the equation (21) reduces to indeterminate form. Therefore, applying L'Hospital rule on (21) and differentiating it w.r.t to one -by- one variable, we get the results

$$-\lambda_{1L} = -\mu_{1L}G_1 + \mu_{1L}G_4 \quad (22)$$

$$-\lambda_{1H} + \mu_{1H} = \mu_{1H}G_1 \quad (23)$$

$$-\alpha_{12}\mu_{1H} + \alpha_2\mu_2 = G_1\alpha_{12}(-\mu_{1H} + \mu_{1L}) + \mu_2\alpha_2G_2 - \mu_{1L}\alpha_{12}G_4 \quad (24)$$

$$-\alpha_{13}\mu_{1H} + \alpha_3\mu_3 = G_1\alpha_{13}(-\mu_{1H} + \mu_{1L}) + \mu_3\alpha_3G_3 - \mu_{1L}\alpha_{13}G_4 \quad (25)$$

Solve equations (22) to (25), we get

$$G_1 = 1 - \frac{\lambda_{1H}}{\mu_{1H}} \quad (26)$$

$$G_2 = 1 - \frac{\alpha_{12}(\lambda_{1H}+\lambda_{1L})}{\alpha_2\mu_2} \quad (27)$$

$$G_3 = 1 - \frac{\alpha_{13}(\lambda_{1H}+\lambda_{1L})}{\alpha_3\mu_3} \quad (28)$$

$$G_4 = 1 - \frac{\lambda_{1L}\mu_{1H}+\lambda_{1H}\mu_{1L}}{\mu_{1L}\mu_{1H}} \quad (29)$$

the probability distribution function as

$$P_{\eta1L,\eta1H,\eta2,\eta3} = (1 - G_1)^{\eta1L}(1 - G_2)^{\eta1H}(1 - G_3)^{\eta2}(1 - G_4)^{\eta3}G_1G_2G_3G_4$$
$$=\gamma_1^{\eta1L}\gamma_2^{\eta1H}\gamma_3^{\eta2}\gamma_4^{\eta3}(1-\gamma_1)(1-\gamma_2)(1-\gamma_3)(1-\gamma_4)$$

Wherever, $\gamma_1 = \frac{\lambda_{1H}}{\mu_{1H}}$

$$\gamma_2 = \frac{\alpha_{12}\lambda_{1H} + \lambda_{1L}\,\alpha'_{12}}{\alpha_2\mu_2}$$

$$\gamma_3 = \frac{\alpha_{13}\lambda_{1H} + \alpha'_{13}\lambda_{1L}}{\alpha_3\mu_3}$$

$\gamma_4 = \frac{\lambda_{1L}\mu_{1H}+\lambda_{1H}\mu_{1L}}{\mu_{1L}\mu_{1H}}$ represent the utilization of server and the solution of model exist if utilization of server is less than equal to one.

Average Queue length

$$L = \frac{\gamma_1}{(1-\gamma_1)} + \frac{\gamma_2}{(1-\gamma_2)} + \frac{\gamma_3}{(1-\gamma_3)} + \frac{\gamma_4}{(1-\gamma_4)}$$

Numerical Illustration

For particular values, we get

Table: Crisp Values

m_{1L}= 3	$\lambda_{1L} = 3$	$\mu_{1L} = 10$	$\alpha_{12} = .5$	$\alpha'_{12} = .6)$
m_{1H}= 5	λ_{1H}= 6	$\mu_{1H} = 15$	$\alpha_{13} = .5$	$\alpha'_{13} = .4$
m_2= 4		$\mu_2 = 16$	$\alpha_2 = .6$	
m_3= 6		$\mu_3 = 14$	$\alpha_3 = .7$	

$$\gamma_1 = .4, \gamma_2 = .5, \gamma_3 = .4286, \gamma_4 = .7$$
$$L_1 = .6667, L_2 = 1, L_3 = .7501, L_4 = 2.3333$$

4. FUZZIFIED MODEL:

Let us represent crisp parameters $\lambda_{1L}, \lambda_{1H}, \mu_{1L}, \mu_{1H}, \mu_2, \mu_3, \gamma_1, \gamma_2, \gamma_3, \gamma_4$ in fuzzy numbers by $\widetilde{\lambda_{1L}}, \widetilde{\lambda_{1H}}, \widetilde{\mu_{1L}}, \widetilde{\mu_{1H}}, \widetilde{\mu_2}, \widetilde{\mu_3}, \widetilde{\gamma_1}, \widetilde{\gamma_2}, \widetilde{\gamma_3}, \widetilde{\gamma_4}$ respectively. As per results derived in stochastic environment, fuzzy utilization factors by using α-cut approach are defined as

$$\alpha_{\gamma_1} = \frac{\alpha_{\lambda_{1H}}}{\alpha_{\mu_{1H}}} = \frac{[\alpha({\lambda_{1H}}^2-{\lambda_{1H}}^1)+{\lambda_{1H}}^1, {\lambda_{1H}}^3-\alpha({\lambda_{1H}}^3-{\lambda_{1H}}^2)]}{[\alpha({\mu_{1H}}^2-{\mu_{1H}}^1)+{\mu_{1H}}^1, {\mu_{1H}}^3-\alpha({\mu_{1H}}^3-{\mu_{1H}}^2)]} = \{\frac{\alpha({\lambda_{1H}}^2-{\lambda_{1H}}^1)+{\lambda_{1H}}^1}{{\mu_{1H}}^3-\alpha({\mu_{1H}}^3-{\mu_{1H}}^2)}, \frac{{\lambda_{1H}}^3-\alpha({\lambda_{1H}}^3-{\lambda_{1H}}^2)}{\alpha({\mu_{1H}}^2-{\mu_{1H}}^1)+{\mu_{1H}}^1}\}$$

On taking α =0 and α=1 in the above, we get an approximate TFN as

$$\widetilde{\gamma_1} = (\frac{{\lambda_{1H}}^1}{{\mu_{1H}}^3}, \frac{{\lambda_{1H}}^2}{{\mu_{1H}}^2}, \frac{{\lambda_{1H}}^3}{{\mu_{1H}}^1}) = (\gamma_1^1, \gamma_1^2, \gamma_1^3)$$

$$\alpha_{\gamma_2} = \frac{\alpha_{\lambda_{1H}}\alpha_{\alpha_{12}} + \alpha_{\lambda_{1L}}\alpha_{\alpha'_{12}}}{\alpha_{\alpha_2}\alpha_{\mu_2}} = \{\frac{(\alpha({\lambda_{1H}}^2-{\lambda_{1H}}^1)+{\lambda_{1H}}^1)(\alpha({\alpha_{12}}^2-{\alpha_{12}}^1)+{\alpha_{12}}^1)+(\alpha({\lambda_{1L}}^2-{\lambda_{1L}}^1)+{\lambda_{1L}}^1)(\alpha({\alpha'_{12}}^2-{\alpha'_{12}}^1)+{\alpha'_{12}}^1)}{({\mu_2}^3-\alpha({\mu_2}^3-{\mu_2}^2))({\alpha_2}^3-\alpha({\alpha_2}^3-{\alpha_2}^2))},$$
$$\frac{({\lambda_{1H}}^3-\alpha({\lambda_{1H}}^3-{\lambda_{1H}}^2))({\alpha_{12}}^3-\alpha({\alpha_{12}}^3-{\alpha_{12}}^2))+({\lambda_{1L}}^3-\alpha({\lambda_{1L}}^3-{\lambda_{1L}}^2))({\alpha'_{12}}^3-\alpha({\alpha'_{12}}^3-{\alpha'_{12}}^2))}{(\alpha({\mu_2}^2-{\mu_2}^1)+{\mu_2}^1)(\alpha({\alpha_2}^2-{\alpha_2}^1)+{\alpha_2}^1)}\}$$

We get TFN as while taking α =0 and α=1 in the above

$$\widetilde{\gamma_2} = (\frac{{\alpha_{12}}^1{\lambda_{1H}}^1+{\alpha'_{12}}^1{\lambda_{1L}}^1}{{\alpha_2}^3\,{\mu_2}^3}, \frac{{\alpha_{12}}^2{\lambda_{1H}}^2+{\alpha'_{12}}^2{\lambda_{1L}}^2}{{\alpha_2}^2{\mu_2}^2}, \frac{{\alpha_{12}}^3\,{\lambda_{1H}}^3+{\alpha'_{12}}^3{\lambda_{1L}}^3}{{\alpha_2}^1{\mu_2}^1}) = (\gamma_2^1, \gamma_2^2, \gamma_2^3)$$

$$\alpha_{\gamma_3} = \frac{\alpha_{\lambda_{1H}}\alpha_{\alpha_{13}} + \alpha_{\lambda_{1L}}\alpha_{\alpha'_{13}}}{\alpha_{\alpha_3}\alpha_{\mu_3}} = \{\frac{(\alpha({\lambda_{1H}}^2-{\lambda_{1H}}^1)+{\lambda_{1H}}^1)(\alpha({\alpha_{13}}^2-{\alpha_{13}}^1)+{\alpha_{13}}^1)+(\alpha({\lambda_{1L}}^2-{\lambda_{1L}}^1)+{\lambda_{1L}}^1)(\alpha({\alpha'_{13}}^2-{\alpha'_{13}}^1)+{\alpha'_{13}}^1)}{({\mu_3}^3-\alpha({\mu_3}^3-{\mu_3}^2))({\alpha_3}^3-\alpha({\alpha_3}^3-{\alpha_3}^2))},$$
$$\frac{({\lambda_{1H}}^3-\alpha({\lambda_{1H}}^3-{\lambda_{1H}}^2))({\alpha_{13}}^3-\alpha({\alpha_{13}}^3-{\alpha_{13}}^2))+({\lambda_{1L}}^3-\alpha({\lambda_{1L}}^3-{\lambda_{1L}}^2))({\alpha'_{13}}^3-\alpha({\alpha'_{13}}^3-{\alpha'_{13}}^2))}{(\alpha({\mu_3}^2-{\mu_3}^1)+{\mu_3}^1)(\alpha({\alpha_3}^2-{\alpha_3}^1)+{\alpha_3}^1)}\}$$

We get TFN as while taking α =0 and α=1 in the above

$$\widetilde{\gamma_3} = (\frac{{\alpha_{13}}^1{\lambda_{1H}}^1+{\alpha'_{13}}^1{\lambda_{1L}}^1}{{\alpha_3}^3\,{\mu_3}^3}, \frac{{\alpha_{13}}^2{\lambda_{1H}}^2+{\alpha'_{13}}^2{\lambda_{1L}}^2}{{\alpha_3}^2{\mu_3}^2}, \frac{{\alpha_{13}}^3\,{\lambda_{1H}}^3+{\alpha'_{13}}^3{\lambda_{1L}}^3}{{\alpha_3}^1{\mu_3}^1}) = (\gamma_3^1, \gamma_3^2, \gamma_3^3)$$

$$\alpha_{\gamma_4} = \frac{\alpha_{\lambda_{1H}}\alpha_{\mu_{1L}} + \alpha_{\lambda_{1L}}\alpha_{\mu_{1H}}}{\alpha_{\mu_{1H}}\alpha_{\mu_{1L}}} =$$
$$\{\frac{(\alpha({\lambda_{1H}}^2-{\lambda_{1H}}^1)+{\lambda_{1H}}^1)(\alpha({\mu_{1L}}^2-{\mu_{1L}}^1)+{\mu_{1L}}^1)+(\alpha({\lambda_{1L}}^2-{\lambda_{1L}}^1)+{\lambda_{1L}}^1)(\alpha({\mu_{1H}}^2-{\mu_{1H}}^1)+{\mu_{1H}}^1)}{({\mu_{1H}}^3-\alpha({\mu_{1H}}^3-{\mu_{1H}}^2))({\mu_{1L}}^3-\alpha({\mu_{1L}}^3-{\mu_{1L}}^2))},$$
$$\frac{({\lambda_{1H}}^3-\alpha({\lambda_{1H}}^3-{\lambda_{1H}}^2))({\mu_{1L}}^3-\alpha({\mu_{1L}}^3-{\mu_{1L}}^2))+({\lambda_{1L}}^3-\alpha({\lambda_{1L}}^3-{\lambda_{1L}}^2))({\mu_{1H}}^3-\alpha({\mu_{1H}}^3-{\mu_{1H}}^2))}{(\alpha({\mu_{1L}}^2-{\mu_{1L}}^1)+{\mu_{1L}}^1)(\alpha({\mu_{1H}}^2-{\mu_{1H}}^1)+{\mu_{1H}}^1)}\}$$

We get TFN as while taking α =0 and α=1 in the above

$$\widetilde{\gamma_4} = (\frac{{\mu_{1L}}^1{\lambda_{1H}}^1+{\mu_{1H}}^1{\lambda_{1L}}^1}{{\mu_{1L}}^3{\mu_{1H}}^3}, \frac{{\mu_{1L}}^2{\lambda_{1H}}^2+{\mu_{1H}}^2{\lambda_{1L}}^2}{{\mu_{1L}}^2{\mu_{1H}}^2}, \frac{{\mu_{1L}}^3\,{\lambda_{1H}}^3+{\mu_{1H}}^3{\lambda_{1L}}^3}{{\mu_{1L}}^1{\mu_{1H}}^1}) = (\gamma_4^1, \gamma_4^2, \gamma_4^3)$$

$$\widetilde{L_1} = \frac{\widetilde{\gamma_1}}{1-\widetilde{\gamma_1}}, \widetilde{L_2} = \frac{\widetilde{\gamma_2}}{1-\widetilde{\gamma_2}}, \widetilde{L_3} = \frac{\widetilde{\gamma_3}}{1-\widetilde{\gamma_3}}, \widetilde{L_4} = \frac{\widetilde{\gamma_4}}{1-\widetilde{\gamma_4}}$$

$$\tilde{L} = \widetilde{L_1} + \widetilde{L_2} + \widetilde{L_3} + \widetilde{L_4} \,\&\, \tilde{\lambda} = \widetilde{\lambda_{1L}} + \widetilde{\lambda_{1H}}$$

$$E(\widetilde{w}) = \frac{\tilde{L}}{\tilde{\lambda}}$$

Membership function of $\tilde{L}$ is

$$\mu_{\tilde{L}} = \begin{cases} 0, & L < l_1 \\ \frac{L - l_1}{l_2 - l_1}, & l_1 \le L < l_2 \\ \frac{l_3 - L}{l_3 - l_2}, & l_2 \le L < l_3 \\ 0, & L \ge l_3 \end{cases}$$

Membership function of $E(\widetilde{w})$ is

$$\mu_{\widetilde{W}} = \begin{cases} 0, & W < w_1 \\ \frac{W - w_1}{w_2 - w_1}, & w_1 \le W < w_2 \\ \frac{w_3 - W}{w_3 - w_2}, & w_2 \le W < w_3 \\ 0, & W \ge w_3 \end{cases}$$

To maintain fuzziness of input data, membership function is used to characterized the system performance.

5. NUMERICAL ILLUSTRATION:

Table: Input Fuzzy Dimensions

Customers in number	Arrival Times	Service costs	Probabilities	
m_{1L}= 3	$\widetilde{\lambda_{1L}}$ = (2,3,4)	$\widetilde{\mu_{1L}}$ = (13,14,15)	$\widetilde{\alpha_{12}}$ = (.4,.5,.6)	$\widetilde{\alpha'_{12}}$ = (.5,.6,.7)
m_{1H}= 5	$\widetilde{\lambda_{1H}}$ = (5,6,7)	$\widetilde{\mu_{1H}}$ = (17,18,19)	$\widetilde{\alpha_{13}}$ = (.6,.5,.4)	$\widetilde{\alpha'_{13}}$ = (.5,.4,.3)
m_2= 4		$\widetilde{\mu_2}$ = (20,21,22)	$\widetilde{\alpha_2}$ = (.7,.6,.5)	
m_3= 6		$\widetilde{\mu_3}$ = (21,22,23)	$\widetilde{\alpha_3}$ = (.8,.7,.6)	

Using these numerical values we get,

Utilization factor of the server,

$$\widetilde{\gamma_1} = (.2632, .3333, .4118)$$
$$\widetilde{\gamma_2} = (.2727, .3809, .5)$$
$$\widetilde{\gamma_3} = (.2899, .2727, .2381)$$
$$\widetilde{\gamma_4} = (.3474, .5476, .8190)$$

Average & partial queue lengths

$$\widetilde{L_1} = (.3973, .4999, .6216)$$
$$\widetilde{L_2} = (.4444, .6152, .8148)$$
$$\widetilde{L_3} = (.3939, .3749, .3235)$$
$$\widetilde{L_4} = (.8335, 1.2104, 1.9650)$$

$$\tilde{L} = (2.0691, 2.7004, 3.7249)$$

Average time spent by customer in the system

$$E(\widetilde{w}) = (.2299, .3000, .4139)$$

$$\tilde{\lambda} = (7, 9, 11)$$

To defuzzied the expected waiting time, we have used Yager's formula

$E(\tilde{\lambda}) = 9$

$$E(\widetilde{w}) = .3110$$

Thus, from above calculation it is clear that modular value of mean queue length is fuzzy that never falls below 2.0691 and exceed 3.7249. The possible value of queue length is 2.7004. The server utilization of 1st server by low and high priority customers is 54% & 33% and utilization of 2nd & 3rd server is 38% & 27%. The exact time spent by customer in system is .3000 which never exceed .4139 and fall below .2299.

CONCLUSION:

A queuing model with the idea of priority's heterogeneous server investigated by applying the concept of fuzzy set. The fuzzy system performance measures utilization of server, expected length of queues and time spent by customers in the system have been derived by adopting triangular α – cut method based on Zadeh extension principal. Due to fuzzy theory's greater adaptability when compared to other theories, The performance of many servers in a queueing system can be monitored much more effectively and efficiently using a fuzzy queuing model [5]. Fuzzy queuing models offer greater comprehensive information, which is useful for defining queueing systems. The study further can be applied to a queuing model with more servers.

ACKNOWLEDGEMENT

We are thankful to Dr. T.P. Singh for his valuable suggestions and time.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00017.3
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

SKOLEM ODD VERTEX GRACEFUL SIGNED GRAPHS FOR PATH

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ABSTRACT

In this article, the new concept skolem odd vertex graceful signed graphs on directed graphs have been introduced. A graph G(P, m, n) is a bijective function f:V(G)   {1, 3, 5, 7, …, 2p-1} such that, when each edge uv   E(G) is assigned by f(uv) = f(v) - f(u) the positive edges receive distinct labels from the set {1, 3, 5, …, 2m-1} and the negative edges receive distinct labels from the set {-1, -3, -5, -6, ..., -2n-1}, it is called as a skolem odd vertex graceful signed graphs. In this article, Path graph is investigated under skolem odd vertex graceful labeling for signed graph.

Keywords: Graceful Labeling, Signed Graphs, Additive Labeling, Skolem edge graceful labeling

1. INTRODUCTION

Graph labeling is an assignment of integer to its vertices or edges under certain condition. All Graphs in this paper are finite and directed. The symbols V(G) and E(G) denote the vertex set and edge set of a graph G. The cardinality of the vertex set is called the order of G denoted by p. The cardinality of the edge set is called the size of G denoted by q edges is called a (p,q) graph. A graph labeling is an assignment of integers to the vertices or edges. A sigraph is an ordered pair S = (G, s), where G = (V, E) is a (p, q) graph called its underlying graph and s: E→{+,-} is a function from the set of edges to the set {+, -}, called a signing of G; hence an edge receiving '+' ('-') in the signing is said to be positive (negative). Let $E^+(S)$ and $E^-(S)$ denote respectively, the set of positive and negative edges of S. By a (p, m, n)-sigraph, it means a sigraph S having p vertices, m positive edges and n negative edges. A sigraph is said to be homogeneous, if it is either all-positive sigraph (all-negative sigraph is defined similarly). A sigraph is said to be homogeneous, if it is either all-positive or all-negative and non-homogeneous otherwise. Bloom and Hsu [2] extended the notion of graceful labeling to directed graphs. Further this work can be extended in the field of automata theory cited as Saridha , Rajaretnam [13,14,15,16,17] which has a wide range of application in automata theory. Shalini, Paul Dhayabaran introduced skolem graceful labelings on directed graphs[19,20,21]. Saridha and Haridha Banu [18] discussed a new direction towards plus weighted grammar. Shalini, Paul Dhayabaran introduced different types of labelings[22,23,24]. Different types of labelings have been proved under connected and disconnected graphs[21,22,23,24,25,26,27,28,29,30]. Shalini.P, S.A. Meena [34] introduced "Lehmer -4 mean labeling of graph ".Shalini.P S.Tamizharasi [35,36] studied "Power -3 Heronian Odd Mean Labeling Graphs". Graph Labeling can be further extended to weiner index polynomial is cited as [8,9,10,11,12]

2. MAIN RESULT

Definition 2.1

A graph G(P, m, n) is a bijective function f:V(G) → {1, 3, 5, 7, …, 2p-1} such that, when each edge uv∈ E(G) is assigned by f(uv) = f(v) – f(u) the positive edges receive distinct labels from the set {1, 3, 5, …, 2m-1} and the negative edges receive distinct labels from the set {-1, -3, -5, -7, ..., -2n-1}, it is called as a skolem odd vertex graceful signed graphs.

Definition 2.2

A graph in which every edges associates, either positive or negative sign with distinct even numbers are called a skolem odd vertex graceful signed graphs.

Theorem 2.1

The path P_n is a skolem odd vertex graceful signed graphs for $n \geq 2$.

Proof

Let G be a graph of path P_n.

Let $\{v_1, v_2, v_3, \ldots, v_n\}$ be the vertices of P_n and $\{e_1, e_2, e_3, ..., e_{n-1}\}$ be the edges of P_n.

The path P_n consists of n vertices and n – 1 edges.

Case (i)

Edges receive positive signs with distinct even numbers.

Let us set an arbitrary labeling as follows:

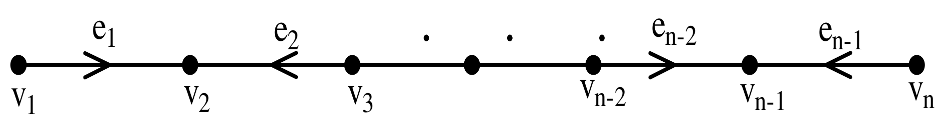

Fig. 2.1 : Path P_n with ordinary labeling when ‘n’ is odd

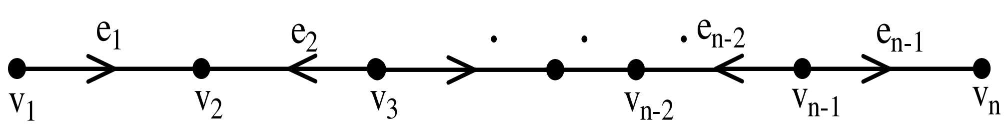

Fig. 2.2 :Path P_n with ordinary labeling when ‘n’ is even

The vertices of P_n are labelled as given below.

Define f: V(G) → {1, 3, 5, 7, ..., 2n – 1} as follows:

$$f(v_i) = \begin{cases} 2i-1 & \text{if i is odd} \\ 2n-i+1 & \text{if i is even} \end{cases}$$

Then the edge labels are:

$f(e_i) = 2n - 2i;\ 1 \leq i \leq n - 1$

Clearly, the edge labels are distinct even numbers with positive signs.

Case (ii)

Edges receive negative signs with distinct even numbers.

Let us set an arbitrary labeling as follows:

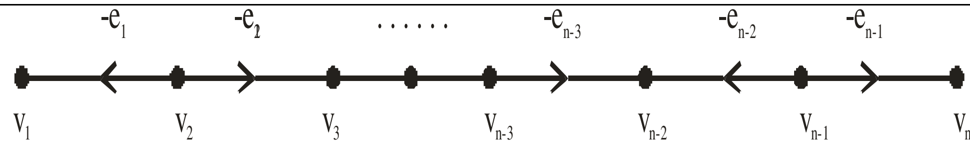

Fig. 2.3 : Path P_n with ordinary labeling when ‘n’ is odd

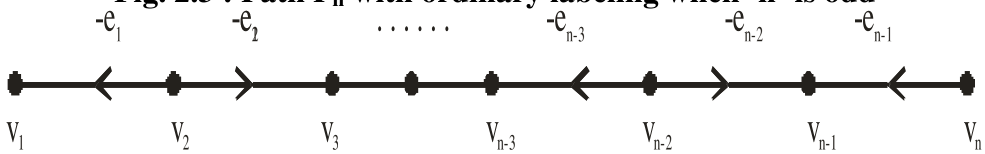

Fig. 2.4 :Path P_n with ordinary labeling when ‘n’ is even

The vertices of P_n are labelled as given below.

Define $f : V(G) \to \{1, 3, 5, 7, ..., 2n - 1\}$ as follows:

$$f(v_i) = \begin{cases} 2i-1 & \text{if i is odd} \\ 2n-i+1 & \text{if i is even} \end{cases}$$

Then the edge labels are:

$f(e_i) = -(2n - 2i)$; $1 \leq i \leq n - 1$

Clearly, the edge labels are distinct even numbers with negative signs.

Case (iii)

Edges receive both positive and negative signs with distinct even numbers.

Let us set an arbitrary labeling as follows:

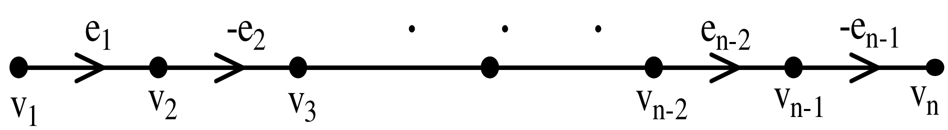

Fig. 2.5 :Path P_n with ordinary labeling when ‘n’ is odd

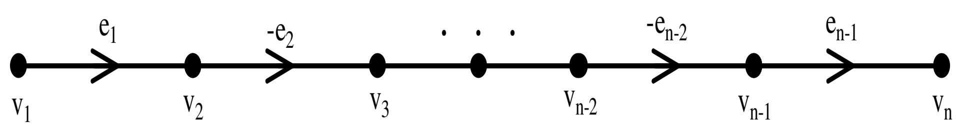

Fig. 2.6 :Path P_n with ordinary labeling when ‘n’ is even

The vertices of P_n are labelled as given below.

Define $f : V(G) \to \{1, 3, 5, 7, ..., 2n - 1\}$ as follows:

$$f(v_i) = \begin{cases} 2i-1 & \text{if i is odd} \\ 2n-i+1 & \text{if i is even} \end{cases}$$

Then the edge labels are:

$$f(e_i) = \begin{cases} 2n-2i & \text{if i is odd} \\ -(2n-2i) & \text{if i is even} \end{cases}$$

Clearly, the edge labels are distinct even numbers with positive and negative signs.

Hence, the path $P_n(n \geq 2)$ is a skolem odd vertex graceful signed graphs.

Example 2.1

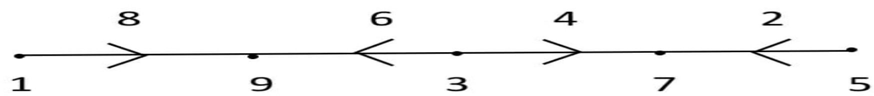

Fig. 2.7 :Path P_5

Example 2.2

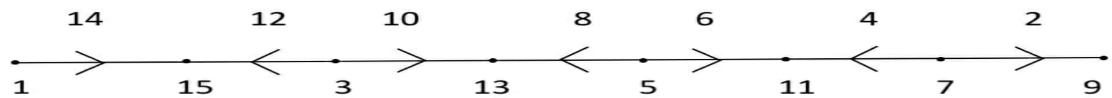

Fig. 2.8 :Path P_8

Example 2.3

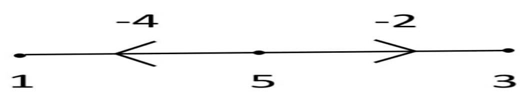

Fig. 2.9 :Path P_3

Example 2.4

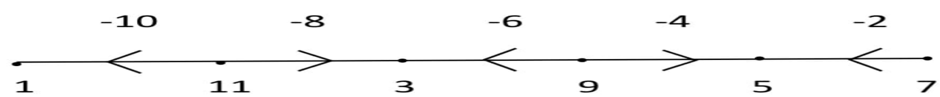

Fig. 2.10 :Path P_6

Example 2.5

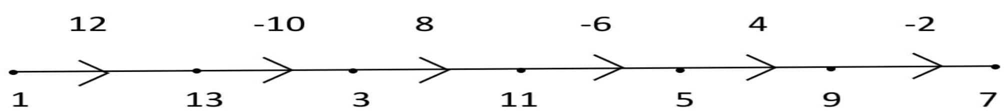

Fig. 2.11 :Path P_7

Example 2.6

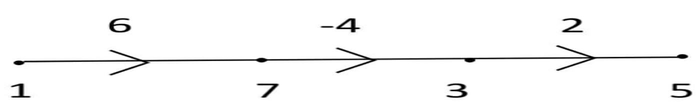

Fig. 2.12 :Path P_4

CONCLUSION

In this article, Skolem odd vertex graceful signed has been discussed. Path graph has been proved under skolem odd vertex graceful labeling for directed graphs.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00018.5
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

HEURISTIC APPROACH FOR REAL TIME FLOW SHOP SCHEDULING UNDER FUZZY ENVIRONMENT

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ABSTRACT

The real time Scheduling characteristics most nearly concerned with uncertainty, the processing time of jobs or tasks and the umbrella category of real situation constraints under which dead line, ready time, job processing period fall. The most obvious place to introduce fuzzy concepts for modeling uncertainty in scheduling/ sequencing problems is with a task execution time. With this intent, the objective of the present paper is to measure the scheduling policies and order of jobs to optimize the total processing time or make span in fuzzy environment including various parameters as weightage of jobs, transportation time and breakdown period.

Keywords: Flow -Shop Scheduling, Fuzzy Set, Triangular Fuzzy Number (TFN), Heuristic Technique etc.

1. INTRODUCTION:

The investigation to deal with problems in which facilities are fixed and the sequence of servicing the waiting jobs is subject to control have given arise to an elegant theory of scheduling. The main goal of scheduling is to maximize the performance of the system as well as to minimize waiting or turnaround time. Schedule may be evaluated according to some criterion of efficiency. The performance regarding the efficiency of scheduling algorithm provided to the request and the optimal utilization of resources in the system is determined by the order in which the request is serviced. Many realistic situations can be cited in which scheduling problems exits such as:

- Technological planning of how the jobs can be completed in a manufacturing plant or unit i.e. production planning problems.
- Scheduling of aircrafts waiting for landing clearance.
- Sequencing related to health services making available the patients to the large multi hospitals. etc.

The study made by Johnson's heuristic approach in (1954) is fundamental in theory of sequencing. Johnson studied n jobs 2 stage flow shop scheduling with the objective minimization of total elapsed time or make span and with some restrictive case 3 -stage flow shop scheduling. Motivated by the study made by Johnson, many scholars extended the work in this direction. Ignall (1965), Campbell, Dudek & Smith (1970), Maggu & Das (1977), Singh T.P.(1985, 1989) are the prominent figures who did a lot of work in the field of flow shop scheduling mainly in deterministic situations. In 1982 Mc. Canon & Lee made an attempt to fuzzify job scheduling problems and tried to obtain the near optimal solution for the real world problems. After that many researchers started to explore some new hope in this direction. Ishibuchiand

Lee (1996) formulated the fuzzy flow shop scheduling with fuzzy processing time. Further Singh T.P. and his research scholars Deepak Gupta (2005, 2008) associated probabilities with the processing time of each machine but the result was not so encouraging as such efforts could not be appreciated by Real World system. In real world situation it has been seen that the processing time of a job on a machine is neither fixed nor it is suitable to associate probabilities with each job, it is because in practical world, a priori data as well as the criteria by which the performance of the system be judged, is far from being precisely specified or having accurately known probability distribution and it is better to take processing or transportation time in fuzzy environment. Moreover, probability measures the likelihood of an event is known before the actual event while fuzzy logic measures the degree to which an outcome belongs to an event does not have a well-defined sharp boundary.

The processing time or transport time vary in many ways, may be due to environmental factor or due to different work places. We find whether in government sector or private sector, the tender of the work is given but the same work may vary in processing time or vary in cost different at different places. At some places the approach is very comfortable, labour is easily available and weather is favourable while at other work place as hilly places, the transportation is costly, the labours difficulty, weather not supportive. The application of fuzzy Set theory is better to take in account.

Generally in scheduling problem most of the researchers have assumed that the jobs are continuously in flow/working i.e. no machine fails and no disturbance occurs in the working of machines. But in actual practical situation this assumption is not correct. It has been observed that sometimes machines may not work continuously due to supply of electricity or due to failure of one or more components suddenly or due to resource break down or due to excessive heating or due to some other external cause. Machines are required to stop for certain time interval. So, in this paper we have considered break down interval criteria.

T.P. Singh & Sunita (2009, 2010) and further T.P. Singh & Meenu (2013, 2014) extended the work of earlier researchers under fuzzy environment including different augmentations & parameters. Singh T.P. did a remarkable work in fuzzy scheduling on tracing different performance measures as satisfaction of demand maker, due date on flow shop, parallel machines and on introducing new parameters as transportation time, priority in jobs, equivalent job concept, weightage in jobs, breakdown interval, rental cost concept etc.

In real sense, it has been observed that the values of all the jobs are not equi-importance in a workshop or manufacturing / production concern. Some jobs are very important while some have less weightage. The reason may be of market demand or different inventory cost associated with the job or it may be some technological, economical constraint, priority changes. Hence weightage concept becomes necessary to be considered in the scheduling problems. In many practical situations of scheduling, it has also been seen that the machines are distantly placed and therefore fixed time is required in transferring the job from one machine to another in form of: (a).loading time of jobs

(b) Moving time of jobs (c) unloading time of jobs. The sum of these times is refereed as transportation time. Generally, the transportation time also varies while earlier researchers considered it definite taking the job from one machine to another.

In this chain, the present study deals with the weightage of jobs in scheduling including variable transportation time and processing time in fuzzy sense. We discuss 3-stage schedule. The first one to complete all jobs in minimum time and second to take care of the job provider and the weightage of jobs which is necessary as it gives an idea of relative importance of the job in comparison to other jobs. In fact, we have extended the study made by Singh T.P. and Sunita (2009, 2010) including the concept of transportation time and triangular fuzzy numbers by using Average High Ranking (AHR) of fuzzy numbers proposed by Yager (1980) The problems have been solved through a heuristic technique which finds optimal or near optimal solution. The objective of the paper is to find the weighted mean flow time and optimal schedule which had the minimum < AHR> under uncertain environment. In this paper we have discussed scheduling of n jobs 3 machines problem. First of all, the preliminaries of Fuzzy set introducing by Zedeh (1965) and fuzzy arithmetic operations have been highlighted.

2. PRELIMINARIES ABOUT FUZZY SET THEORY:

2.1 The Classical or Crisp Set verses The Fuzzy Set:

The classical set is defined in such a way as to partitions the individuals in some given universe of discourse in to two groups: members or non-members (belongs to or not belongs to).

Mathematically, let X be a space of objects and x be generic element of X. A crisp set $A \subseteq X$ is defined by the collection of elements or objects $x \in X$, such that each x either belong to or not belong to the set A. We can represent crisp set A by a set of ordered pairs(x, 0) or (x, 1) which implies $x \notin A$ or $x \in A$, where 0 or 1 are “characteristic function” or membership functions for each element $x \in X$.

However many classification concepts do not exhibit this characteristic. For example; the set of tall people, set of sunny days etc.

2.2 Concept of Fuzzy Set:

Fuzzy set was introduced by Zedeh (1965). It expresses the degree to which an element belongs to a set. The characteristic or membership function of a fuzzy set is allowed to have values between 0 and 1, which is the degree of membership function of an element x in a given set.

Mathematically X is collection of objects denoted by x, then a ‘fuzzy set’ $\tilde{A}$ in X is defined as a set of ordered pairs:

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$, where $\mu_{\tilde{A}}(x)$ is called ‘degree or membership’ (MF) for the fuzzy set $\tilde{A}$. The MF maps each element of X to a membership function grade between 0 and 1. In fact the fuzzy set is a simple extension or generalization of a classical set. If values of $\mu_{\tilde{A}}$ (x) is restricted to 0 or 1, then Ã is reduced to classical set.

2.2.1 '$\propto$- cut' or '$\propto$ level set' of a fuzzy set Ã is a crisp set defined by :

$A_{\propto} = \{x: \mu_A(x) \geq \propto\}$.

'Strong $\propto$- cut' is defined as:

$A'_{\propto} = \{x: \mu_A(x) > \propto\}$.

- The $\propto$- cut of a Fuzzy set A is the crisp set that contains all the elements of the universal set X whose membership grades in A are greater than or equal to the specified values $\propto$, while Strong $\propto$- cut contains all elements whose membership grades in A are only greater than the specified value of $\propto$.

Remark: Unlike Statistics and Probability the degree is not describing probabilities that the item is in the set, but instead describes to what extent the item is in the set.

2.2.2 Containment:

The fuzzy set Ã is contained in set $\bar{B}$(or equivalently Ã ⊆ $\bar{B}$) iff$\mu_A(x) \leq \mu_B(x)$ for all x.

3. KINDS OF MEMBERSHIP FUNCTION:

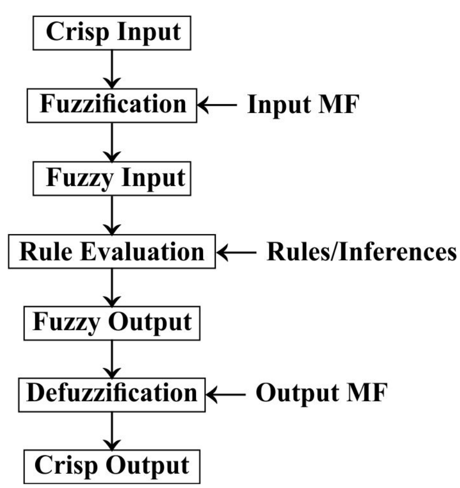
3.1 Triangular Membership Function:

A 'triangular MF' is specified by three parameters <a , b , c > as under:

$$y = triangle\ (x, a, b, c) = \begin{cases} 0 & \text{if } x \leq a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ \frac{c-x}{c-b} & \text{if } b \leq x \leq c \\ 0 & \text{if } x \geq c \end{cases}$$

The parameters < a,b,c >(with a< b<c) determine the x coordinates of the corners of triangle MF.

3.2 Operation On Fuzzy System :



4. METHODOLOGY:

In this paper, a new approach is taken to rank the jobs to make span or total processing times of jobs in a flow-shop problem. The approach is based on the concept of triangular fuzzy numbers. First of all we have defined the processing time of each job as a triangular fuzzy number, which has its value in three different situations; may be small, average and high. In order to find an optimal sequence, modified form of Johnson's Rank is applied in fuzzy environment with a new approach by converting the fuzzy processing time of jobs in a modal value by calculating $<$A H R$>$ of fuzzy numbers. Then apply Johnson's rule to find the optimal sequence of jobs. In addition of fuzzy processing time, various parameters like transportation time, weightage of jobs and brakedown interval etc. have been included in our study.

5. ALGORITHM:

Step 1. If the problem is of n jobs × 3 Machines:

First check the constraint

Min p_{i1} ≥ Max p_{i2} for all i

Or Min p_{i3} ≥ Max p_{i2} for all i or both gives optimal sequence in each case. On satisfying the condition proceed as:

(i) Find < A H R > of the fuzzy processing time of each job using Yager's (1980) formula: $< \text{A H R} > = \frac{\{3b+(c-a)\}}{3}$ for fuzzy number <a,b,c>

Now, proceeding on the line of Singh T.P. (1985)

Step 2. Modify the problem in two fictitious machines G_i and H_i as

$G_i = A_i + t_i + B_i + g_i$

$H_i = t_i + B_i + g_i + C_i$

Step3. Find Min (G_i , H_i)

(a) If Min (G_i , H_i) = G_i , then define

$G_i^{'} = G_i - W_i$ and $H_i^{'} = H_i$

(b) If Min (G_i , H_i) = H_i, then define

$G_i^{'} = G_i$ and $H_i^{'} = H_i + W_i$

Step 4. Define new reduced problem $G_i^{''} = \frac{G_i^{'}}{W_i}$ and $H_i^{''} = \frac{H_i^{'}}{W_i}$ as per Singh T.P. (1991)

Step 5. Find optimal sequence using Johnson's Algorithm for this reduced obtained in Step 4.

Step 6. Now check the effect of break down interval (a,b) on all jobs by taking the optimal sequence as per Step 5. Formulate again new problem with the processing time $A^{'}$ = A + (b - a), $B^{'}$ = B + (b - a) and

$C^{´}$ = C + (b - a) if (a, b) has effect on jobs. Otherwise $A^{´}$ = A, $B^{´}$ = B and $C^{´}$ = C if (a, b) has no effect on jobs.

Step 7. Repeat the process to get weighted mean flow time and make span of the job. We get optimal or near optimal sequence for the original problem covering all the parameters.

We shall clarify the above mentioned Algorithm through **Numerical Examples**:

We consider 5 jobs 3 machines A, B, C problem, whose processing times A_i, $B_i C_i$ are given in fuzzy environment and 30-35 is the breakdown interval; t_i and g_i represents transportation time; w_i represents weightage of the jobs as given in table 5.1 given below:

Machines	A		B		C	
Jobs	A_i	t_i	B_i	g_i	C_i	w_i
1	<6,8,9>	2	<2,4,5>	3	<4,5,6>	1
2	<10,11,12>	3	<1,3,5>	2	<5,6,8>	4
3	<7,8,9>	5	<1,2,4>	2	<4,6,7>	2
4	<6,7,9>	4	<3,5,6>	4	<3,5,6>	5
5	<8,9,12>	2	<1,1,4>	5	<5,7,8>	3

Table 5.1

The objective is to optimize the total elapsed time or make span and to find mean flow-time.

Solution:

From the given table it is clear that min time on A≥ max time on B. As the condition is satisfied, as per step 1 of Algorithm find < A H R > of process time of all the jobs as shown in table 5.2

Jobs	A_i	t_i	B_i	g_i	C_i	w_i
1	27/3	2	15/3	3	17/3	1
2	35/3	3	13/3	2	21/3	4
3	26/3	5	9/3	2	21/3	2
4	24/3	4	18/3	4	18/3	5
5	31/3	2	6/3	5	24/3	3

Table 5.2

As per step 2, convert the problem in two fictitious machines G_i , H_i such that

$$G_i = A_i + t_i + B_i + g_i$$

$$H_i = t_i + B_i + g_i + C_i$$

The results can be summarized in table 5.3

Jobs	G_i	H_i	w_i
1	57/3	47/3	1
2	63/3	49/3	4
3	56/3	51/3	2
4	66/3	60/3	5
5	58/3	51/3	3

Table 5.3

As per step 3: Find min (G_i, H_i) , we get the result table 5.4 with modified processing times G_i' , H_i' as:

Jobs	G_i'	H_i'	W_i
1	57/3	50/3	1
2	63/3	61/3	4
3	56/3	57/3	2
4	66/3	75/3	5
5	58/3	60/3	3

Table 5.4

As per step 4, we define a New Reduced Problem as :

$G_i'' = \frac{G_i'}{w_i}$ and $H_i'' = \frac{H_i'}{w_i}$ as per Singh T.P.(1991)

Jobs	G_i''	H_i''
1	57/3	50/3
2	63/12	61/12
3	56/6	57/6
4	66/15	75/15
5	58/9	60/9

Table 5.5

As per step 5, we apply Johnsons' technique for n jobs 2 machines problem and find optimal sequence as:

4	5	3	1	2

Step 5 : By taking this optimal sequence , we find the flow time of jobs on machines for original problem as given below in table 5.6

Jobs	A_i In — Out	t_i	B_i In — Out	g_i	C_i In — Out
4	0 — 24/3	4	36/3—54/3	4	66/3—84/3
5	24/3 — 55/3	2	61/3—67/3	5	**84/3---108/3**
3	55/3 — 81/3	5	**96/3----105/3**	2	111/3—132/3
1	**81/3 ---108/3**	2	114/3—129/3	3	138/3—155/3
2	108/3—143/3	3	152/3—165/3	2	171/3—192/3

Table 5.6

As per step 6, we have to check the effect of break down interval (30, 35).

The break down interval will affect the flow time of jobs only shown in **bold/Red letters** shown in table 5.6.

Here, l = b - a = 35 - 30 = 5 units = 15/3 units.

Hence final reduced Flow –Time can be presented in table 5.7 as given below:

Jobs	A_i In —— Out	t_i	B_i In —— Out	g_i	C_i In—— Out
4	0 – 24/3	4	36/3 – 54/3	4	66/3— 84/3
5	24/3 — 55/3	2	61/3 — 67/3	5	84/3— 123/3
3	55/3 — 81/3	5	105/3—114/3	2	123/3 —144/3
1	81/3 —123/3	2	129/3—144/3	3	153/3—170/3
2	123/3—158/3	3	167/3—180/3	2	186/3—207/3

Table 5.7

From Table 5.7, Total elapsed time = 207/3= 69 units.

As per Step 7, we find weighted mean of net flow-time:

Jobs	Net flow-time (f_i)	Weights(w_i)	$f_i w_i$
4	84/3	5	420/3
5	99/3	3	297/3
3	89/3	2	178/3
1	89/3	1	89/3
2	84/3	4	336/3
		$\sum w_i$=15	$\sum f_i w_i$=1320/3

Table 5.8

Weighted Mean Flow-Time = $\frac{\sum f_i w_i}{\sum w_i} = \frac{1320/3}{15}$ =29.33 units.

CONCLUSION:

In this paper we have considered 3- stage flow scheduling under fuzzy environment. Various constraints along with weightage of jobs and break down interval have been considered. The problem can proceed for n-stage scheduling making the problem more general. Here the transportation time going jobs from one machine to another can also be considered in fuzzy situation.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00019.7
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

INVENTORY MODEL HAVING POLYNOMIAL TIME CUBIC FUNCTION DEMAND WITH WEIBULL DISTRIBUTION AS DETERIORATION RATE

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ABSTRACT
This study examines an inventory model for products or objects that deteriorate over time with a cubic demand and a Weibull degradation rate. The model allows for shortages. Additionally, it demonstrates how the cubic demand function is convex and produces the optimal outcome. The convexity of this model is depicted graphically in three dimensions. In order to confirm the model, an illustration is made. The findings of a sensitivity analysis of the optimal solution with regard to the key variables were provided.
Keywords: Deterioration, cubic demand, Shortages, Total inventory cost.

INTRODUCTION:

The stock of goods kept on hand to enable the effective and seamless functioning of a trade or business is known as inventory. It also aids in the expansion of the business. It is utilized by businesses, schools, farms, hospitals, and institutes of higher learning. Both retailers and wholesalers must maintain a minimal inventory of goods or products. There are multiple diverging criteria that make up this entity that can be modeled. These conditions could include deterioration, time-varying demand, deterministic demand, and others.

The concept that demand rates do not change for numerous inventory items, such as clothing, dairy products, fruits and vegetables, and so forth, is incorrect; rather, demand rates may depend on time, stock, and price. Degrading items are things or things that change with time. The deterioration of goods or products causes numerous problems for the inventory system.

The rate of change in demand could also be linear, meaning that demand could rise or fall linearly over time. As a demand function for linear demand, we use a linear polynomial. Demand should occasionally vary significantly, meaning that it is growing quickly as time goes on.

By Liang-Yuh Ouyang, Kun-Shan WU, and Mei-Chuan CHENG, a mathematical model of inventory for deteriorating products with exponentially deteriorating demand was put out (2005). Alfares (2007) examined the inventory management strategy for a good whose demand rate depends on the amount of stock and whose time spent in storage depends on the cost of retaining it. C. K. Tripathy* and U. Mishra (2010) developed a listing model for the situation when the degradation rate follows Weibull distributions. R. Amutha and Dr. E. Chandrasekaran presented a list model for decaying products (2012).

In this concept, demand was meant to follow a straight line. A linear function of time is used to determine the cost of holding. The production model is demonstrated by Ravish Kumar Yadav and Ms. Pratibha Yadav (2013) under the premise of a cubic demand rate. Using a Weibull distribution, Garima Sharma and Bhawna Vyas (2018) proposed an inventory model for products that degrade over time. A listing model with a time-dependent demand rate employing multiple parameters was proposed by Ganesh Kumar, Sunita, and Ramesh Inaniyan in 2020. The demand rate is thought to be a three-degree polynomial of your time.

The deterioration rate varies over time, and the demand rate is modeled as a cubic polynomial of time in this study. It is assumed that the order's cost is fixed and does not change over time. With the use of a three-dimensional graphical representation, the convexity of this model is examined. Also, an illustration is made to confirm the model.

NOTATIONS AND ASSUMPTIONS

Notations

$C_{D'C'}$	:	Deterioration cost.
$C_{S'C'}$	:	Shortage cost per unit per time.
$C_{O'C'}$	:	Ordering cost per order.
$C_{H'C'}$	:	Holding cost.
W	:	For each ordering cycle, the maximum inventory level.
S	:	The maximum amount of inventory.
Q	:	The order quantity (Q=W + S)
$I'(t)$	:	Inventory level at time t.
t_1	:	Time at which shortages start.
$T'I'C'$	:	Total inventory cost.
T	:	The total length of each ordering cycle.
A'	:	Holding cost parameter.
B'	:	Holding cost parameter.
α	:	Weibull distribution Parameter
β	:	Weibull distribution Parameter

Assumptions

- The inventory management system presumes only one product.
- Lead Time is Zero.
- The demand rate $f(t)$ at time t is assumed as $f(t) = (a' + b't + c't^2 + d't^3)$ where a', b', c', d' are constants.
- Shortage occurs at time interval $t_1 \le t \prec T$.
- $\theta(t) = \alpha\beta t^{\beta-1}$ is deterioration rate

MATHEMATICAL FORMULATION:

The graph below (Figure 1) shows how inventory changes over time. The ideal order quantity, Q, and the total optimal inventory cost, T'I'C', are shown in this diagram.

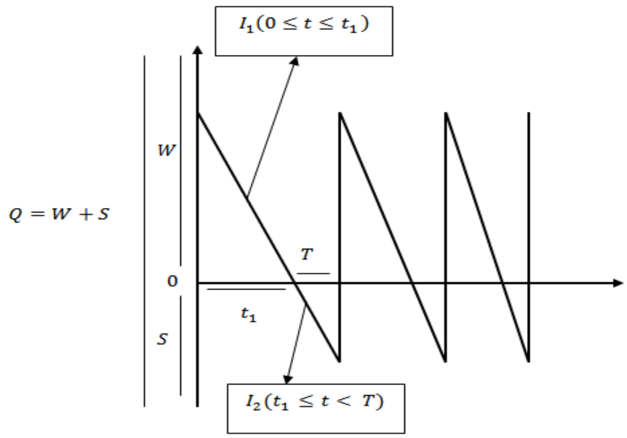


Figure 1: Inventory level (Q) vs time

Now till the shortages are allowed at interval $[0, t_1]$, As shown by, the differential equation is

$$\frac{dI_1^{'}(t)}{dt} + \alpha\beta t^{\beta-1} I_1^{'}(t) = -(a^{'} + b^{'}t + c^{'}t^2 + d^{'}t^3); 0 \le t \le t_1 \quad (1)$$

And during the interval $[t_1, T]$, the shortage occurs, so As shown by, the differential equation is

$$\frac{dI_2^{'}(t)}{dt} = -(a^{'} + b^{'}t + c^{'}t^2 + d^{'}t^3); t_1 \le t \prec T \quad (2)$$

With the boundary conditions: $t = 0, I^{'}(0) = W$,

$t = t_1; I^{'}(t_1) = 0$

$t = T; I^{'}(T) = S$

Now, by solving above equations (4.1), we get:

$$I.F. = e^{\int \alpha\beta t^{\beta-1} dt} = e^{\alpha t^{\beta}} = 1 + \alpha t^{\beta} + ... \quad \text{(By ignoring higher terms values)}$$

$$I.F. = 1 + \alpha t^{\beta}$$

$$I_1^{'}(t)\left(1 + \alpha t^{\beta}\right) = -\int \left(1 + \alpha t^{\beta}\right)\left(a^{'} + b^{'}t + c^{'}t^2 + d^{'}t^3\right) dt$$

$$I_1^{'}(t)\left(1 + \alpha t^{\beta}\right) = -\left[a^{'}t + \frac{b^{'}t^2}{2} + \frac{c^{'}t^3}{3} + \frac{d^{'}t^4}{4} + \frac{a^{'}\alpha t^{\beta+1}}{\beta+1} + \frac{b^{'}\alpha t^{\beta+2}}{\beta+2} + \frac{c^{'}\alpha t^{\beta+3}}{\beta+3} + \frac{d^{'}\alpha t^{\beta+4}}{\beta+4}\right] + D$$

Using condition at $t = t_1$, $I_1(t) = 0$

$$D = a^{'}t_1 + \frac{b^{'}t_1^2}{2} + \frac{c^{'}t_1^3}{3} + \frac{d^{'}t_1^4}{4} + \frac{a^{'}\alpha t_1^{\beta+1}}{\beta+1} + \frac{b^{'}\alpha t_1^{\beta+2}}{\beta+2} + \frac{c^{'}\alpha t_1^{\beta+3}}{\beta+3} + \frac{d^{'}\alpha t_1^{\beta+4}}{\beta+4}$$

By putting value of D in equation, we get

$$I_1^{'}(t)=\begin{pmatrix} a^{'}(t_1-t)+\frac{b^{'}}{2}(t_1^{\,2}-t^2)+\frac{c^{'}}{3}(t_1^{\,3}-t^3)+\frac{d^{'}}{4}(t_1^{\,4}-t^4)+\frac{a^{'}\alpha}{\beta+1}(t_1^{\beta+1}-t^{\beta+1}) \\ +\frac{b^{'}\alpha}{\beta+2}(t_1^{\beta+2}-t^{\beta+2})+\frac{c^{'}\alpha}{\beta+3}(t_1^{\beta+3}-t^{\beta+3})+\frac{d^{'}\alpha}{\beta+4}(t_1^{\beta+4}-t^{\beta+4})-a^{'}\alpha(t_1t^{\beta}-t^{\beta+1}) \\ -\frac{b^{'}\alpha}{2}(t_1^{\,2}t^{\beta}-t^{\beta+2})-\frac{c^{'}\alpha}{3}(t_1^{\,3}t^{\beta}-t^{\beta+3})-\frac{d^{'}\alpha}{4}(t_1^{\,4}t^{\beta}-t^{\beta+4})-\frac{a^{'}\alpha^2}{\beta+1}(t_1^{\beta+1}t^{\beta}-t^{2\beta+1}) \\ -\frac{b^{'}\alpha^2}{\beta+2}(_1^{\beta+2}t^{\beta}-t^{2\beta+2})-\frac{c^{'}\alpha^2}{\beta+3}(t_1^{\beta+3}t^{\beta}-t^{2\beta+3})-\frac{d^{'}\alpha^2}{\beta+4}(t_1^{\beta+4}t^{\beta}-t^{2\beta+4}) \end{pmatrix} \quad (3)$$

Now applying boundary conditions i.e., at $t=0, I_1^{'}(t)=W$

$$W=a^{'}t_1+\frac{b^{'}t_1^2}{2}+\frac{c^{'}t_1^3}{3}+\frac{d^{'}t_1^4}{4}+\frac{a^{'}\alpha t_1^{\beta+1}}{\beta+1}+\frac{b^{'}\alpha t_1^{\beta+2}}{\beta+2}+\frac{c^{'}\alpha t_1^{\beta+3}}{\beta+3}+\frac{d^{'}\alpha t_1^{\beta+4}}{\beta+4} \quad (4)$$

By solving equation (2),

$$I_2^{'}(t)=-\int(a+bt+ct^2+dt^3)dt$$

$$I_2^{'}(t)=-(at+\frac{bt^2}{2}+\frac{ct^3}{3}+\frac{dt^4}{4})+E$$

By using condition at $t=t_1, I_2^{'}(t)=0,$ we get

$$E=a^{'}t_1+\frac{b^{'}t_1^{\,2}}{2}+\frac{c^{'}t_1^{\,3}}{3}+\frac{d^{'}t_1^{\,4}}{4}$$

By putting value of E, we get

$$I_2^{'}(t)=a^{'}(t_1-t)+\frac{b^{'}}{2}\left(t_1^{\,2}-t^2\right)+\frac{c^{'}}{3}\left(t_1^{\,3}-t^3\right)+\frac{d^{'}}{4}\left(t_1^{\,4}-t^4\right) \quad (5)$$

Again by applying boundary condition, at $t=T, I_2^{'}(t)=-S$

$$S=-\left[a^{'}(t_1-T)+\frac{b^{'}}{2}\left(t_1^{\,2}-T^2\right)+\frac{c^{'}}{3}\left(t_1^{\,3}-T^3\right)+\frac{d^{'}}{4}\left(t_1^{\,4}-T^4\right)\right] \quad (6)$$

The order quantity per cycle is: $\mathrm{Q}=\mathrm{W}+\mathrm{S}$

$$Q=\begin{bmatrix} \left(a^{'}t_1+\frac{b^{'}t_1^2}{2}+\frac{c^{'}t_1^3}{3}+\frac{d^{'}t_1^4}{4}+\frac{a^{'}\alpha t_1^{\beta+1}}{\beta+1}+\frac{b^{'}\alpha t_1^{\beta+2}}{\beta+2}+\frac{c^{'}\alpha t_1^{\beta+3}}{\beta+3}+\frac{d^{'}\alpha t_1^{\beta+4}}{\beta+4}\right) \\ -\left(a^{'}(t_1-T)+\frac{b^{'}}{2}\left(t_1^{\,2}-T^2\right)+\frac{c^{'}}{3}\left(t_1^{\,3}-T^3\right)+\frac{d^{'}}{4}\left(t_1^{\,4}-T^4\right)\right) \end{bmatrix} \quad (7)$$

Holding cost per unit per unit time is:

$$H^{'}C^{'}=C_{H^{'}C^{'}}\int_0^{t_1}I_1(t)dt$$

$$H^{'}C^{'}=C_{H^{'}C^{'}}\left(\begin{array}{l}\frac{a^{'}t_1^2}{2}+\frac{b^{'}t_1^3}{3}+\frac{c^{'}t_1^4}{4}+\frac{d^{'}t_1^5}{5}+\frac{a^{'}\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{b^{'}\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}\\+\frac{c^{'}\alpha\beta t_1^{\beta+4}}{(\beta+1)(\beta+4)}+\frac{d^{'}\alpha\beta t_1^{\beta+5}}{(\beta+1)(\beta+5)}-\frac{a^{'}\alpha^2 t_1^{2\beta+2}}{(\beta+1)(2\beta+2)}\\-\frac{b^{'}\alpha^2 t_1^{2\beta+3}}{(\beta+1)(2\beta+3)}-\frac{c^{'}\alpha^2 t_1^{2\beta+4}}{(\beta+1)(2\beta+4)}-\frac{d^{'}\alpha^2 t_1^{2\beta+5}}{(\beta+1)(2\beta+5)}\end{array}\right) \tag{8}$$

Shortage cost per unit per unit time is:

$$S^{'}C^{'}=C_{S^{'}C^{'}}\int_{t_1}^{T} I_2^{'}(t)dt$$

With the help of equation (5)

$$S^{'}C^{'}=C_{S^{'}C^{'}}\left[\begin{array}{l}a^{'}\left(t_1T-\frac{T^2}{2}-\frac{t_1^{\,2}}{2}\right)+b^{'}\left(\frac{t_1^{\,2}T}{2}-\frac{T^3}{6}-\frac{t_1^{\,3}}{3}\right)+c^{'}\left(\frac{t_1^{\,3}T}{3}-\frac{T^4}{12}-\frac{t_1^{\,4}}{4}\right)\\+d^{'}\left(\frac{t_1^{\,4}T}{4}-\frac{T^5}{20}-\frac{t_1^{\,5}}{5}\right)\end{array}\right] \tag{9}$$

Ordering cost per unit per unit time is:

$$O^{'}C^{'}=C_{O^{'}C^{'}} \tag{10}$$

Deterioration cost per unit per unit time is:

$$D^{'}C^{'}=C_{D^{'}C^{'}}\left[W-\int_0^{t_1} Q(t)dt\right]$$

$$D^{'}C^{'}=C_{D^{'}C^{'}}\left[W-\int_0^{t_1}\left(a^{'}+b^{'}t+c^{'}t^2+d^{'}t^3\right)dt\right]$$

$$D^{'}C^{'}=C_{D^{'}C^{'}}\left(\frac{a^{'}\alpha t_1^{\beta+1}}{\beta+1}+\frac{b^{'}\alpha t_1^{\beta+2}}{\beta+2}+\frac{c^{'}\alpha t_1^{\beta+3}}{\beta+3}+\frac{d^{'}\alpha t_1^{\beta+4}}{\beta+4}\right) \tag{11}$$

Total Inventory Cost (TIC) given by:

$$T^{'}I^{'}C^{'}=\frac{1}{T}\left[H^{'}C^{'}+S^{'}C^{'}+O^{'}C^{'}+D^{'}C^{'}\right]$$

$$T'I'C' = \frac{1}{T}\left\{\begin{array}{l} C_{H'C'}\left(\begin{array}{l} \frac{a't_1^2}{2}+\frac{b't_1^3}{3}+\frac{c't_1^4}{4}+\frac{d't_1^5}{5}+\frac{a'\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{b'\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)} \\ +\frac{c'\alpha\beta t_1^{\beta+4}}{(\beta+1)(\beta+4)}+\frac{d'\alpha\beta t_1^{\beta+5}}{(\beta+1)(\beta+5)}-\frac{a'\alpha^2 t_1^{2\beta+2}}{(\beta+1)(2\beta+2)} \\ -\frac{b'\alpha^2 t_1^{2\beta+3}}{(\beta+1)(2\beta+3)}-\frac{c'\alpha^2 t_1^{2\beta+4}}{(\beta+1)(2\beta+4)}-\frac{d'\alpha^2 t_1^{2\beta+5}}{(\beta+1)(2\beta+5)} \end{array}\right) \\ +C_{S'C'}\left[\begin{array}{l} a'\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+b'\left(\frac{t_1^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right)+c'\left(\frac{t_1^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right) \\ +d'\left(\frac{t_1^4T}{4}-\frac{T^5}{20}-\frac{t_1^5}{5}\right) \end{array}\right] \\ +C_{O'C'}+C_{D'C'}\left(\frac{a'\alpha t_1^{\beta+1}}{\beta+1}+\frac{b'\alpha t_1^{\beta+2}}{\beta+2}+\frac{c'\alpha t_1^{\beta+3}}{\beta+3}+\frac{d'\alpha t_1^{\beta+4}}{\beta+4}\right) \end{array}\right\} \tag{12}$$

This is a must in order to reduce the overall cost of the inventory.

$$\frac{d\left(T'I'C'\right)}{dt_1} = \frac{1}{T}\left\{\begin{array}{l} C_{H'C'}\left[\begin{array}{l} a't_1+b't_1^2+c't_1^3+d't_1^4+\frac{a'\alpha\beta t_1^{\beta+2}}{(\beta+1)t_1}+\frac{b'\alpha\beta t_1^{\beta+3}}{(\beta+1)t_1}+\frac{c'\alpha\beta t_1^{\beta+4}}{(\beta+1)t_1} \\ +\frac{d'\alpha\beta t_1^{\beta+5}}{(\beta+1)t_1}-\frac{a'\alpha^2 t_1^{2\beta+2}}{(\beta+1)t_1}-\frac{b'\alpha^2 t_1^{2\beta+3}}{(\beta+1)t_1}-\frac{c'\alpha^2 t_1^{2\beta+4}}{(\beta+1)t_1}-\frac{d'\alpha^2 t_1^{2\beta+5}}{(\beta+1)t_1} \end{array}\right] \\ -C_{S'C'}\left[a'\left(T-t_1\right)+b'\left(t_1T-t_1^2\right)+c'\left(t_1^2T-t_1^3\right)+d'\left(t_1^3T-t_1^4\right)\right] \\ +C_{D'C'}\left(\frac{a'\alpha t_1^{\beta+2}}{t_1}+\frac{b'\alpha t_1^{\beta+3}}{t_1}+\frac{c'\alpha t_1^{\beta+4}}{t_1}+\frac{d'\alpha t_1^{\beta+5}}{t_1}\right) \end{array}\right\} \tag{13}$$

$$\frac{d\left(T'I'C'\right)}{dT} = -\frac{1}{T}\left\{C_{S'C'}\left[a'(t_1-T)+b'\left(\frac{t_1^2}{2}-\frac{T^2}{2}\right)+c'\left(\frac{t_1^3}{3}-\frac{T^3}{3}\right)+d'\left(\frac{t_1^4}{4}-\frac{T^4}{4}\right)\right]\right\}$$

$$-\frac{1}{T^2}\left\{\begin{array}{l} C_{H'C'}\left(\begin{array}{l}\frac{a't_1^2}{2}+\frac{b't_1^3}{3}+\frac{c't_1^4}{4}+\frac{d't_1^5}{5}+\frac{a'\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{b'\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}\\ +\frac{c'\alpha\beta t_1^{\beta+4}}{(\beta+1)(\beta+4)}+\frac{d'\alpha\beta t_1^{\beta+5}}{(\beta+1)(\beta+5)}-\frac{a'\alpha^2 t_1^{2\beta+2}}{(\beta+1)(2\beta+2)}\\ -\frac{b'\alpha^2 t_1^{2\beta+3}}{(\beta+1)(2\beta+3)}-\frac{c'\alpha^2 t_1^{2\beta+4}}{(\beta+1)(2\beta+4)}-\frac{d'\alpha^2 t_1^{2\beta+5}}{(\beta+1)(2\beta+5)}\end{array}\right)\\ +C_{S'C'}\left[\begin{array}{l}a'\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+b'\left(\frac{t_1^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right)+c'\left(\frac{t_1^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right)\\ +d'\left(\frac{t_1^4T}{4}-\frac{T^5}{20}-\frac{t_1^5}{5}\right)\end{array}\right]\\ +C_{O'C'}+C_{D'C'}\left(\frac{a'\theta t_1^3}{6}+\frac{b'\theta t_1^4}{8}+\frac{c'\theta t_1^5}{10}+\frac{d'\theta t_1^6}{12}\right)\end{array}\right\} \qquad (14)$$

The optimal values of t_1 and T from equation (13) & (14) by MAPLE 15

Numerical Example:

Using Maple 15, we will resolve the example. The inventory system's following parameters can be used to numerically illustrate the model:

$a'=10, b'=4, c'=4, d'=3, C_{H'C'}=4, C_{S'C'}=15, C_{O'C'}=100, C_{D'C'}=10, \alpha=0.1, \beta=0.5$

Under the above-given parameters, by using Maple 15 get the optimal shortage value $t_1=1.234454529$ per unit time and the optimal length of the ordering cycle is $T=1.559308506$ unit time. The total inventory cost is $T'I'C'=176.8809149$.

2D Representation

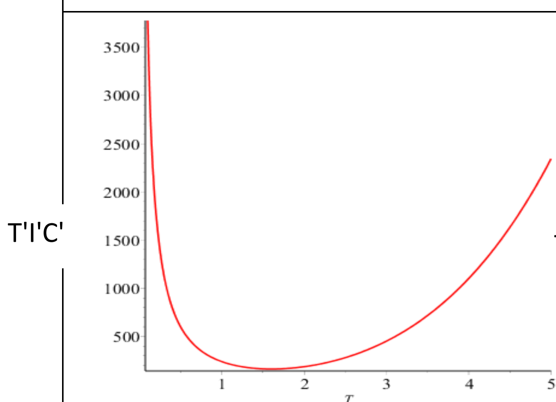


Figure 2 (Total Inventory Cost vs Time)

3D Representation

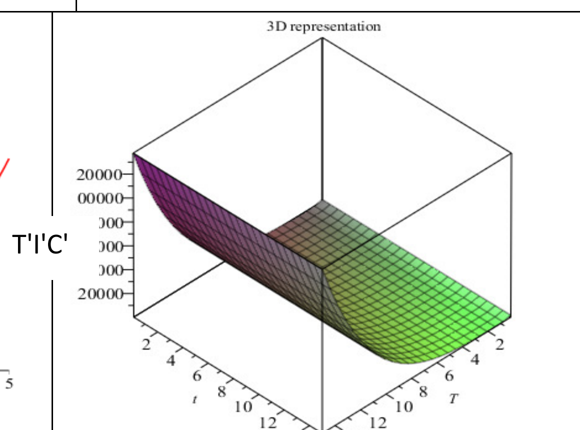


Figure 3 (3D Representation of Total Inventory Cost vs Time)

SENSITIVE ANALYSIS OF PARAMETERS

Here, we examine the impact of modifying the model's parameters such that $a', b', c', d', C_{H'C'}, C_{S'C'}, C_{O'C'}, C_{D'C'}, \alpha,$ and β the optimal order quantity and optimal total inventory cost per unit time. The outcomes are given in below table:

Parameter	% change	Change in			
		T	t_1	Q	$T'I'C'$
a'	+20%	1.603896271	1.289146952	36.81623890	191.8371917
	+10%	1.568618139	1.216711017	32.56914302	180.8997758
	-10%	1.537839289	1.208227222	29.04572767	169.0347097
	-20%	1.517034484	1.182951026	26.64390636	161.2846331
b'	+20%	1.539550707	1.212124022	31.74566643	178.8959541
	+10%	1.549380965	1.223202067	31.64593728	177.8119517
	-10%	1.569331489	1.245882449	31.43124085	175.5993973
	-20%	1.579447871	1.257486732	31.31578652	174.4701583
c'	+20%	1.529580785	1.201760482	31.35119014	178.9090809
	+10%	1.544189173	1.217788367	31.44598056	177.8277506
	-10%	1.574966130	1.251800094	31.63662222	175.5633731
	-20%	1.591191112	1.269869746	31.73191830	174.3759446
d'	+20%	1.513545373	1.186213620	30.54994917	179.3761816
	+10%	1.535509911	1.209408379	31.45575446	179.3439794
	-10%	1.59875912	1.28756974	32.11259704	175.2869791
	-20%	1.613746592	1.291436610	32.74815908	173.7565270
$C_{H'C'}$	+20%	1.479911573	1.091073046	28.31101181	183.1714517
	+10%	1.518505997	1.161619901	29.84438202	180.1794500
	-10%	1.603096218	1.310575724	33.45177555	172.7003266
	-20%	1.650807746	1.391122643	35.64302355	168.0487238
$C_{S'C'}$	+20%	1.604687345	1.347707128	33.63994747	188.8085173
	+10%	1.581764910	1.293568251	32.57225675	182.9825936
	-10%	1.537901867	1.169430727	30.56672565	169.9449504
	-20%	1.518470657	1.097310232	29.67942154	162.6079893
$C_{O'C'}$	+20%	1.611818504	1.284955997	33.71077833	189.3228638
	+10%	1.586564268	1.260866443	32.65301787	183.0701644
	-10%	1.529580629	1.205109266	30.36288365	170.2391594
	-20%	1.496687219	1.171910476	29.09960476	163.6314152
$C_{D'C'}$	+20%	1.542710635	1.203614726	30.83734305	178.7133475
	+10%	1.551012432	1.219072723	31.18768414	177.7250143
	-10%	1.567605704	1.249771493	31.89822821	175.6776823
	-20%	1.575910508	1.265034184	32.25905017	174.6181473
α	+20%	1.541152022	1.200863436	31.07218564	178.8350784
	+10%	1.550191264	1.217632503	31.30781452	177.7900824
	-10%	1.568517381	1.251349728	31.77267919	175.6034615
	-20%	1.577831636	1.268338043	32.00252961	174.4597498
β	+20%	1.552778559	1.224758923	31.22345685	176.5596111
	+10%	1.559874563	1.234868441	31.58745964	176.6569845
	-10%	1.562762624	1.239576280	31.71076676	176.7933240
	-20%	1.566354255	1.244897028	31.88819454	176.8757253

Table 1 (Sensitive Analysis of all Parameters)

The findings presented above lead to the following conclusions:

- With the increase in a', b', d'; T'I'C' and Q increase..
- With the increase in c', T'I'C' increases and Q decreases.
- If $C_{H'C'}$ and $C_{D'C'}$, increases, T'I'C' increases, and Q will decrease.
- If $C_{S'C'}$ (Shortage Cost), $C_{O'C'}$ (Ordering Cost) increases, Q and T'I'C' will increases.
- If α, β increases, Q decreases.

Here we consider two cases

- **An inventory model with Weibull distribution as deterioration rate and variable holding cost**

We develop an inventory model with assumption of variable holding cost.

$$H'C_{V'} = \int_0^{t_1} \left(A' + B't\right) I_1'(t)\,dt$$

With the help of equation (4.3), we get

$$H'C_{V'} = A'\left(\begin{array}{l} a'(t_1 t - \frac{t^2}{2}) + \frac{b'}{2}(t_1{}^2 t - \frac{t^3}{3}) + \frac{c'}{3}(t_1{}^3 t - \frac{t^4}{4}) + \frac{d'}{4}(t_1{}^4 t - \frac{t^5}{5}) + \frac{a'\alpha}{\beta+1}(t_1^{\beta+1} t - \frac{t^{\beta+2}}{\beta+2}) \\ + \frac{b'\alpha}{\beta+2}(t_1^{\beta+2} t - \frac{t^{\beta+3}}{\beta+3}) + \frac{c'\alpha}{\beta+3}(t_1^{\beta+3} t - \frac{t^{\beta+4}}{\beta+4}) + \frac{d'\alpha}{\beta+4}(t_1^{\beta+4} t - \frac{t^{\beta+5}}{\beta+5}) \\ -a'\alpha(\frac{t_1 t^{\beta+1}}{\beta+1} - \frac{t^{\beta+2}}{\beta+2}) - \frac{b'\alpha}{2}(\frac{t_1{}^2 t^{\beta+1}}{\beta+1} - \frac{t^{\beta+3}}{\beta+3}) - \frac{c'\alpha}{3}(\frac{t_1{}^3 t^{\beta+1}}{\beta+1} - \frac{t^{\beta+4}}{\beta+4}) \\ -\frac{d'\alpha}{4}(\frac{t_1{}^4 t^{\beta+1}}{\beta+1} - \frac{t^{\beta+5}}{\beta+5}) - \frac{a'\alpha^2}{\beta+1}(\frac{t_1^{\beta+1} t^{\beta+1}}{\beta+1} - \frac{t^{2\beta+2}}{2\beta+2}) - \frac{b'\alpha^2}{\beta+2}(\frac{t_1^{\beta+2} t^{\beta+1}}{\beta+1} - \frac{t^{2\beta+3}}{2\beta+3}) \\ -\frac{c'\alpha^2}{\beta+3}(\frac{t_1^{\beta+3} t^{\beta+1}}{\beta+1} - \frac{t^{2\beta+4}}{2\beta+4}) - \frac{d'\alpha^2}{\beta+4}(\frac{t_1^{\beta+4} t^{\beta+1}}{\beta+1} - \frac{t^{2\beta+5}}{2\beta+5}) \end{array}\right)_0^{t_1}$$

$$+B'\left[\begin{array}{l} a'\left(\frac{t_1 t^2}{2} - \frac{t^3}{3}\right) + b'\left(\frac{t_1^2 t^2}{2} - \frac{t^4}{4}\right) + c'\left(\frac{t_1^3 t^2}{2} - \frac{t^5}{5}\right) + d'\left(\frac{t_1^4 t^2}{2} - \frac{t^6}{6}\right) + \frac{a'\alpha}{\beta+1}(\frac{t_1^{\beta+1} t^2}{2} - \frac{t^{\beta+3}}{\beta+3}) \\ + \frac{b'\alpha}{\beta+2}(\frac{t_1^{\beta+2} t^2}{2} - \frac{t^{\beta+4}}{\beta+4}) + \frac{c'\alpha}{\beta+3}(\frac{t_1^{\beta+3} t^2}{2} - \frac{t^{\beta+5}}{\beta+5}) + \frac{d'\alpha}{\beta+4}(\frac{t_1^{\beta+4} t^2}{2} - \frac{t^{\beta+6}}{\beta+6}) \\ -a'\alpha(\frac{t_1 t^{\beta+2}}{\beta+2} - \frac{t^{\beta+3}}{\beta+3}) - \frac{b'\alpha}{2}(\frac{t_1{}^2 t^{\beta+2}}{\beta+2} - \frac{t^{\beta+4}}{\beta+4}) - \frac{c'\alpha}{3}(\frac{t_1{}^3 t^{\beta+2}}{\beta+2} - \frac{t^{\beta+5}}{\beta+5}) \\ -\frac{d'\alpha}{4}(\frac{t_1{}^4 t^{\beta+2}}{\beta+2} - \frac{t^{\beta+6}}{\beta+6}) - \frac{a'\alpha^2}{\beta+1}(\frac{t_1^{\beta+1} t^{\beta+2}}{\beta+2} - \frac{t^{2\beta+3}}{2\beta+3}) - \frac{b'\alpha^2}{\beta+2}(\frac{t_1^{\beta+2} t^{\beta+2}}{\beta+2} - \frac{t^{2\beta+4}}{2\beta+4}) \\ -\frac{c'\alpha^2}{\beta+3}(\frac{t_1^{\beta+3} t^{\beta+2}}{\beta+2} - \frac{t^{2\beta+5}}{2\beta+5}) - \frac{d'\alpha^2}{\beta+4}(\frac{t_1^{\beta+4} t^{\beta+2}}{\beta+2} - \frac{t^{2\beta+6}}{2\beta+6}) \end{array}\right]_0^{t_1}$$

$$H'C_{V'} = A'\begin{bmatrix} \frac{a't_1^2}{2}+\frac{b't_1^3}{3}+\frac{c't_1^4}{4}+\frac{d't_1^5}{5}+\frac{a'\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{b'\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)} \\ +\frac{c'\alpha\beta t_1^{\beta+4}}{(\beta+1)(\beta+2)}+\frac{d'\alpha\beta t_1^{\beta+5}}{(\beta+1)(\beta+2)}-\frac{a'\alpha^2 t_1^{2\beta+2}}{(\beta+1)(2\beta+2)} \\ -\frac{b'\alpha^2 t_1^{2\beta+3}}{(\beta+1)(2\beta+3)}-\frac{c'\alpha^2 t_1^{2\beta+4}}{(\beta+1)(2\beta+4)}-\frac{d'\alpha^2 t_1^{2\beta+5}}{(\beta+1)(2\beta+5)} \end{bmatrix}$$

$$+B'\begin{bmatrix} \frac{a't_1^3}{6}+\frac{b't_1^4}{8}+\frac{c't_1^5}{10}+\frac{d't_1^6}{12}+\frac{a'\alpha t_1^{\beta+3}}{(\beta+2)(\beta+3)}+\frac{b'\alpha t_1^{\beta+4}}{(\beta+2)(\beta+4)} \\ +\frac{c'\alpha t_1^{\beta+5}}{(\beta+2)(\beta+5)}+\frac{d'\alpha t_1^{\beta+6}}{(\beta+2)(\beta+6)}-\frac{a'\alpha^2 t_1^{2\beta+3}}{(\beta+2)(2\beta+3)} \\ -\frac{b'\alpha^2 t_1^{2\beta+4}}{(\beta+2)(2\beta+4)}-\frac{c'\alpha^2 t_1^{2\beta+5}}{(\beta+2)(2\beta+5)}-\frac{d'\alpha^2 t_1^{2\beta+6}}{(\beta+2)(2\beta+6)} \end{bmatrix} \quad (15)$$

Total Inventory Cost is:

$$T'I'C' = \frac{1}{T}\left(H'C_{V'} + S'C' + O'C' + D'C'\right)$$

$$T'I'C' = \frac{1}{T}\left\{\begin{array}{l} A'\begin{bmatrix} \frac{a't_1^2}{2}+\frac{b't_1^3}{3}+\frac{c't_1^4}{4}+\frac{d't_1^5}{5}+\frac{a'\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{b'\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)} \\ +\frac{c'\alpha\beta t_1^{\beta+4}}{(\beta+1)(\beta+2)}+\frac{d'\alpha\beta t_1^{\beta+5}}{(\beta+1)(\beta+2)}-\frac{a'\alpha^2 t_1^{2\beta+2}}{(\beta+1)(2\beta+2)} \\ -\frac{b'\alpha^2 t_1^{2\beta+3}}{(\beta+1)(2\beta+3)}-\frac{c'\alpha^2 t_1^{2\beta+4}}{(\beta+1)(2\beta+4)}-\frac{d'\alpha^2 t_1^{2\beta+5}}{(\beta+1)(2\beta+5)} \end{bmatrix} \\ +B'\begin{bmatrix} \frac{a't_1^3}{6}+\frac{b't_1^4}{8}+\frac{c't_1^5}{10}+\frac{d't_1^6}{12}+\frac{a'\alpha t_1^{\beta+3}}{(\beta+2)(\beta+3)}+\frac{b'\alpha t_1^{\beta+4}}{(\beta+2)(\beta+4)} \\ +\frac{c'\alpha t_1^{\beta+5}}{(\beta+2)(\beta+5)}+\frac{d'\alpha t_1^{\beta+6}}{(\beta+2)(\beta+6)}-\frac{a'\alpha^2 t_1^{2\beta+3}}{(\beta+2)(2\beta+3)} \\ -\frac{b'\alpha^2 t_1^{2\beta+4}}{(\beta+2)(2\beta+4)}-\frac{c'\alpha^2 t_1^{2\beta+5}}{(\beta+2)(2\beta+5)}-\frac{d'\alpha^2 t_1^{2\beta+6}}{(\beta+2)(2\beta+6)} \end{bmatrix} \\ +C_{S'C'}\begin{bmatrix} a'\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+b'\left(\frac{t_1^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right)+c'\left(\frac{t_1^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right) \\ +d'\left(\frac{t_1^4T}{4}-\frac{T^5}{20}-\frac{t_1^5}{5}\right) \end{bmatrix} \\ +C_{O'C'}+C_{D'C'}\left(\frac{a'\alpha t_1^{\beta+1}}{\beta+1}+\frac{b'\alpha t_1^{\beta+2}}{\beta+2}+\frac{c'\alpha t_1^{\beta+3}}{\beta+3}+\frac{d'\alpha t_1^{\beta+4}}{\beta+4}\right) \end{array}\right\} \quad (16)$$

This is a must in order to reduce the overall cost of the inventory.

$$
\frac{d\left(T'I'C'\right)}{dt_1}=\frac{1}{T}\left\{\begin{aligned}
&A'a't_1+3\left(\frac{A'b'}{3}+\frac{B'a'}{6}\right)t_1^2+4\left[\frac{A'c'}{4}+\frac{B'b'}{8}\right]t_1^{3}+5\left(\frac{A'd'}{5}+\frac{B'c'}{10}\right)t_1^{4}\\
&+\frac{B'd'}{2}t_1^{5}+\frac{1}{t_1}\left[\frac{A'b'}{\beta+1}+\frac{B'a'}{2\beta+4}\right]\alpha\beta t_1^{\beta+3}+\frac{1}{t_1}\left[\frac{A'c'}{\beta+1}+\frac{B'b'}{2\beta+4}\right]\alpha\beta t_1^{\beta+4}\\
&+\frac{1}{t_1}\left[\frac{A'd'}{\beta+1}+\frac{B'c'}{2\beta+4}\right]\alpha\beta t_1^{\beta+5}+\frac{B'd'\alpha\beta t_1^{\beta+6}}{(2\beta+4)t_1}-\frac{A'a'\alpha^2t_1^{2\beta+2}}{(\beta+1)t_1}\\
&-\frac{1}{t_1}\left[\frac{A'b'}{\beta+1}+\frac{B'a'}{\beta+2}\right]\alpha^2t_1^{2\beta+3}-\frac{1}{t_1}\left[\frac{A'c'}{\beta+1}+\frac{B'b'}{\beta+2}\right]\alpha^2t_1^{2\beta+4}-\frac{B'd'\alpha^2t_1^{2\beta+6}}{(\beta+1)t_1}\\
&-\frac{1}{t_1}\left[\frac{A'd'}{\beta+1}+\frac{B'c'}{\beta+2}\right]\alpha^2t_1^{2\beta+5}\\
&-C_{S'C'}\left[\begin{aligned}&a'\left(T-t_1\right)+b'\left(t_1T-t_1^2\right)\\&+d'\left(t_1^3T-t_1^4\right)+c'\left(t_1^2T-t_1^3\right)\end{aligned}\right]+C_{D'C'}\left(\begin{aligned}&\frac{a'\alpha t_1^{\beta+1}}{t_1}+\frac{b'\alpha t_1^{\beta+2}}{t_1}\\&+\frac{c'\alpha t_1^{\beta+3}}{t_1}+\frac{d'\alpha t_1^{\beta+4}}{t_1}\end{aligned}\right)
\end{aligned}\right\} \tag{17}
$$

$$
\begin{aligned}
\frac{d\left(T'I'C'\right)}{dT}=-\frac{1}{T^2}&\left\{\begin{aligned}
&A'\left[\begin{aligned}&\frac{a't_1^2}{2}+\frac{b't_1^3}{3}+\frac{c't_1^4}{4}+\frac{d't_1^5}{5}+\frac{a'\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{b'\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}\\
&+\frac{c'\alpha\beta t_1^{\beta+4}}{(\beta+1)(\beta+2)}+\frac{d'\alpha\beta t_1^{\beta+5}}{(\beta+1)(\beta+2)}-\frac{a'\alpha^2t_1^{2\beta+2}}{(\beta+1)(2\beta+2)}\\
&-\frac{b'\alpha^2t_1^{2\beta+3}}{(\beta+1)(2\beta+3)}-\frac{c'\alpha^2t_1^{2\beta+4}}{(\beta+1)(2\beta+4)}-\frac{d'\alpha^2t_1^{2\beta+5}}{(\beta+1)(2\beta+5)}\end{aligned}\right]\\
&+B'\left[\begin{aligned}&\frac{a't_1^3}{6}+\frac{b't_1^4}{8}+\frac{c't_1^5}{10}+\frac{d't_1^6}{12}+\frac{a'\alpha t_1^{\beta+3}}{(\beta+2)(\beta+3)}+\frac{b'\alpha t_1^{\beta+4}}{(\beta+2)(\beta+4)}\\
&+\frac{c'\alpha t_1^{\beta+5}}{(\beta+2)(\beta+5)}+\frac{d'\alpha t_1^{\beta+6}}{(\beta+2)(\beta+6)}-\frac{a'\alpha^2t_1^{2\beta+3}}{(\beta+2)(2\beta+3)}\\
&-\frac{b'\alpha^2t_1^{2\beta+4}}{(\beta+2)(2\beta+4)}-\frac{c'\alpha^2t_1^{2\beta+5}}{(\beta+2)(2\beta+5)}-\frac{d'\alpha^2t_1^{2\beta+6}}{(\beta+2)(2\beta+6)}\end{aligned}\right]\\
&+C_{S'C'}\left[\begin{aligned}&a'\left(t_1T-\frac{T^2}{2}-\frac{t_1^{2}}{2}\right)+b'\left(\frac{t_1^{2}T}{2}-\frac{T^3}{6}-\frac{t_1^{3}}{3}\right)+c'\left(\frac{t_1^{3}T}{3}-\frac{T^4}{12}-\frac{t_1^{4}}{4}\right)\\
&+d'\left(\frac{t_1^{4}T}{4}-\frac{T^5}{20}-\frac{t_1^{5}}{5}\right)\end{aligned}\right]\\
&+C_{O'C'}+C_{D'C'}\left(\left(\frac{a'\alpha t_1^{\beta+1}}{\beta+1}+\frac{b'\alpha t_1^{\beta+2}}{\beta+2}+\frac{c'\alpha t_1^{\beta+3}}{\beta+3}+\frac{d'\alpha t_1^{\beta+4}}{\beta+4}\right)\right)
\end{aligned}\right\}\\
&-\frac{1}{T}\left\{C_{S'C'}\left[a'(t_1-T)+b'\left(\frac{t_1^{2}}{2}-\frac{T^2}{2}\right)+c'\left(\frac{t_1^{3}}{3}-\frac{T^3}{3}\right)+d'\left(\frac{t_1^{4}}{4}-\frac{T^4}{4}\right)\right]\right\}
\end{aligned} \tag{18}
$$

- **Numerical Example**

Using Maple 15, we will resolve the example. The inventory system's following parameters can be used to numerically illustrate the model:

$a^{'}=10, b^{'}=4, c^{'}=4, d^{'}=3, C_{S'C'}=15, C_{O'C'}=100, C_{D'C'}=10, \alpha=0.1, \beta=0.5, A^{'}=1, B^{'}=1$

Under the above-given parameters, by using Maple 15, we get the optimal shortage value $t_1=1.884644653$ per unit time. The optimal length of ordering cycle is $T=1.959480184$ unit time and Total inventory cost is $T^{'}I^{'}C^{'}=60.36718330$.

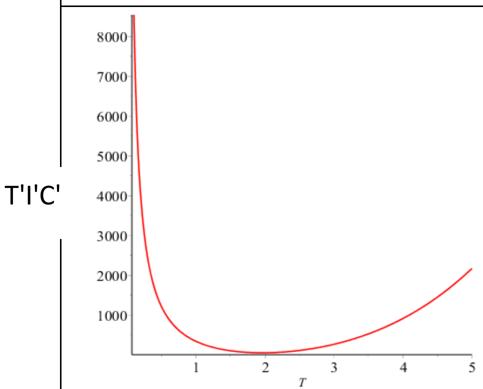
<table>
<tr><th>2D Representation</th><th>3D Representation</th></tr>
<tr><td>

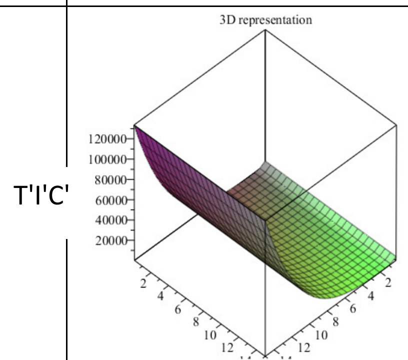
Figure 4 (Total Inventory Cost vs Time)</td>
<td>

Figure 5 (3D Representation of Total Inventory Cost vs Time)</td></tr>
</table>

- **An inventory model with constant deterioration and variable ordering cost**

$$O^{'}C_{V^{'}}=\frac{C_{O'C'}}{T} \tag{19}$$

Total Inventory Cost is:

$$T^{'}I^{'}C^{'}=\frac{1}{T}\left(H^{'}C^{'}+O^{'}C_{V^{'}}+S^{'}C^{'}+D^{'}C^{'}\right)$$

$$T^{'}I^{'}C^{'}=\frac{1}{T}\left\{\begin{array}{l} C_{H'C'}\left(\begin{array}{l}\frac{a^{'}t_1^2}{2}+\frac{b^{'}t_1^3}{3}+\frac{c^{'}t_1^4}{4}+\frac{d^{'}t_1^5}{5}+\frac{a^{'}\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{b^{'}\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}\\ +\frac{c^{'}\alpha\beta t_1^{\beta+4}}{(\beta+1)(\beta+4)}+\frac{d^{'}\alpha\beta t_1^{\beta+5}}{(\beta+1)(\beta+5)}-\frac{a^{'}\alpha^2 t_1^{2\beta+2}}{(\beta+1)(2\beta+2)}\\ -\frac{b^{'}\alpha^2 t_1^{2\beta+3}}{(\beta+1)(2\beta+3)}-\frac{c^{'}\alpha^2 t_1^{2\beta+4}}{(\beta+1)(2\beta+4)}-\frac{d^{'}\alpha^2 t_1^{2\beta+5}}{(\beta+1)(2\beta+5)}\end{array}\right)+\frac{C_{O'C'}}{T}\\ +C_{S'C'}\left[\begin{array}{l}a^{'}\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+b^{'}\left(\frac{t_1^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right)\\ +c^{'}\left(\frac{t_1^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right)+d^{'}\left(\frac{t_1^4T}{4}-\frac{T^5}{20}-\frac{t_1^5}{5}\right)\end{array}\right]+C_{D'C'}\left(\begin{array}{l}\frac{a^{'}\alpha t_1^{\beta+1}}{\beta+1}+\frac{b^{'}\alpha t_1^{\beta+2}}{\beta+2}\\ +\frac{c^{'}\alpha t_1^{\beta+3}}{\beta+3}+\frac{d^{'}\alpha t_1^{\beta+4}}{\beta+4}\end{array}\right)\end{array}\right\} \tag{20}$$

This is a must in order to reduce the overall cost of the inventory.

$$\frac{d\left(T'I'C'\right)}{dT}=-\frac{1}{T^2}\left\{\begin{array}{l} A'\left[\begin{array}{l}\frac{a't_1^2}{2}+\frac{b't_1^3}{3}+\frac{c't_1^4}{4}+\frac{d't_1^5}{5}+\frac{a'\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{b'\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}\\ +\frac{c'\alpha\beta t_1^{\beta+4}}{(\beta+1)(\beta+2)}+\frac{d'\alpha\beta t_1^{\beta+5}}{(\beta+1)(\beta+2)}-\frac{a'\alpha^2 t_1^{2\beta+2}}{(\beta+1)(2\beta+2)}\\ -\frac{b'\alpha^2 t_1^{2\beta+3}}{(\beta+1)(2\beta+3)}-\frac{c'\alpha^2 t_1^{2\beta+4}}{(\beta+1)(2\beta+4)}-\frac{d'\alpha^2 t_1^{2\beta+5}}{(\beta+1)(2\beta+5)}\end{array}\right]\\ +B'\left[\begin{array}{l}\frac{a't_1^3}{6}+\frac{b't_1^4}{8}+\frac{c't_1^5}{10}+\frac{d't_1^6}{12}+\frac{a'\alpha t_1^{\beta+3}}{(\beta+2)(\beta+3)}+\frac{b'\alpha t_1^{\beta+4}}{(\beta+2)(\beta+4)}\\ +\frac{c'\alpha t_1^{\beta+5}}{(\beta+2)(\beta+5)}+\frac{d'\alpha t_1^{\beta+6}}{(\beta+2)(\beta+6)}-\frac{a'\alpha^2 t_1^{2\beta+3}}{(\beta+2)(2\beta+3)}\\ -\frac{b'\alpha^2 t_1^{2\beta+4}}{(\beta+2)(2\beta+4)}-\frac{c'\alpha^2 t_1^{2\beta+5}}{(\beta+2)(2\beta+5)}-\frac{d'\alpha^2 t_1^{2\beta+6}}{(\beta+2)(2\beta+6)}\end{array}\right]\\ +C_{S'C'}\left[\begin{array}{l}a'\left(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2}\right)+b'\left(\frac{t_1^2T}{2}-\frac{T^3}{6}-\frac{t_1^3}{3}\right)\\ +c'\left(\frac{t_1^3T}{3}-\frac{T^4}{12}-\frac{t_1^4}{4}\right)+d'\left(\frac{t_1^4T}{4}-\frac{T^5}{20}-\frac{t_1^5}{5}\right)\end{array}\right]\\ +\frac{C_{O'C'}}{T}+C_{D'C'}\left(\left(\frac{a'\alpha t_1^{\beta+1}}{\beta+1}+\frac{b'\alpha t_1^{\beta+2}}{\beta+2}+\frac{c'\alpha t_1^{\beta+3}}{\beta+3}+\frac{d'\alpha t_1^{\beta+4}}{\beta+4}\right)\right)\end{array}\right\}$$

$$-\frac{1}{T}\left\{C_{S'C'}\left[a'(t_1-T)+b'\left(\frac{t_1^2}{2}-\frac{T^2}{2}\right)+c'\left(\frac{t_1^3}{3}-\frac{T^3}{3}\right)+d'\left(\frac{t_1^4}{4}-\frac{T^4}{4}\right)\right]-\frac{C_{O'C'}}{T^2}\right\} \tag{21}$$

$$\frac{d\left(T'I'C'\right)}{dt_1}=\frac{1}{T}\left\{\begin{array}{l} C_{H'C'}\left[\begin{array}{l}a't_1+b't_1^2+c't_1^3+d't_1^4+\frac{a'\alpha\beta t_1^{\beta+2}}{(\beta+1)t_1}+\frac{b'\alpha\beta t_1^{\beta+3}}{(\beta+1)t_1}+\frac{c'\alpha\beta t_1^{\beta+4}}{(\beta+1)t_1}\\ +\frac{d'\alpha\beta t_1^{\beta+5}}{(\beta+1)t_1}-\frac{a'\alpha^2 t_1^{2\beta+2}}{(\beta+1)t_1}-\frac{b'\alpha^2 t_1^{2\beta+3}}{(\beta+1)t_1}-\frac{c'\alpha^2 t_1^{2\beta+4}}{(\beta+1)t_1}-\frac{d'\alpha^2 t_1^{2\beta+5}}{(\beta+1)t_1}\end{array}\right]\\ -C_{S'C'}\left[\begin{array}{l}a'\left(T-t_1\right)+b'\left(t_1T-t_1^2\right)\\ +c'\left(t_1^2T-t_1^3\right)+d'\left(t_1^3T-t_1^4\right)\end{array}\right]+C_{D'C'}\left(\begin{array}{l}\frac{a'\alpha t_1^{\beta+2}}{t_1}+\frac{b'\alpha t_1^{\beta+3}}{t_1}\\ +\frac{c'\alpha t_1^{\beta+4}}{t_1}+\frac{d'\alpha t_1^{\beta+5}}{t_1}\end{array}\right)\end{array}\right\} \tag{22}$$

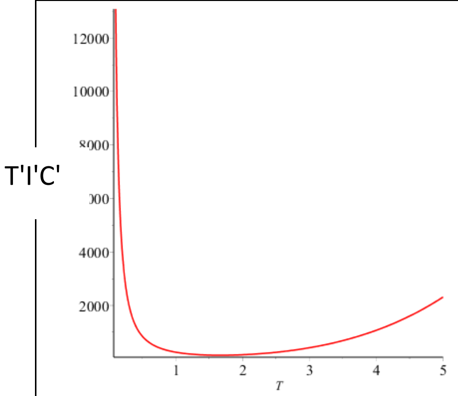
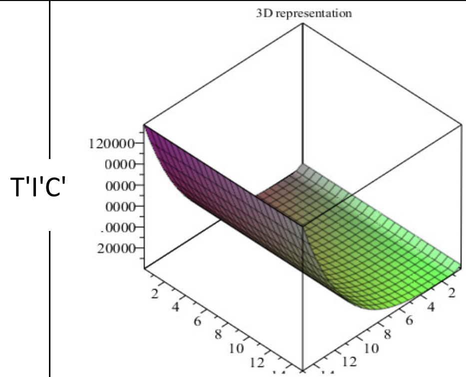
We get the optimal values of t_1 and T by solving equation (21) & (22) by using MAPLE 15.

- **Numerical Illustration**

To verify that the solution is the best one, we now use a numerical example. Using Maple 15, we will resolve the example. The inventory system's following parameters can be used to numerically illustrate the model:

$a'=10, b'=4, c'=4, d'=3, C_{H'C'}=4, C_{S'C'}=15, C_{O'C'}=100, C_{D'C'}=10, \alpha=0.1, \beta=0.5$

Under the above-given parameters, by using Maple 15, we get the optimal shortage value $t_1=1.292803768$ per unit time. After applying maple software for the above parameters, we get the optimal length of ordering cycle is $T=1.620124477$ unit timeand Total inventory cost is $T'I'C'=153.4621820$.

<table>
<tr><th>2D Representation</th><th>3D Representation</th></tr>
<tr><td>
Figure 6 (Total Inventory Cost vs Time)</td><td>
Figure 7 (3D Representation of Total Inventory Cost vs Time)</td></tr>
</table>

CONCLUSION:

This present chapter highlight the development of an inventory model having cubic polynomial demand with Weibull distribution as deterioration rate. The inventory model is verified with numerical illustration to study the effect of cubic demand and Weibull distribution as deterioration rate. Authors also show the convexity of model with help of graphical representation. Authors also analyzed that the cubic demand with Weibull distribution as deterioration rate find minimum Total Inventory Cost (T'I'C') in case of Variable Holding cost. Further this model can be extended for different forms of demand rate and deterioration rate.

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Aryabhatta Journal of Mathematics & Informatics Vol. 15, No. 1, Jan.-June, 2023
Double-Blind Peer Reviewed Refereed International Journal ©www.abjmi.com
DOI No. : 10.5958/2394-9309.2023.00020.3
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

PROPER COLOURINGS IN *r*-REGULAR ZAGREB INDEX GRAPH

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ABSTRACT

In this article, the new concept proper colourings in r - regular Zagreb index graph has been introduced. The first and second Zagreb indices are introduced. New inequalities on chromatic number related with first and second Zagreb indices are being established.

Keywords: Regular graph, Proper Colouring, Zagreb index, Chromatic number.

1. INTRODUCTION

In this article, we consider only finite, simple and undirected graphs. The symbols V(G) and E(G) denote the vertex set and edge set of a graph G. The cardinality of the vertex set is called the order of G denoted by p. The cardinality of edge set is called the size of G denoted by q edges is called a (p,q) graph. If G is a r-regular graph, then $M_1(G) = nr^2$ and

$M_2(G) = mr^2 = \frac{1}{2}nr^3$. Proper colourings in r-regular Zagreb index graph is extended by the result proper colourings in magic and anti-magic graphs[17]. Many results and theorems are proved under Zagreb index[1,8,9,10]. This work can be extended to domination which is related with domatic number and Zagreb index [4,5,6]. Further this work can be extended in the field of automata theory [11,12,13,14,15,16,] which has a wide range of application in automata theory. There are many applications in graph labeling under undirected [21,22,23,24,25,26] and directed graph [18,19,20]

2. MAIN RESULTS

Definition 2.1

For a graph, $G = (V(G), E(G))$, the first and the second Zagreb indices were defined as $M_1(G) = \sum_{v \in V(G)} (d(v))^2$ and $M_2(G) = \sum_{uv \in E(G)} (d(u)d(v))$ respectively, where $d(v)$ denote the degree of the vertex v in G.

Theorem 2.1:

If G is a r-regular Zagreb index graph then the chromatic number satisfies the inequality $\frac{K-1}{r(V+E)} \le \psi(G) \le \frac{1}{2}nr$, r ≥ 2.

PROOF

Case (i)

Let G be a graph of cycle C_n, $'n'$ be an odd integer.

Let C_n be r-regular with n vertices and m edges then $M_1(C_n) = nr^2$, $M_2(C_n) = mr^2$.

Let C_n be a cycle graph with Zagreb index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 3. Therefore, $\psi(G) = 3$.

Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G.

The following inequality is obtained.

$$\frac{K-1}{r(V+\in)} \leq \psi(G). \qquad \text{......... (1)}$$

The general condition of r-regular graph is denoted by as $\frac{1}{2}nr$.

$$\text{Therefore } \psi(G) \leq \frac{1}{2}nr. \qquad \text{......... (2)}$$

From the equations (1) and (2) it is easily verify that

$$\frac{K-1}{r(V+\in)} \leq \psi(G) \leq \frac{1}{2}nr.$$

Hence the odd cycle satisfies $\frac{K-1}{r(V+\in)} \leq \psi(G) \leq \frac{1}{2}nr$ for 2 - regular graph.

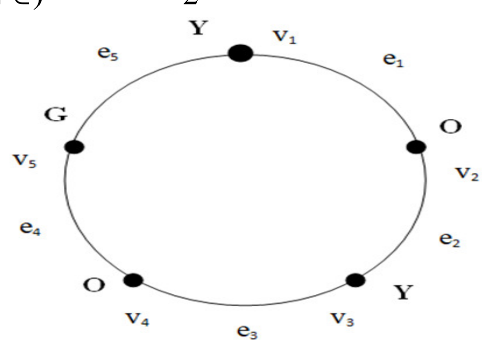

Fig 2.1

ZAGREB INDEX NUMBER FOR ODD CYCLE

$k = M_1(C_5) = 5(2^2) = 20.$

$k = M_2(C_5) = 5(2^2) = 20.$

$$\frac{K-1}{r(V+\in)} \leq \psi(G) \leq \frac{1}{2}nr.$$

$0.95 \leq 3 \leq 5.$

Case (ii)

Let G be a graph of cycle C_n, $'n'$ be an even integer.

Let C_n be r-regular with n vertices and m edges then $M_1(C_n) = nr^2$, $M_2(C_n) = mr^2$.

Let C_n be a cycle graph with Zagreb index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 2 .Therefore, $\psi(G) = 2$.

Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G .

The following inequality is obtained.

$$\frac{K-1}{r(V+\in)} \leq \psi(G). \qquad \text{........ (3)}$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr.$

Therefore $\psi(G) \leq \frac{1}{2}nr.$ .... (4)

From the equations (3) and (4) it is easily verify that

$$\frac{K-1}{r(V+\in)} \leq \psi(G) \leq \frac{1}{2}nr.$$

Hence the even cycle satisfies $\frac{K-1}{r(V+\in)} \leq \psi(G) \leq \frac{1}{2}nr$ for 2 - regular graph.

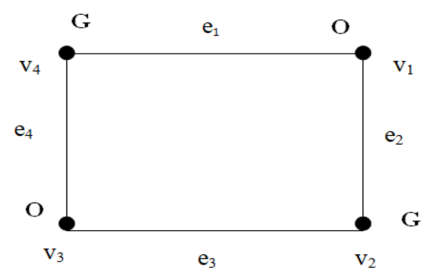

Fig 2.2

ZAGREB INDEX NUMBER FOR EVEN CYCLE

$K = M_1(C_4) = 4(2^2) = 16$.

$K = M_2(C_4) = 4(2^2) = 16$.

$$\frac{K-1}{r(V+\in)} \leq \psi(G) \leq \frac{1}{2}nr.$$

$0.94 \leq 2 \leq 4.$

Case (iii)

Let the graph G be Generalized Petersen Graph, here $'n'$ is an even integer.

Let $V(p) = \{v_1, v_2, ..., v_{10}\}$ be the vertices and $E(p) = \{e_1, e_2, ..., e_{15}\}$ be the edges of $P(n,m)$ then $M_1(P) = nr^2$ and $M_2(P) = mr^2$.

Let $P(n,m)$ be a Generalized Petersen Graph with Zagreb index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is 3 .Therefore, $\psi(P) = 3$.

Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G .

The following inequality is obtained.

$$\frac{K-1}{r(V+\in)} \le \psi(G). \qquad .... (5)$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr.$

Therefore $\psi(G) \le \frac{1}{2}nr.$ .... (6)

From the equations (5) and (6) it is easily verify that

$$\frac{K-1}{r(V+\in)} \le \psi(G) \le \frac{1}{2}nr.$$

Hence the Generalized Petersen Graph satisfies $\frac{K-1}{r(V+\in)} \le \psi(G) \le \frac{1}{2}nr$ for 3 - regular graphs.

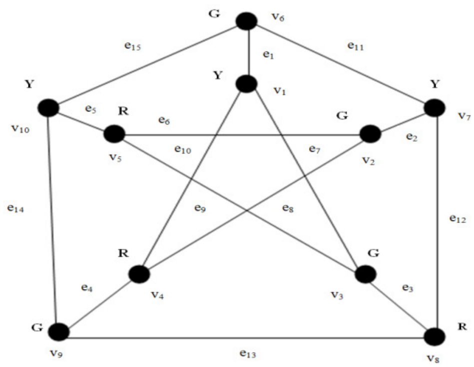

Fig 2.3

ZAGREB INDEX NUMBER FOR GENERALISED PETERSEN GRAPH

$k = M_1(P) = 10(3^2) = 90.$

$k = M_2(P) = 15(3^2) = 135.$

$$\frac{K-1}{r(V+\in)} \le \psi(G) \le \frac{1}{2}nr.$$

The inequality of first Zagreb index for generalized Petersen graph is $1.19 \le 3 \le 15$.

The inequality of second Zagreb index for generalized Petersen graph is $1.79 \le 3 \le 15$.

Case (iv)

Let G be a complete graph, $'n'$ be an any integer.

Let $V = \{v_1, v_2, ..., v_n\}$ be the vertices and $E = \{e_1, e_2, ..., e_n\}$ be the edges of k_n, then $M_1(K_n) = nr^2$ and $M_2(K_n) = mr^2$.

Let k_n be a complete Graph with Zagreb index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is n.

Therefore, $\psi(k_n) = n$.

Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G.

The following inequality is obtained.

$$\frac{K-1}{r(V+\in)} \le \psi(G). \qquad .... (7)$$

The general condition of r-regular graph is denoted by as $\frac{1}{2}nr$.

Therefore $\psi(G) \le \frac{1}{2}nr$. .... (8)

From the equations (7) and (8) it is easily verify that

$$\frac{K-1}{r(V+\in)} \le \psi(G) \le \frac{1}{2}nr.$$

Hence the complete Graph satisfies $\frac{K-1}{r(V+\in)} \le \psi(G) \le \frac{1}{2}nr$ for n - regular graphs.

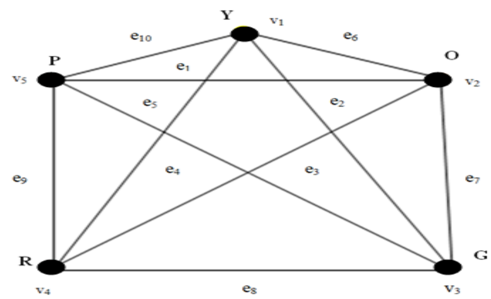

Fig 2.4

ZAGREB INDEX NUMBER FOR COMPLETE GRAPH

$k = M_1(k_5) = 5(4^2) = 80.$

$k = M_2(k_5) = 10(4^2) = 160.$

$$\frac{K-1}{r(V+\in)} \le \psi(G) \le \frac{1}{2}nr.$$

The inequality of first Zagreb index for complete graph is $1.32 \le 5 \le 10$.
The inequality of second Zagreb index for complete graph is $2.65 \le 5 \le 10$.

CONCLUSION

In this article, new inequality has been established. Further, it has been verified for first and second Zagreb indices. Finally, we conclude that new inequality on chromatic number related with Zagreb indices.

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Aryabhatta Journal of Mathematics & Informatics
Double-Blind Peer Reviewed Refereed International Journal © www.abjmi.com
ISSN (Print) : 0975-7139
ISSN (Online) : 2394-9309

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Published in accordance in Form -1, Rule-3 of the Press and Registration of News Paper/Journals/Books Act, 1867.

1.	Place of Publication	:	2255, Sector-17, Urban Estate, HUDA, JAGADHRI (Yamuna Nagar) - 135 003 Haryana - INDIA
2.	Periodicity of Publication	:	Twice a year
3.	Printer's Name	:	Dr. T.P. Singh
	Nationality	:	Indian
	Address	:	2255, Sector-17, HUDA, JAGADHRI (Yamuna Nagar) - 135 003 Haryana - INDIA
4.	Publisher's Name	:	Aryans Research & Educational Trust
	Nationality	:	Indian
	Address	:	2255, Sector-17, HUDA, JAGADHRI (Yamuna Nagar) - 135 003 Haryana - INDIA
5.	Editor's Name	:	Dr. T.P. Singh
	Nationality	:	Indian
	Address	:	2255, Sector-17, HUDA, JAGADHRI (Yamuna Nagar) - 135 003 Haryana - INDIA
6.	Name and Address of the owner	:	Aryans Research & Educational Trust (Regd. No. 2655)

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